Lesson: Solving Linear Equations, A Review of the Basics

To begin, let's review some of what we've learned about working with equations and using graphs to solve them.

Linear Equation in One Variable
An equation that can be written in the form ax+b=0 where a and b are real numbers

Solution
A value that when substituted into the variable of an equation makes the equation true

Example 1
Determine if $x = -1$ is a solution to the following equations.
a. $5 - 4x = 9$

b. $2x + 3 = 1 - 3x$

We will now work with some basic ideas of solving linear equations. There are two key concepts we must understand to do this.

The Addition Principle
Let a, b, and c be real numbers. If a=b then:

Example 1

Use the addition principle to solve the following:

Choose at least one from each category. Label the examples you choose as a, b, c, etc to keep the class organized.

a. $x - 4 = 2$

b. $x + 3 = 3$

c. $x - \frac{1}{3} = \frac{1}{2}$

The Multiplication Principle

Let a, b, c be real numbers. If a=b then:

Example 2

Use the multiplication principle to solve the following:

a. $3x = 15$

b. $5x = 0$

c. $-\frac{2}{5}\mathrm{x} = 10$

Example 3

Solve the following examples:

a. $5 - 4x = 33$

b. $3x + 2 = 2 + 4x - 8$

c. $3(4 - 2x) = 3x + 6$

Lesson: Solving Linear Equations, No Solution and Infinite Solutions

There are 2 special outcomes we have to watch out for:

An Example Where There Is No Solution

$3(2x - 1) = 2(2x + 1) - 2(3 - x)$

An Example Where There Are Infinite Solutions

$4x + 5 = 3(2x + 1) - 2(x - 1)$

A Summary of the 3 Possible Solution Types for Linear Equations

Examples: Solving Linear Equations, Basic Examples with No Fractions or Decimals

a. $\mathbf{3x - 5 = 7}$

b. $\mathbf{6 - 2x = 6}$

c. $\mathbf{4x + 7 = 2x - 9}$

d. $\mathbf{3 - 4x - 7 + 2x = 4 - 6x - 8 + 4x}$

e. $\mathbf{4x + 7 + 6x = 7(1 - 2x)}$

f. $\mathbf{3(5 - 2x) - 6(3 - 2x) = 5(1 + 2x)}$

g. $\mathbf{4(2x - 1) - 2(6 - 3x) = 2(3x - 4)}$

h. $\mathbf{3(2x - 5) - 4x + 1 = 4(x - 3) - 2(1 - 3x)}$

Lesson: Solving Linear Equations with Fractions

To begin, we will look at 3 examples to demonstrate how to clear fractions without parentheses. Take notes as shown in the video.

a. $\frac{1}{2}x + \frac{1}{5} = \frac{6}{5}$

b. $\frac{1}{2}x + \frac{1}{5} - \frac{1}{10}x = 1 + \frac{3}{10}x - \frac{2}{5}$

c. $\frac{1}{2}x + 1 + \frac{1}{4}x = \frac{3}{4} + \frac{2}{3} + \frac{1}{3}x$

Now we will look at how to clear fractions in equations with parentheses.

a. $\frac{3}{4}(3+2x)-\frac{1}{2}(5-3x)=\frac{5}{4}$

b. $\frac{1}{2}(x+2)+\frac{1}{3}(2x-5)=\frac{1}{3}(4+2x)$

Examples: Solving Linear Equations with Fractions (No Parentheses)

a. $\frac{1}{5}x - \frac{1}{3} = \frac{2}{3}$

b. $\frac{1}{3}x - 1 + \frac{1}{5}x + \frac{2}{5} = \frac{2}{5}x + \frac{1}{5}$

c. $\frac{1}{4}x - \frac{1}{10} + \frac{1}{5}x - \frac{4}{5} = \frac{3}{20}x + \frac{1}{10} + \frac{3}{10}$

d. $\frac{2}{3}x + \frac{1}{3} - \frac{1}{2}x = \frac{5}{6}x - \frac{1}{6} - \frac{2}{3}x + \frac{1}{2}$

Examples: Solving Linear Equations with Fractions (With Parentheses)

a. $\frac{1}{2}(3x-2)-\frac{2}{5}(2x-3)=\frac{1}{5}(x+1)$

b. $\frac{3}{4}(4x+5)+\frac{1}{2}(2-3x)=\frac{3}{4}(2x+1)$

c. $\frac{1}{2}(2x+3)-\frac{1}{3}(2x-1)=\frac{1}{4}(x+7)$

d. $\frac{3}{4}(2x-1)+\frac{5}{8}(2-4x)=\frac{1}{4}(2-4x)$

Absolute Value Equations
Video 1: The Basics

Absolute value equations look like this:

$$|x+2| = 7 \quad \text{or} \quad |3x-5|+4 = 2$$

To understand how to solve these types of equations, first think about: what are the solutions to $|x| = 5$? Explain why.

From here, we can see that we must consider 2 possible solution sets to find all solutions to absolute value equations. Write down the general strategy for solving absolute value equations:

Solve: $|2x+3| = 9$

Examples Solve the following:

a. $|3x - 9| = 6$

b. $|7 - 4x| = 3$

To solve $|5x - 4| + 2 = 13$, what must we do first?

Solve:

Examples Solve:

a. $|3x - 6| + 4 = 10$

b. $\left|\frac{1}{2}x + \frac{1}{3}\right| + 5 = 10$

The next video will discuss a special situation and a more complicated equation type.

Absolute Value Equations
Video 2: More Complicated Situations

In this video, we will discuss how to solve equations like:

$$|x+2|=-7 \quad \text{or} \quad |3x-5|=|2x+7|$$

What can we say about the solutions to $|x|=-5$?

Examples Solve the following:

a. $|2x-7|=-8$

b. $|3x+5|+7=2$

Absolute value equations can also look like this: $|3x+1|=|2x-5|$. Explain how to solve this.

Examples Solve the following:

a. $|5x - 10| = |10x + 15|$

b. $|3x - 5| = |5x + 7|$

c. $|4x - 6| = |3x - 8|$

Example/Question: Why Do You Only Make One Side Negative with Solving $|\boldsymbol{ax+c}| = |\boldsymbol{bx+d}|$ type problems?

What is the approach for solving $|x+1| = 5$?

What is key to notice here?

How would you solve $|x+1| = |5x|$?

How would you solve $|x+1| = |5x-3|$?

Lesson: Solving a Linear Equation with Multiple Variables

<u>Examples</u>

a. Solve for x: $x + y = 7$

b. Solve for y: $4y = z$

c. Solve for a: $3a - 6b = 2c$

d. Solve for b: $4a + 8b = 16c$

e. Solve for k: $\frac{1}{3}k = m$

f. Solve for a: $3(a - 2b) = 6c$

g. Solve for b: $\frac{2}{3}(3a + 2b) = c$

h. Solve for a: $-\frac{2}{5}(2a - 4b) = 4$

Lesson: Number Lines, Interval Notation, and Set Notation

Show how to set up the number line, interval notation, and set notation for the following:

a. $x > 5$

b. $x \leq 7$

c. $x \geq 0$

d. $-3 < x < 5$

e. $-7 \leq x < -1$

f. $3 < x \leq 8$

Lesson: How to Solve Linear Inequalities in One Variable

Linear inequalities in one variable have the form $ax + b < 0$. Alternatively, they could use the symbol $\leq, >, \geq$.

Solving linear inequalities has some similarities and some differences when compared to solving for linear equations.

The Addition and Subtraction Properties

Let a, b, c be real numbers. If $a < b$, then:

This also holds for $\leq . >, \geq$

<u>Examples</u> Solve the following. Put your final answer both on a number line and in interval notation.

a. $x + 4 > 9$

b. $x - 8 \leq 12$

The Multiplication and Division Properties

Let a, b, c be real numbers. If $a < b$, then:

- if c is positive ($c > 0$):

- if c is negative $(c < 0)$

This also holds for $\leq . >, \geq$

Why is this rule different? Use the following example.

To begin, is $-2 < 4$ a true statement? True False

Now multiply both sides of the inequality by 3 and write down the new result below:

This new statement is True False

Now multiply both sides of the inequality by -3 and write down the new result below:

This new statement is True False

Now divide both sides of the inequality by -2 and write down the new result below:

This new statement is True False

Examples Solve the following. Put your final answer both on a number line and in interval notation.

a. $3x \leq 15$

b. $-2x > 8$

c. $5x < -10$

d. $-4x \geq -12$

e. $-9x \leq 27$

From here, you can start to combine the rules to solve more types of inequalities.

Examples Solve the following. Put your final answer both on a number line and in interval notation.

a. $2x - 4 < 6$

b. $3x + 7 > 5x - 9$

c. $\frac{1}{2}x + \frac{1}{5} > \frac{3}{4}x + \frac{7}{10}$

Examples: Solving Linear Inequalities – Basic Examples

Solve the following. Put your final answer both on a number line and in interval notation.

a. $2x + 7 \geq 13$

b. $4x - 6 < 6x - 10$

c. $3(5 - 2x) > 5(3 + 2x)$

d. $2(4 - 2x) - 3(4 - 4x) \leq 4(3x + 3)$

Examples: Solving Linear Inequalities – Examples with Fractions Decimals

Solve the following. Put your final answer both on a number line and in interval notation.

a. $\frac{1}{3}x - \frac{1}{5} \leq \frac{1}{3} + \frac{2}{5}x$

b. $0.03x + 0.2 > 0.02$

c. $\frac{3}{5}(2x - 1) - \frac{3}{4}(2 - 3x) < \frac{1}{10}(4 - 3x)$

d. $0.2(2x - 4) + 0.03(6 - 10x) \geq 0.06(x + 9)$

Lesson: Three-Part Inequalities

Show how to solve the following three-part inequalities. Put your final answer both on a number line and in interval notation.

a. $4 < x + 2 \leq 10$

b. $-3 \leq 2x - 1 < 11$

c. $-5 < 3x + 4 < 19$

d. $4 \leq -1 - 5x < 14$

Lesson: The Basics of Compound Inequalities

The word “and” has special implications in mathematic. One place it is often used is with sets, and we represent “and” with the intersection symbol ∩.

The word “or” also has special implications in mathematic. It is also used is with sets, and we represent “or” with the intersection symbol ∪.

<u>Examples</u> Given $A = \{1, 2, 3, 4, 5\}, B = \{2, 4, 6, 8, 10\}, C = \{1, 3, 5, 7\}$, find the following:

a. $A \cap B$

b. $A \cap C$

c. $A \cup C$

d. $B \cup C$

We use similar ideas with compound inequalities.

Show how to represent a solution to $x \geq -2$ and $x < 5$

<u>Example</u> Represent the following with a number line and interval notation.

a. $x > 0$ and $x \leq 7$

b. $x > 8$ and $x \leq -2$

Show how to represent a solution to $x \geq 4$ or $x < -3$

Example Represent the following with a number line and interval notation.

a. $x \leq 0$ or $x > 5$

b. $x > 0$ or $x \leq 5$

We can create more complicated problems where we first have to isolate x, but then we can represent in the same manner as shown above.

Example Represent the following with a number line and interval notation.

$3x - 1 > 5$ and $5x + 4 \leq 24$

Examples: Compound Inequalities

Put your final answer both on a number line and in interval notation.

a. $x - 4 > 8$ or $x + 2 < -3$

b. $2x + 1 \geq 7$ and $5x - 10 < 20$

c. $4x + 2 > 18$ or $3x - 1 < 17$

d. $2 - 3x > 8$ and $5x - 2 > 12$

e. $5x + 9 < 16$ or $4 - 3x > 14$

f. $3 - 2x > 9$ and $5 - x < -12$

A review of number lines, interval notation, and set notation

This is a general review of these concepts. We will look at how to represent solutions for problems like:

$$x > 5 \quad \text{or} \quad -3 \le x < 4 \quad \text{or} \quad x > 2 \text{ or } x < -3$$

Use the example $x < 4$ to describe the 3 types of inequality notation. Write down any special symbols associated with it.

Number line:

The special signs/symbols are:

Graph $x < 4$ on a number line:

Interval Notation:

The special signs/symbols are:

Write $x < 4$ in interval notation:

Set Notation

The format of this is always:

Write $x < 4$ in set notation:

<u>Examples</u> Write the following inequalities on a number line, in interval notation, and in set notation.

a. $x \geq 5$

b. $x < -2.5$

c. $-4 \leq x < 7$

d. $x > 4$ or $x < -5$

<u>Examples</u> Write the following inequalities on a number line, in interval notation, and in set notation.

a. $x \leq -3$

b. $x > 12$

c. $0 < x \leq 8$

d. $x > 3$ or $x < 1$

Lesson: Absolute Value Inequalities

Absolute value inequalities have 2 key aspects to solving them. What are those 2 key aspects?

Explain the setup to solve an absolute value inequality of the form $|P| \leq k$ (or $< k$).

Explain the setup to solve an absolute value inequality of the form $|P| \geq k$ (or $> k$).

Let’s try to understand why these are setup this way.

Explain the solutions to $|x| < 3$. Summarize what was explained in the video.

Explain the solutions to $|x| \geq 3$. Summarize what was explained in the video.

Examples Solve the following inequalities. Graph your solution set on a number line and express your solution in interval notation.

a. $|2x - 4| < 6$

b. $|3x + 6| \geq 9$

c. $|x - 5| - 4 \leq 6$

d. $5 \geq |3x - 4|$

e. $7 < 3 + |x - 4|$

f. $6 \leq 4 + |8 - 2x|$

Lesson: Special Absolute Value Inequalities

We will discuss some special situations. Explain the reasoning for each of these situations:

a. $|2x+3|+7<4$

b. $|3x-2|>-3$

c. $|5x+1|\geq 0$

d. $|2x-4|<0$

e. $|6x+2|\leq 0$

Lesson: An Intro to Functions

Define a relation:

Define a domain:

Define a range:

<u>Example</u> In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, define the domain and range of the relation.

Define a function:

What is the visual explanation that explains what a function is or is not?

Example In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, is this a function?

Example Are the following functions?

a. $\{(4,-1),(3,2),(7,-1)\}$

b. $\{(2,4),(3,5),(2,-1)\}$

What is the notation for a function? How do we describe/use this notation?

Example In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, what is $f(4)$?

Example Suppose $f(x) = 2x + 1$ and $g(x) = x^2 - 3$. Find the value of the following:

a. $f(2)$

b. $g(1)$

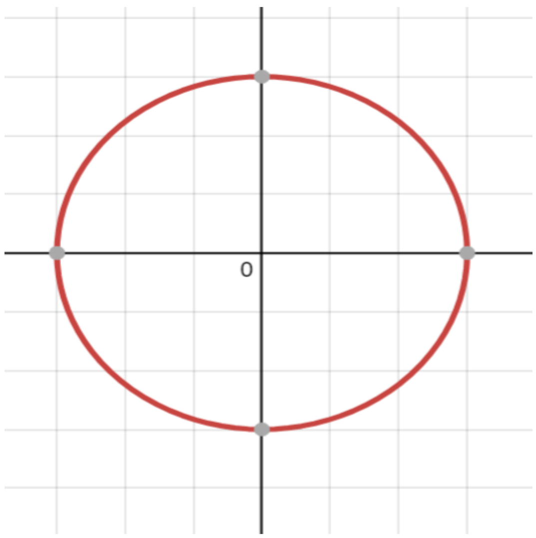
c. $f(a)$

d. $g(h + k)$

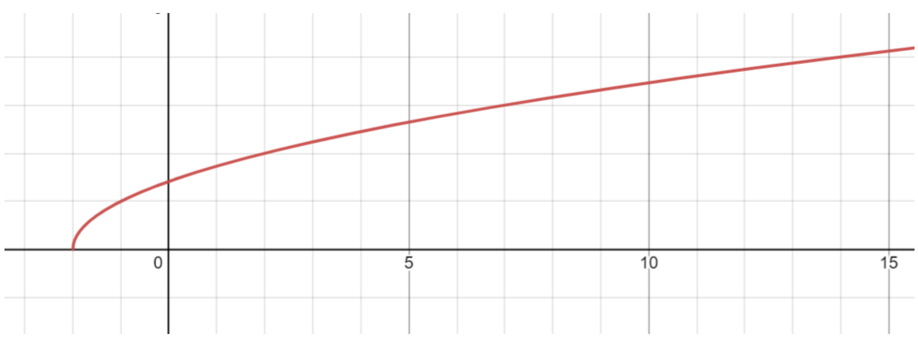
What is the vertical line test?

Example Do the following graphs represent a function?

a.



b.



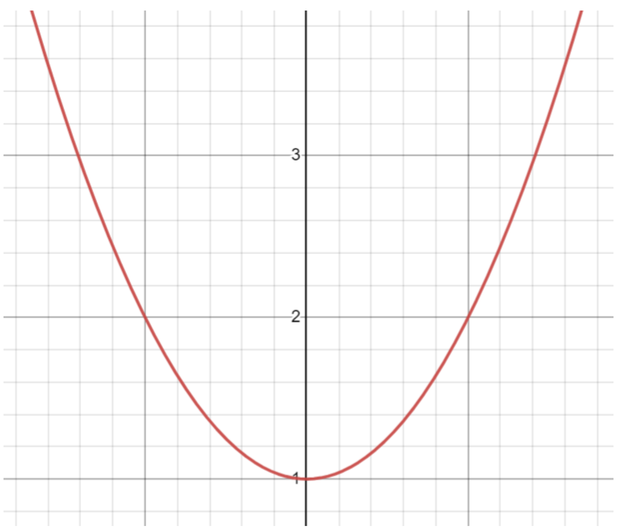
Lesson: Determining the Domain and Range of Functions In General

How do you determine the domain and range of a set of points like $\{(2,4),(5,8),(4,6)\}$?

What is the domain and range of that set of points?

In general, it is easiest to determine the domain and range when we have the graph in front of us.

For the following graph, write out how you should determine the domain and range.



Write a sentence on how you should find the domain:

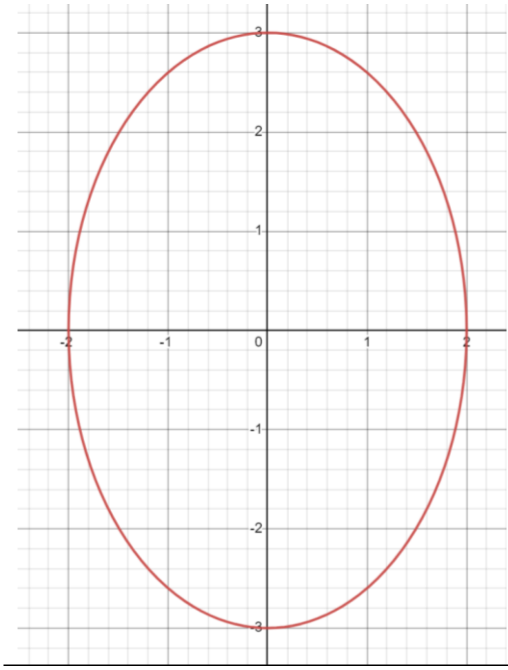
State the domain:

Write a sentence on how you should find the range:

State the range:

Examples Determine if the following graphs represent functions, and then state the domain and range.

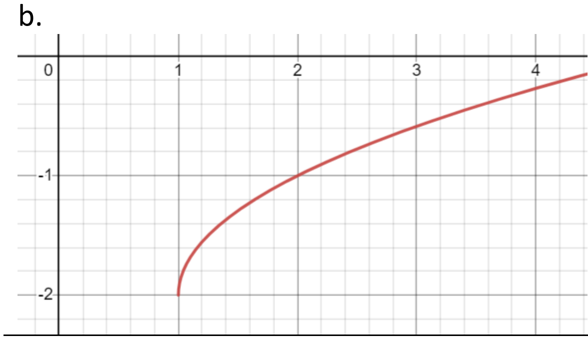
a.



Domain:

Range:

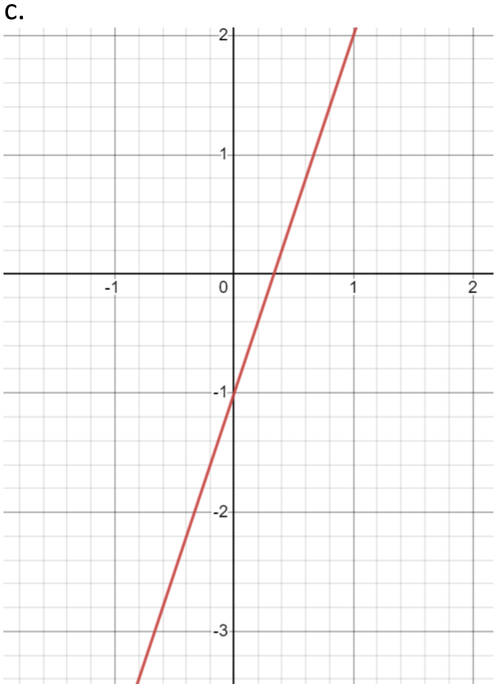
b.



Domain:

Range:

c.



Domain:

Range

Lesson: The Domain of Some Specific Types of Functions

What is a polynomial? Write some examples.

What is the domain of any polynomial function?

What is a rational function?

What is the domain of any rational function?

Examples Determine the domain of the following.

a. $f(x) = 3x - 1$

b. $g(x) = \frac{4}{x+2}$

c. $h(x) = 3x^3 - 2x^2 + 4x - 1$

d. $f(x) = \frac{2x+1}{x^2-9}$

What is a square root function?

What is the domain of a square root function?

<u>Examples</u> Find the domain of the following:

a. $f(x) = \sqrt{x - 2}$

b. $g(x) = \sqrt{4x - 1}$

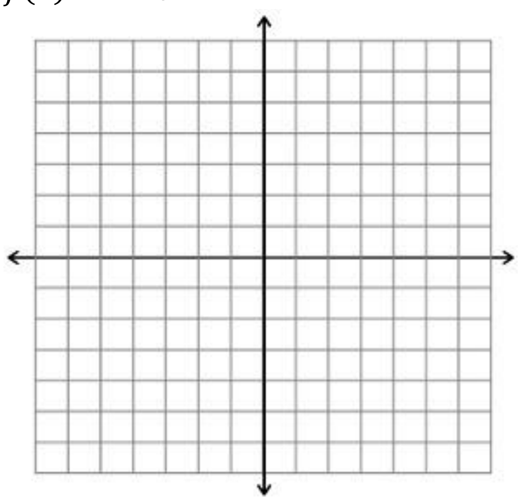
c. $h(x) = \sqrt{4 - x}$

Lesson: How to Graph a Function

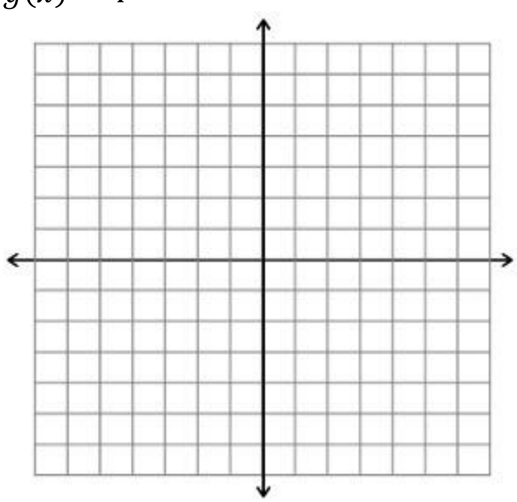
How do you graph a function?

<u>Examples</u> Graph the following functions.

a. $f(x) = 3x + 1$



b. $g(x) = 4$



Lesson: Intro to Linear Equations in Two Variables, Graphs, and Plotting Points

A **linear equation in two variables** is an equation of the form ax+by=c, where a, b, and c are real numbers, and both a and b are not zero at the same time.

Solutions for equations with two variables come as an ordered pair. Unless otherwise stated, you generally view the ordered pair in the alphabetical order of the equation.

<u>Examples</u> Decide if the following ordered pairs are solutions to the equations.

a. $\left(2, -\frac{1}{2}\right)$ for $3x - 2y = 4$

b. $(-5, -1)$ for $3a + 2b = 10$

c. $(3, 2)$ for $y = 5 - x$

<u>Examples</u> Complete the ordered pairs for the following equations

a. $(2, b)$ for $4a - 6b = 2$

b. $(x, -3)$ for $y = 2x + 1$

Another way to represent ordered pairs is with a table.

Examples Complete the following tables for the linear equations:

a. $y = 2x + 4$

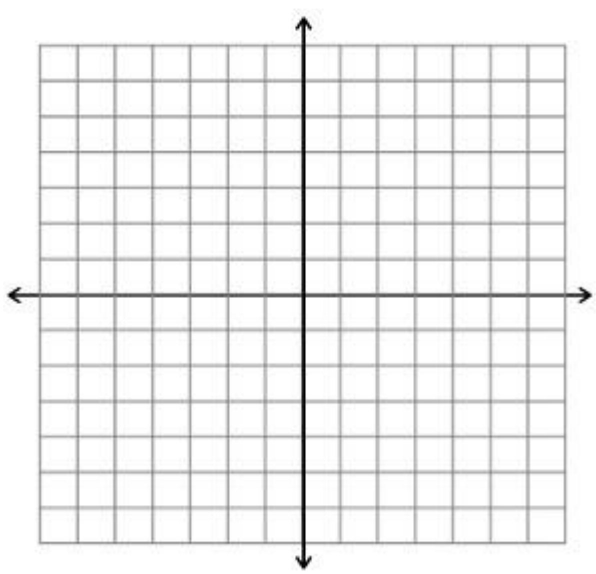
x	y
-3	
	10
0	

b. $3a - 2b = 12$

x	y
	0
0	
3	

Order pairs are often represented in coordinate systems. Below, draw the Cartesian (aka rectangular) coordinate system and label its parts.

Examples Plot the following points on the graph:



a. $(-2, 3)$

b. $(4, -1)$

c. $(1, 4)$

d. $(0, -4)$

Lesson: Graphing Equations by Using Intercepts

What are x-intercepts? What are y-intercepts? Draw an illustration.

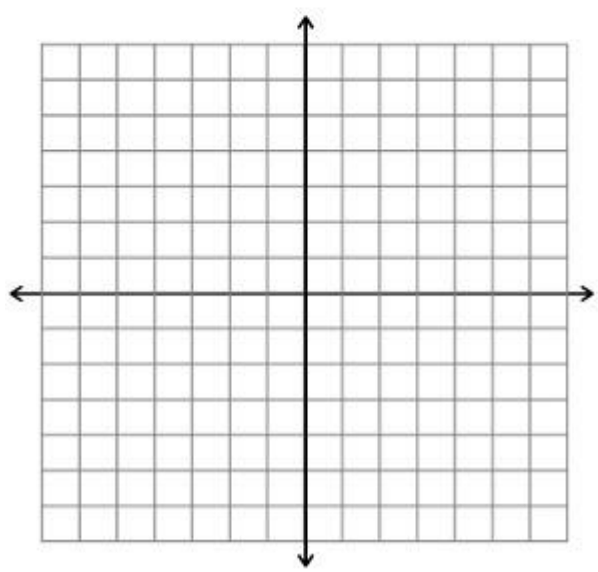
How do you find x-intercepts?

How do you find y-intercepts?

<u>Examples</u> Find the x- and y-intercepts of the following equations. Then find a third point point. Plot these three points to make a line.

$3x - 2y = 6$

x-intercept



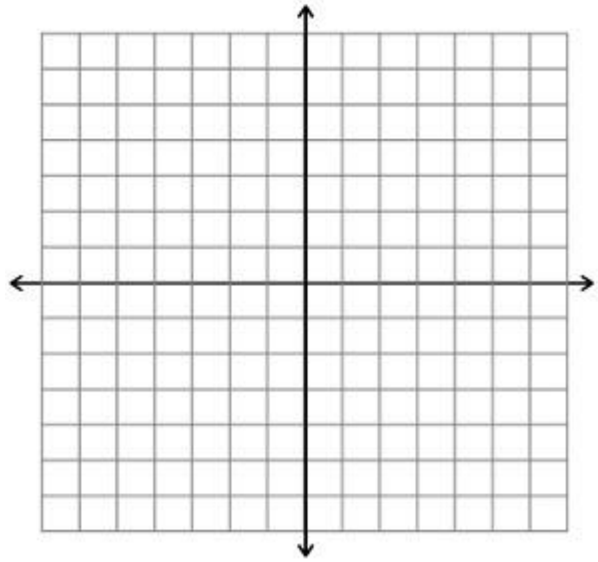
y-intercept

Third point:

Examples Find the x- and y-intercepts of the following equations. Then find a third point point. Plot these three points to make a line.

$x - 4y = 4$

x-intercept



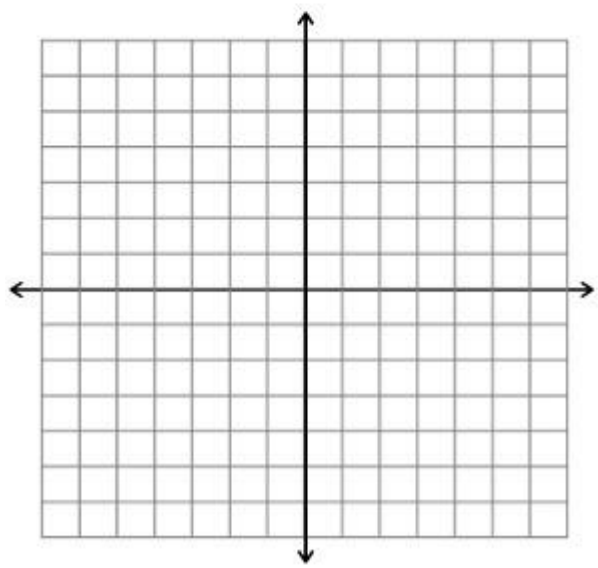
y-intercept

Third point:

Examples Find the x- and y-intercepts of the following equations. Then find a third point point. Plot these three points to make a line.

$y = \frac{1}{2}x - 2$

x-intercept



y-intercept

Third point:

Lesson: Review of Functions

How do you represent a function?

What is a function often interchangeable with? How can you rewrite $f(x) = 3x + 8$?

So what IS a function? What does it actually represent?

What's another handy feature of functions?

What are the domain and range?

Does domain and range only work for functions?

Examples Are the following functions, and what is the domain and range?

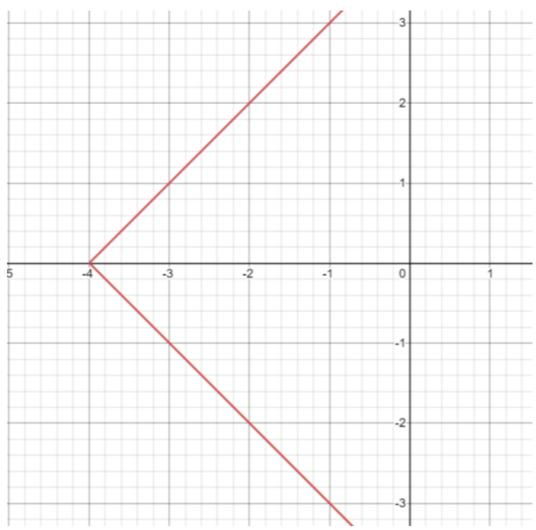
a. $\{(2,3),(5,-1),(-3,7)\}$

b. $\{(-1,5),(-8,3),(-1,6)\}$

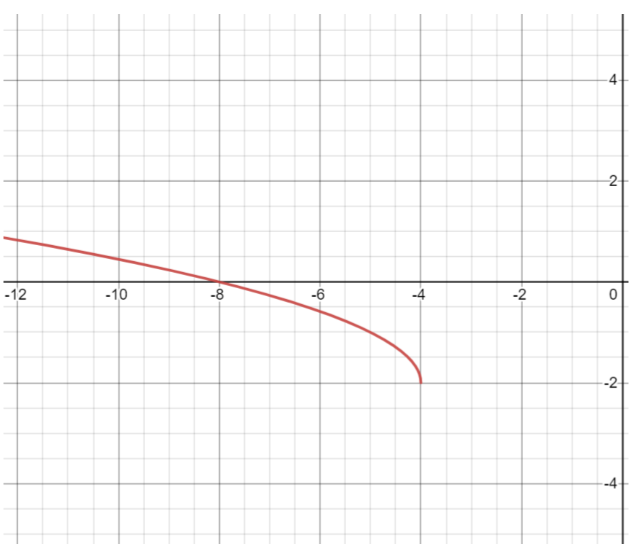
What is the Vertical Line Test, and how does it help you determine if you have a function?

Example State the domain and range of the following, and use the vertical line test to determine if the graph represents a function.

a.



b.



Examples Determine if the following represent functions.

a. $y = 3x^2$

b. $x = y^2$

c. $y = \frac{5}{x^2}$

Example Write the following in function notation: $3x + 2y = 6$

Example Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x - 1$ and $h(x) = \{(2,5), (6,-3), (-1,-4)\}$. Find the following:

a. $f(7) =$

b. $g(3) =$

c. $f(k) =$

d. $g(x + h) =$

e. $h(6) =$

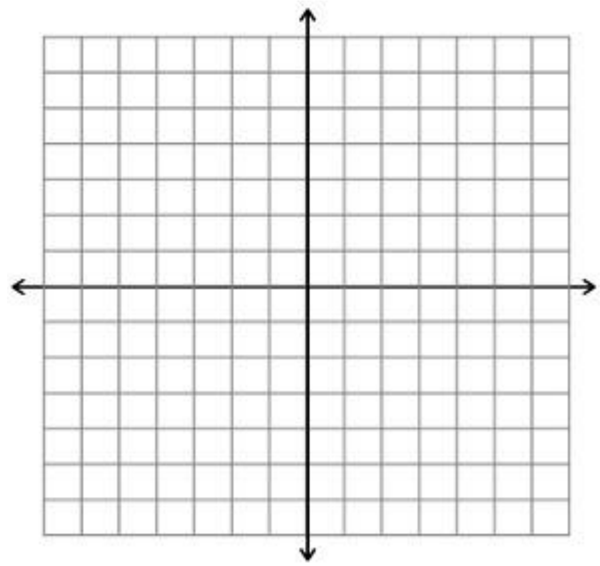
Lesson: Graphing Equations by Using a Table

One way to graph lines is by finding three points, and the plotting those points.

Examples Complete the following tables of ordered pairs, and the plot the points to form a line.

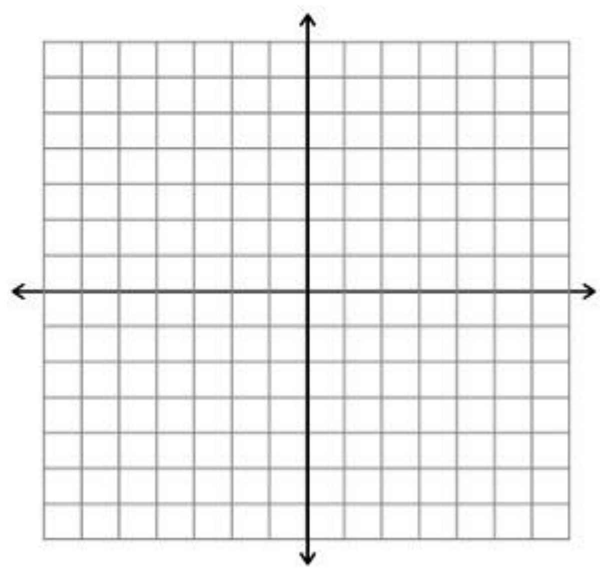
a. $y = 2x - 1$

x	y
2	
-2	
	0



b. $-2x + 6y = 12$

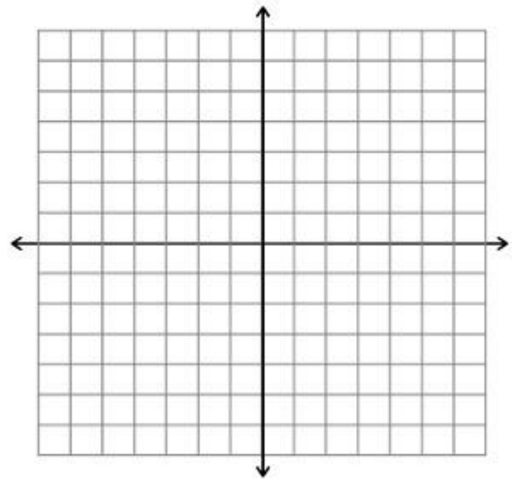
x	y
-3	
	0
3	



Examples Find three points for the following equations, then plot those points to make a line.

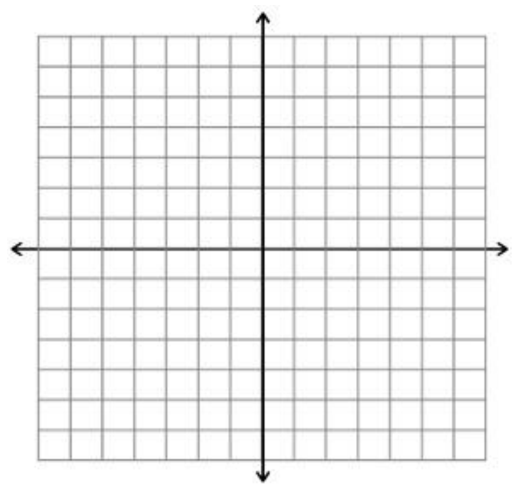
a. $y = -3x + 2$

x	y



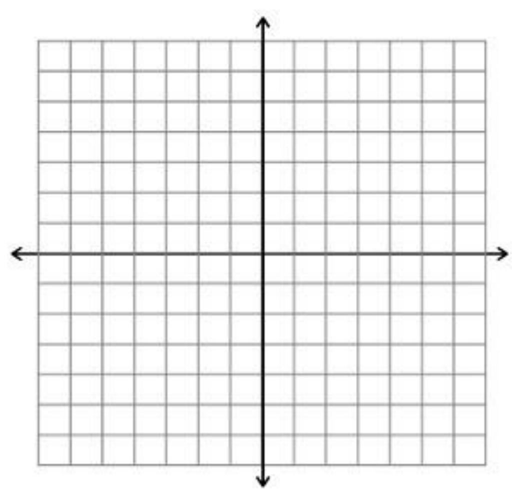
b. $y = -\frac{1}{2}x + 3$

x	y



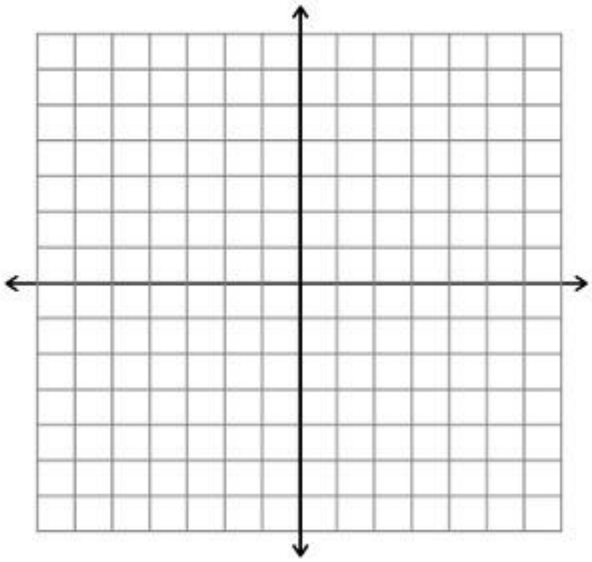
c. $3x - 5y = 15$

x	y

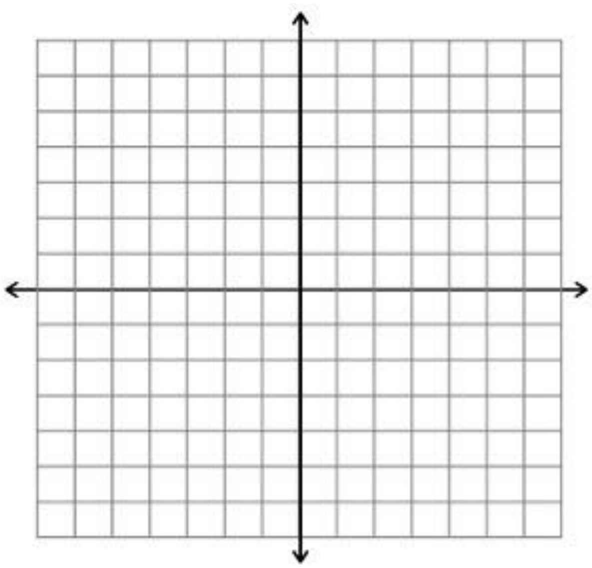


Lesson: How to Graph Vertical and Horizontal Lines

Show how to graph $y = 3$



Show how to graph $x = -2$

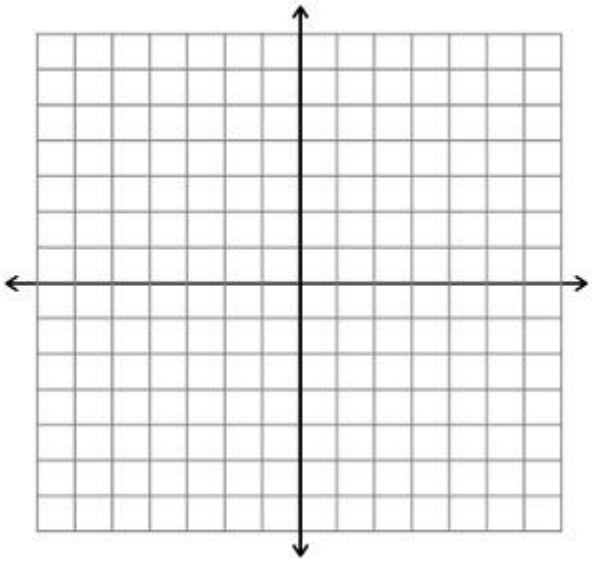


How do you know if you are working with a vertical line? How do you graph it?

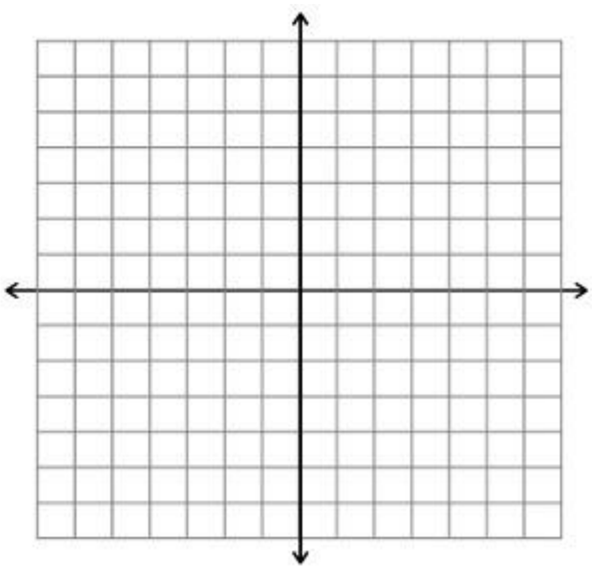
How do you know if you are working with a horizontal line? How do you graph it?

Examples Graph the following:

a. $x = 4$



b. $y = -1$



Lesson: Putting Equations in Standard Form

What is the standard form of a line?

<u>Examples</u> Write the following equations in standard form:

a. $y = 2x - 4$

b. $y = -5x + 3$

c. $y = -\frac{1}{2}x + 1$

d. $y = \frac{2}{3}x - 2$

Lesson: The Slope of a Line

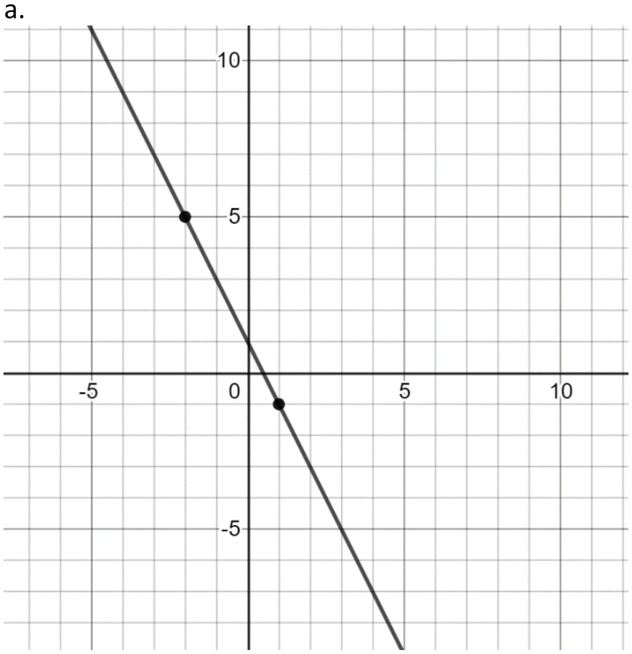
What does slope represent?

Draw the various possible slopes of lines:

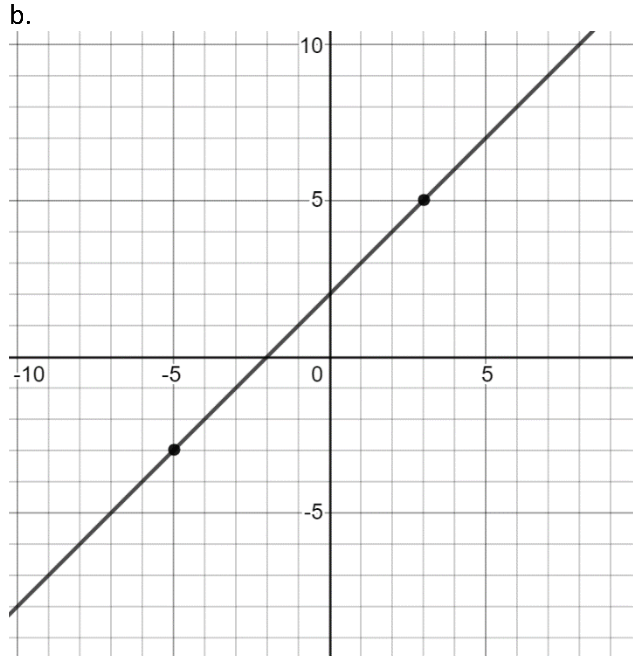
How do find the slope of a line?

<u>Examples</u> Find the slope of the following lines:

a.



b.



What is the slope formula?

Examples Use the slope formula to find the slope between the pairs of points:

a. $(3, 5)$ and $(4, 8)$

b. $(6, -2)$ and $(-2, 4)$

c. $(7, 2)$ and $(7, 4)$

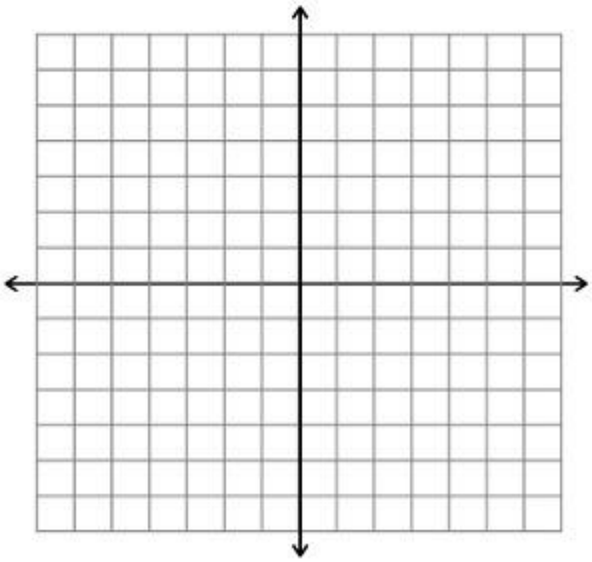
d. $(2, 4)$ and $(-1, 4)$

Lesson: Using Slope to Graph Lines; Intro to Slope-Intercept Form

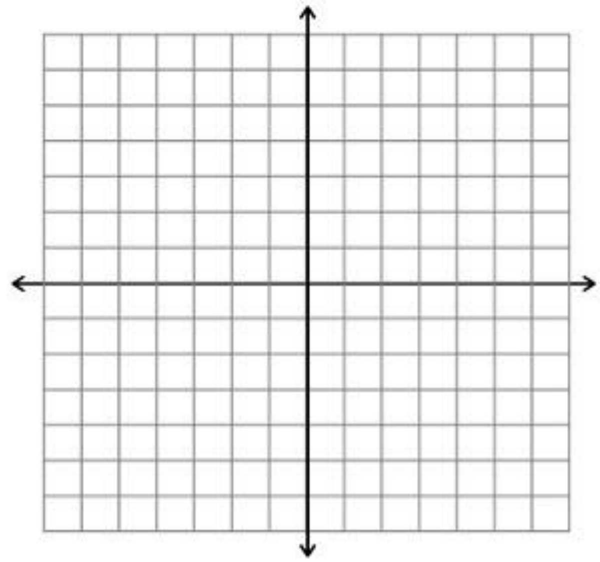
When the slope of a line and a point are known, a line can be easily graphed.

<u>Examples</u> Given the slope and the point, graph the following lines:

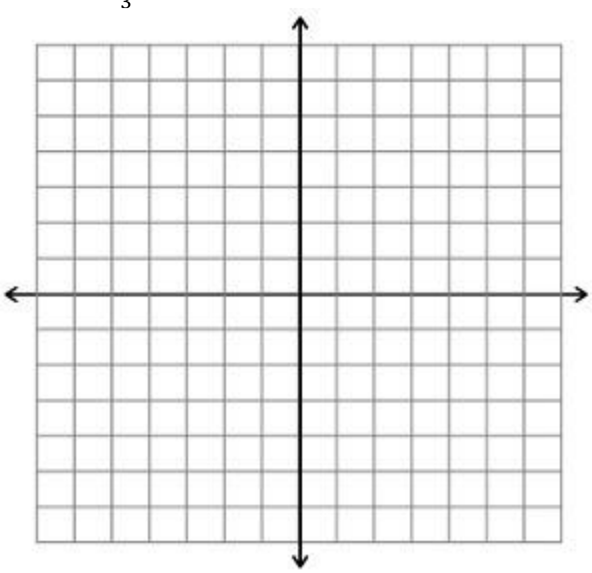
a. $m = 2$ and $(-1, 2)$



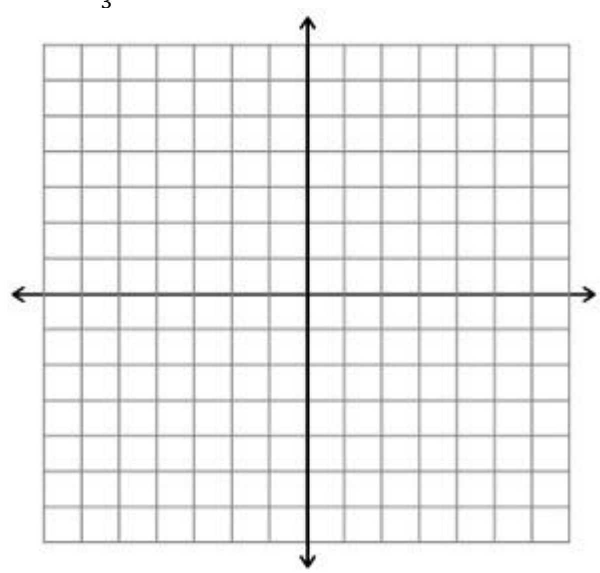
c. $m = 1$ and $(2, -1)$



b. $m = -\frac{1}{3}$ and $(-3, 4)$



d. $m = \frac{2}{3}$ and $(0, -3)$



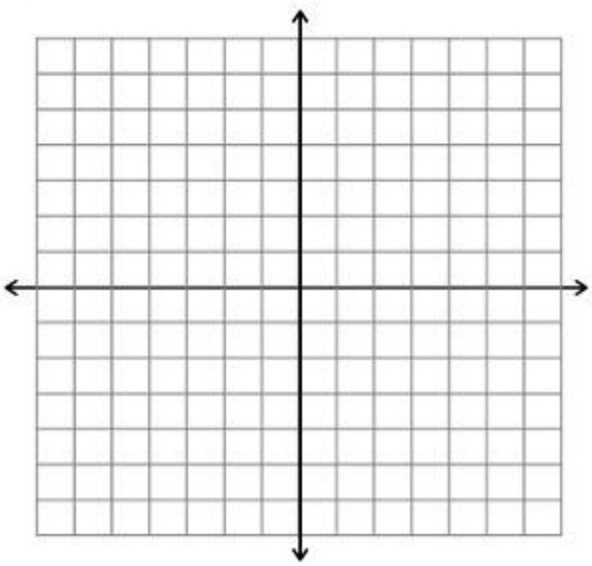
What is the slope-intercept form of a line?

Examples Identify the slope m and the y-intercept $(0, b)$ in each of the following equations:

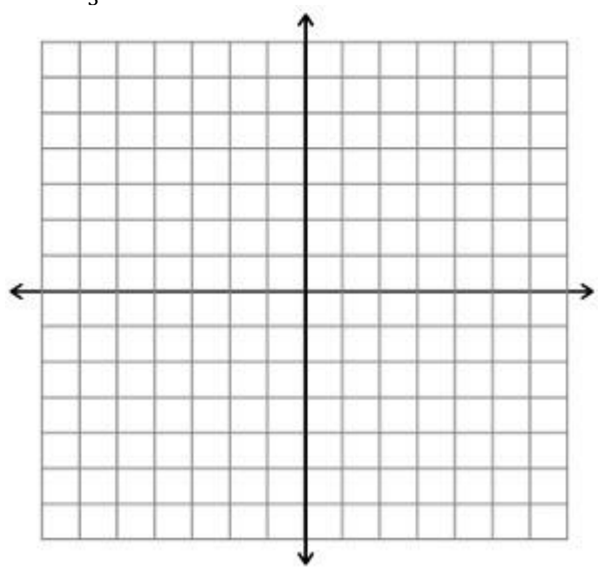
Equation	Slope, m	What stands for b	State the y-intercept (as a point)
a. $y = -2x + 3$			
b. $y = \frac{1}{4}x - 7$			
c. $y = 3x + 9$			
d. $y = x$			

Examples Graph the following lines:

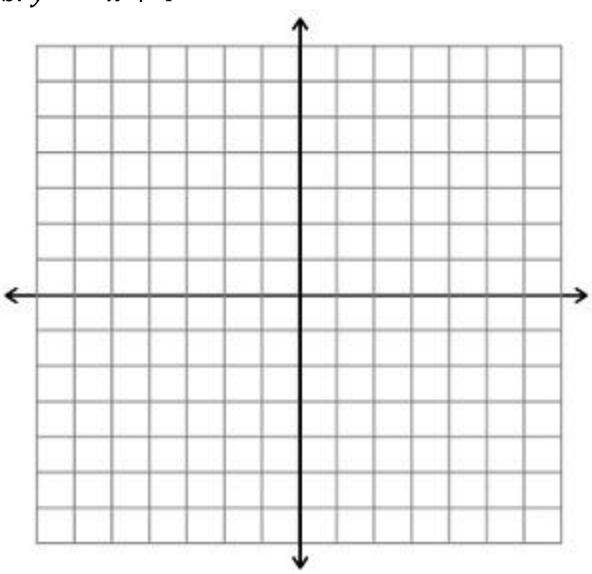
a. $y = 2x + 3$



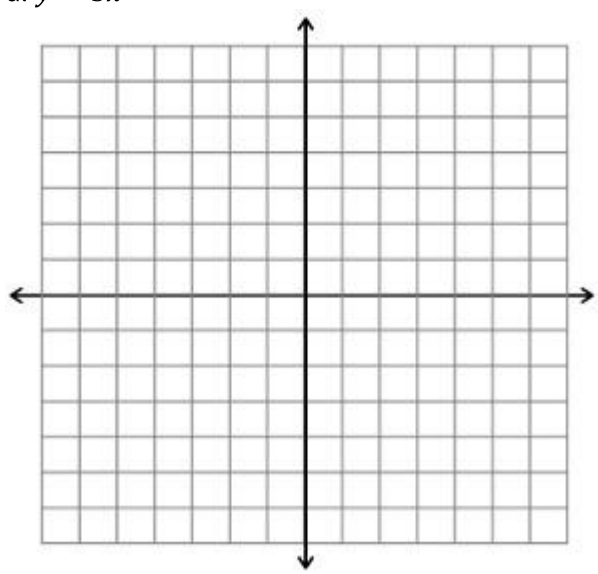
c. $y = \frac{1}{3}x - 4$



b. $y = -x + 4$



d. $y = 3x$



Lesson: Rewriting Linear Equations into Slope-Intercept Form of a Line

What is the slope-intercept form of a line?

Explain how to put the equation $2x - 4y = 8$ into slope-intercept form.

<u>Examples</u> Put the following equations in slope-intercept form:

a. $6y - 3x = 12$

b. $-5x + 2y = -4$

Lesson: A Few Basic Graphs

There are a few graphs that are helpful to immediately know as we move forward. Let's break these down.

The Parabola: $f(x) = x^2$

Create table of relevant points and then sketch your own graph.

Absolute Value: $f(x) = |x|$

Create table of relevant points and then sketch your own graph.

X cubed: $f(x) = x^3$

Create table of relevant points and then sketch your own graph.

Square Root: $f(x) = \sqrt{x}$

Create table of relevant points and then sketch your own graph.

Cube Root: $f(x) = \sqrt[3]{x}$

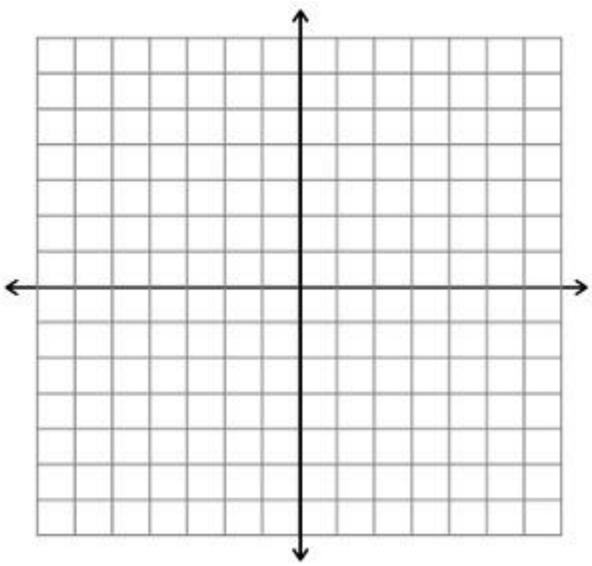
Create table of relevant points and then sketch your own graph.

Lesson: An Intro to Translations and Reflections of Graphs

We are going to develop the intuition for this concept before we actually graph anything.

Vertical Translations

Draw the base graph of $f(x) = x^2$ in one color and then use some other colors to show how the translations work. Label the graphs.

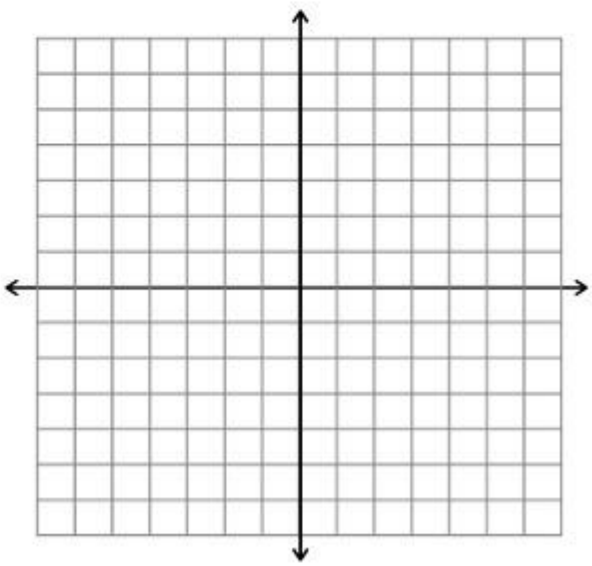


Explain the general way that vertical shifts work:

<u>Example</u> How does $f(x) = \sqrt{x} + 6$ shift?

Horizontal Translations

Draw the base graph of $f(x) = |x|$ in one color and then use some other colors to show how the translations work. Label the graphs.

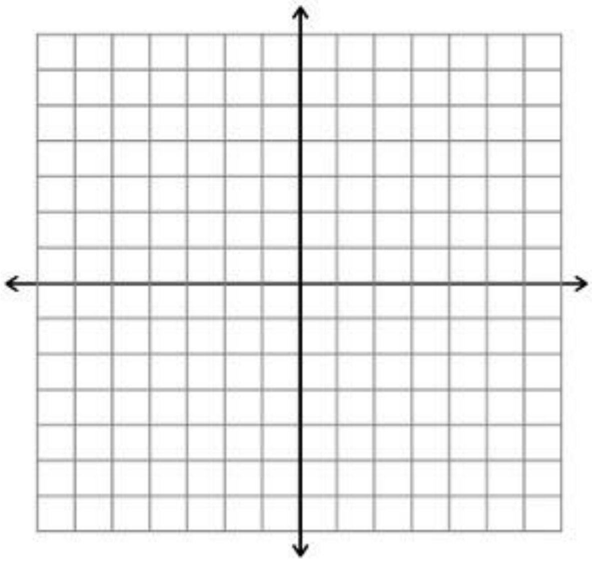


Explain the general way that horizontal shifts work:

<u>Example</u> How does $f(x) = \sqrt[3]{x+3}$ shift?

Reflections Over the X-Axis

Draw the base graph of $f(x) = \sqrt{x}$ and then graph the reflection in another color. Label the graph of the reflection. Label the graphs.



Explain the general way that reflections over the x-axis work:

Example How does $f(x) = -x^2$ look?

Example Describe the translations and reflections of the following.

a. $f(x) = -x^2 + 3$

b. $g(x) = \sqrt{x+2} - 1$

c. $h(x) = (x-5)^3 + 2$

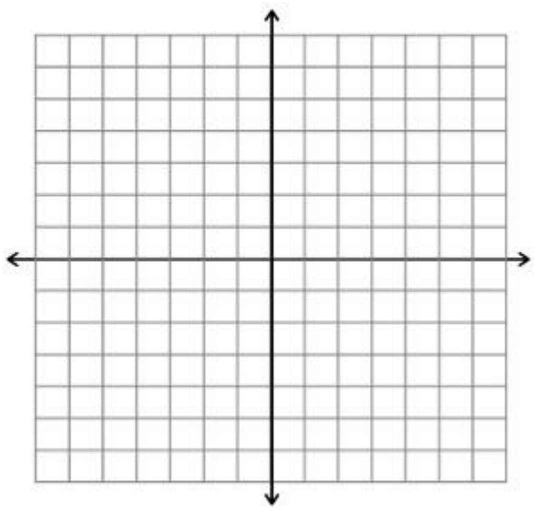
d. $f(x) = -|x+2| + 5$

e. $g(x) = -\sqrt[3]{x-1} - 4$

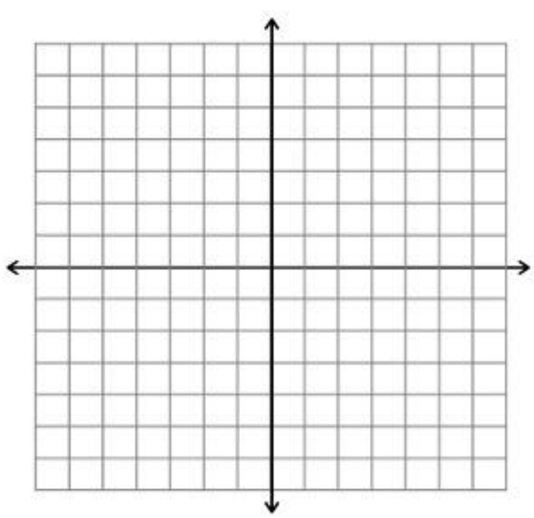
Lesson: Graphing with Translations and Reflections

Examples For each of the following, describe the translations and reflections. Then graph with 2-3 points.

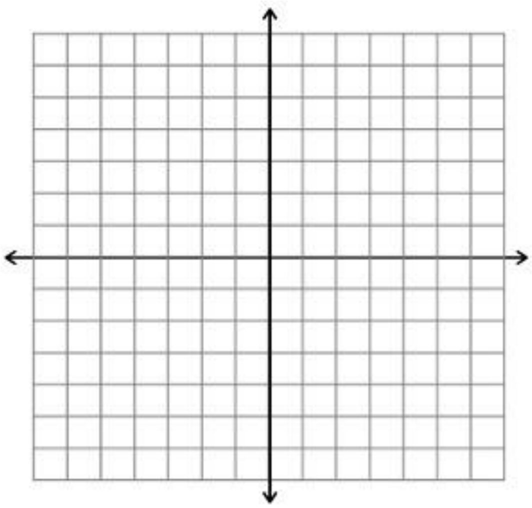
a. $f(x) = x^2 + 1$
Describe any translations or reflections:



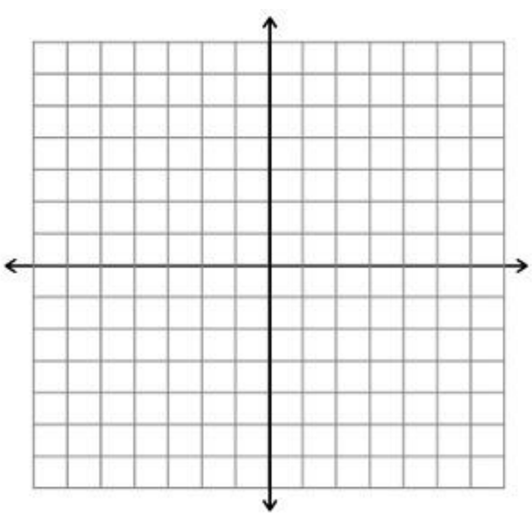
b. $g(x) = \sqrt{x-2}$
Describe any translations or reflections:



c. $f(x) = (x+2)^3 - 1$
Describe any translations or reflections:

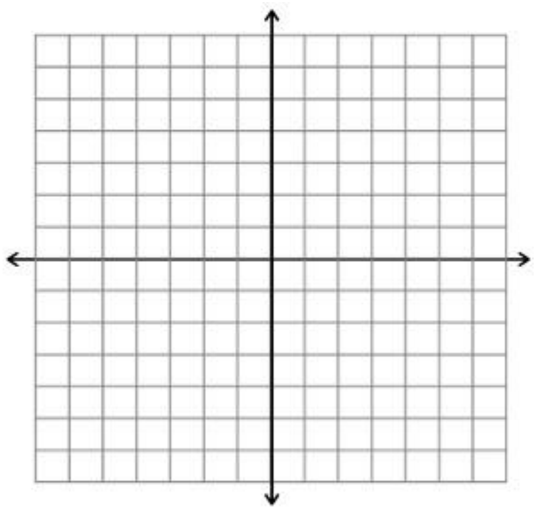


d. $h(x) = |x-3| + 2$
Describe any translations or reflections:



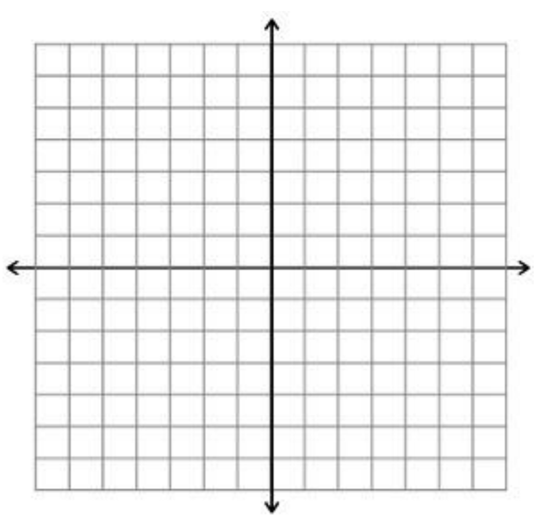
e. $f(x) = -|x - 3| - 2$

Describe any translations or reflections:



f. $f(x) = -\sqrt{x + 3} + 1$

Describe any translations or reflections:



Lesson: Writing Equations of Functions with Translations and Reflections

Example Write the equation of a function $g(x)$ where the following transformations are performed on the graph of $f(x)$

a. $f(x) = \sqrt[3]{x}$ is shifted 3 units down and 2 units right

b. $f(x) = x^2$ is shifted 2.5 units up and π units left

c. $f(x) = e^x$ is reflected about the x-axis, shifted 4 units down and 1 unit left

Lesson: Determining Whether Two Lines are Parallel or Perpendicular

What makes 2 lines parallel?

<u>Examples</u> Are the equations $y = \frac{2}{3}x + 5$ and $2x - 3y = 6$ parallel?

What makes 2 lines perpendicular?

<u>Examples</u> Determine the negative reciprocal of each of the following:

a. $\frac{2}{3}$

c. -7

b. $-\frac{1}{5}$

d. 1

Example Are the equations $y = \frac{3}{5}x + 2$ and $-3x + 5y = -15$ perpendicular?

Examples Are the following pairs of lines parallel, perpendicular, or neither?

a. $3x - 2y = 6$ and $4y = -6x - 8$

b. $y - 3x = 0$ and $12x - 4y = 24$

c. $y = 3$ and $x = -2$

Examples Lines A and B contain the given points. Determine whether the lines are parallel, perpendicular, or neither.

a. Line A: $(2, 11), (5, 10)$ Line B: $(1, 3), (-6, -18)$

b. Line A: $(-5, 5), (-7, -2)$ Line B: $(0, 11), (-4, 9)$

Lesson: Writing Equations of Lines with The Point-Slope Form

What is the point-slope form of a line?

<u>Examples</u> Given the point and the slope, write the point-slope form of a line.

a. $m = -2, (1, 3)$

b. $m = -\frac{2}{3}, (5, -2)$

What is the slope-intercept form of a line?

What is the standard form of a line?

Examples Given the point and the slope, write the equation of the line in the desired format.

a. $m = -1, (2, -5)$ in slope-intercept form

b. $m = 3, (5, -4)$ in standard form

c. $m = -\frac{1}{2}, (6, 3)$

d. $m = \frac{2}{3}, (-5, -1)$

Lesson: Writing the Equation of a Line from Two Points

Write the equation of the line that contains the following 2 points in the desired format.

a. $(4, 2)$ and $(5, 5)$ in slope-intercept form

b. $(-2, 4)$ and $(-4, -2)$ in standard form

c.$(5, -3)$ and $(3, 5)$ in slope-intercept form

d. $(-1, -3)$ and $(1, 5)$ in standard form

Lesson: Writing the Equation of a Line Parallel/Perpendicular to Another Line

What makes 2 lines parallel?

What makes 2 lines perpendicular?

Examples Write the equation of a line with the given conditions.

a. Parallel to $y = 3x + 1$ that goes through $(-2, -5)$ in slope-intercept form

b. Perpendicular to $y = -\frac{2}{3}x - 5$ that goes through $(6, -2)$ in standard form

c. Parallel to $2x - 3y = 6$ that goes through $(1, -2)$ in slope-intercept form

Examples Write the equation of a line with the given conditions.

a. Perpendicular to $2x - 4y = 8$ that goes through $(-4, -2)$ in standard form

b. Parallel to $3x - y = 12$ that goes through $(-1, -5)$ in slope-intercept form

c. Perpendicular to $-2x + 3y = 12$ that goes through $(2, -5)$ in standard form

Lesson: Arithmetic Sequences

What is an arithmetic sequence?

Give some examples of an arithmetic sequence.

<u>Example</u> Determine the common difference, d, for the arithmetic sequence:

$$-7, -5, -3, -1, \ldots$$

Example Find the first five terms of each arithmetic sequence.

a. The first term is 3, and the common difference is -2.

b. $a_1 = -5, d = 7$

How do we define the general nth term of an arithmetic sequence?

Example Determine a_{11} and a_n for the arithmetic sequence $-2, 2, 6, 10, \ldots$

Example Determine a_n and a_{20} for the arithmetic sequence having $a_3 = 9$, and $a_4 = 16$.

Example An arithmetic sequence has $a_7 = -15$ and $a_{14} = -35$. Determine a_1.

Lesson: Arithmetic Series

When we sum up the terms of an arithmetic sequence, how can we define that sum? (sum of an arithmetic sequence)

<u>Example</u> Consider the arithmetic sequence $-7, -3, 1, 5, \ldots$

a. Evaluate S_{10}

b. Evaluate the sum of the first 50 positive integers.

Example The sum of the first 15 terms of an arithmetic sequence is 345. If $a_{15} = 65$, find a_1 and d.

Examples Evaluate each sum.

a. $\Sigma_{i=1}^{19}(-3 - 5i)$

b. $\Sigma_{k=4}^{15}(2 + 4k)$

Lesson: Solving Systems of Equations by Graphing

We will now turn our attention to systems of equations.

System of Equations

A system of equations refers to 2 or more equations that share 2 or more variables.

Solution of a System of Equations

A solution of a system of equations is an ordered pair that is true in all equations.

Determine if the following pairs are solutions to the systems.

a. $(1,3)$
$x - 2y = -5$
$2x - 3y = -4$

b. $(-4, 3)$
$x + 3y = 5$
$2x + 3y = 1$

There are several ways to solve systems. The first method we will focus on is how to solve by graphing.

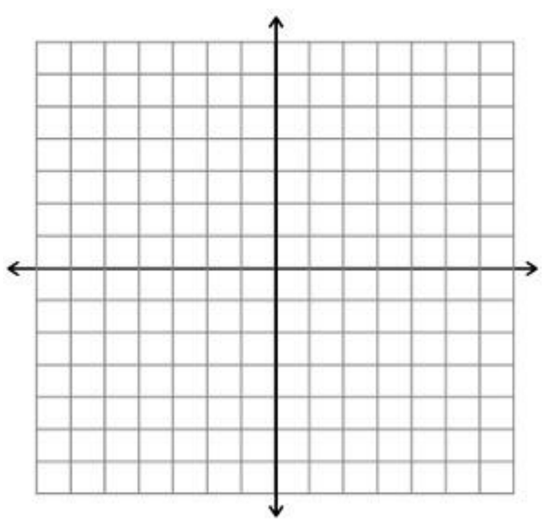
Solving by Graphing

In this method, we graph both lines and determine the point of intersection if possible. We will examine all the possibilities.

Example

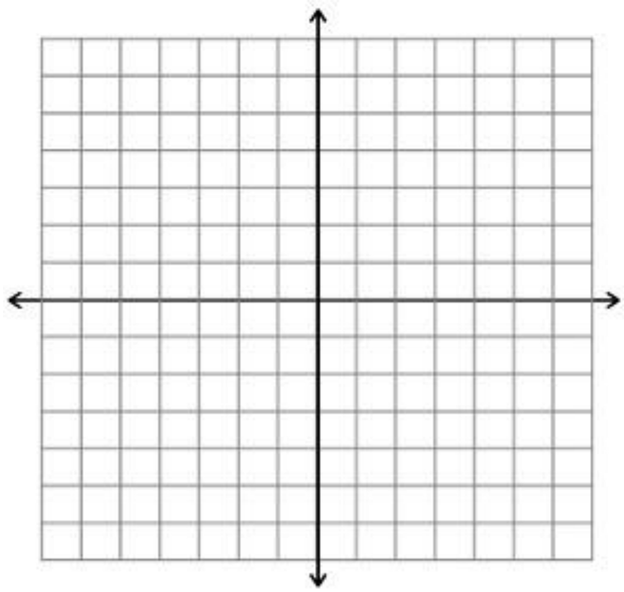
Graph each line and determine the point of intersection to find the solution.

$y = 2x + 3$
$y = -\frac{1}{2}x - 2$

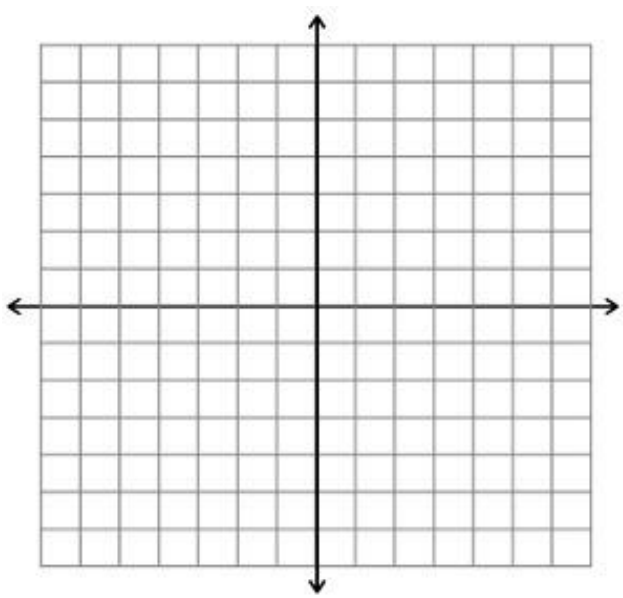


Example

a. $y = -3x + 1$
$y = 2x - 4$



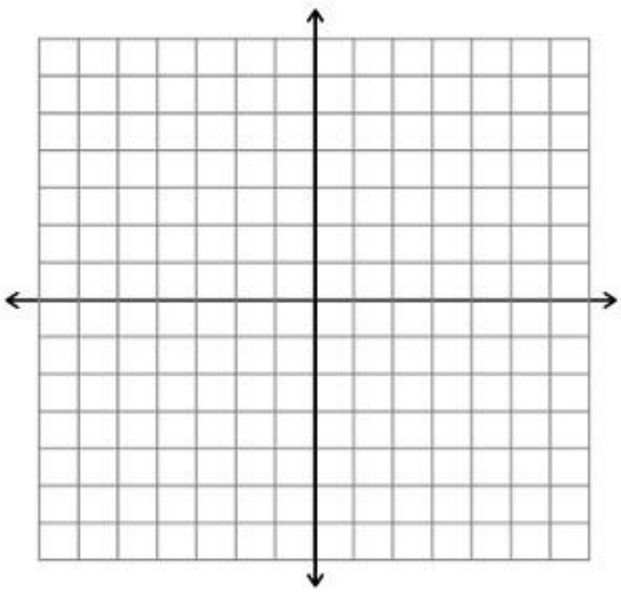
b. $y - \frac{1}{2}x = -4$
$y + 2x = 1$



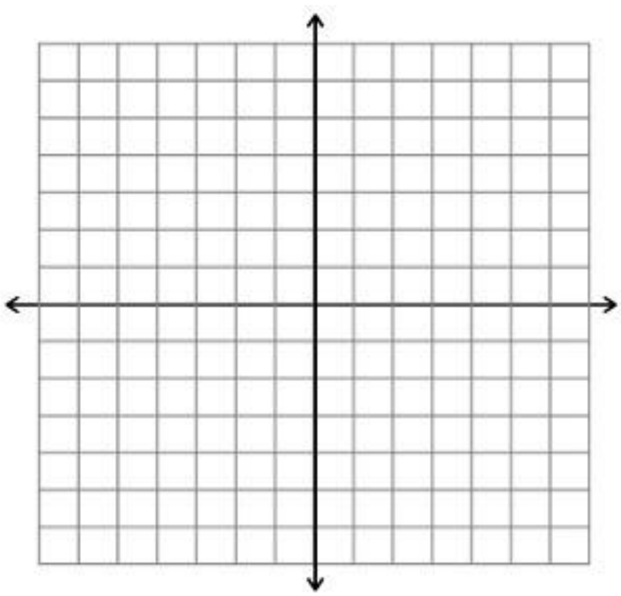
There are two special situations that can occur with graphing.

Example

a. $y = -3x + 2$
$9x + 3y = 6$



b. $y = -\frac{1}{2}x + 4$
$2x + 4y = 12$



Lesson: Solving Systems of Equations by Substitution

In this lesson, we will focus on the second method for solving systems: substitution.

Show how to use substitution with this example:

$2x + y = 6$

$3x + 5y = 9$

<u>Example</u> Solve the system using substitution

$x - 2y = 10$

$5x - 7y = 20$

Below are 2 more complicated problems using substitution. Outline the steps shown in the video:

$-2x + 4y = 4$

$-2x + 6y = 6$

$2x - 5y = -4$

$8x - 9y = 6$

Now let’s see what happens when we get a problem with no solution or infinite solutions.

a. $9y - 18x = 5$

$2x - y = 3$

b. $x + 2y = -3$

$4x + 12 = -8y$

Lesson: Solving Systems of Equations by Elimination

In this lesson, we will focus on the third method for solving systems: elimination.

Show how to use elimination with this example:
$3x - y = 8$

$2x + y = -3$

Example Use elimination to solve the system.

$4x + 2y = -6$
$3x - 2y = 6$

Now let's examine a more complicated case of elimination. Write down how to approach these problems:

a. $3x - 6y = 12$

$5x - 2y = -4$

b. $5x - 10y = 5$

$2x - 3y = 6$

Now let's investigate what happens in the situation where there is no solution or infinite solutions.

a. $5x - 2y = 1$
$-10x + 4y = 3$

b. $3x + y = 5$
$-9x - 3y = -15$

Lesson: Solving Systems of Equations that Require Clearing of Fractions

Examples

a. $\frac{1}{6}x - \frac{1}{3}y = \frac{1}{2}$
$\frac{2}{5}x - \frac{3}{5}y = -\frac{6}{5}$

b. $-\frac{1}{2}x - \frac{3}{4}y = 1$
$\frac{1}{3}x + \frac{3}{2}y = -\frac{2}{3}$

c. $\frac{1}{2}x + \frac{3}{5}y = 6$

$\frac{1}{4}x - \frac{1}{2}y = -9$

Lesson: Solving Systems of Equations that Require Clearing of Decimals

Examples

a. $0.02x + 0.1y = 0.05$
$4x + 20y = 10$

b. $0.1x - 0.05y = 0.2$
$0.3x - 0.25y = 0.2$

c. $-0.4x + 1.3y = 1.$
$1.6x - 0.9y = 6.9$

Lesson: Linear Inequalities in Two Variables

What is a linear inequality in 2 variables?

Show how to graph $y \leq 2x + 1$

Examples Graph the following:

a. $3x + 2y > -6$

b. $5x - 2y \leq 10$

c. $-3x + 4y < -12$

Lesson: Compound Linear Inequalities in Two Variables

What is a compound linear inequality?

What does “and” mean in compound inequalities?

What does “or” mean in compound inequalities?

Show how to graph $y > 2x + 1$ and $y \leq -x - 3$

Show how to graph $y \leq -2x + 1$ or $y > x + 2$

<u>Examples</u> Graph the following:

a. $3x - 2y > 6$ and $2x + y \leq 3$

b. $5x - 2y \leq 10$ or $2x + 6y > 12$

<u>Examples</u> Graph the following:

a. $y \geq 4$ and $y < -2x + 1$

b. $x < 3$ or $4x - 2y \leq 8$

c. $x > -1$ and $y \geq 2x + 2$

Lesson: Basic Exponent Rules – Product, Power, and Quotient Rules

When working with rules of exponents, it is very important to be able to identify the base of each exponent. To begin, we will practice this.

<u>Example 1</u>
State the base for each exponent in the problems below.

Expression	Exponent	Base for this Exponent
5^3		
-5^4		
$-5x^6$		
$3y^7$		

<u>Product Rule</u>
First we want to remember what the actual definition of an exponent means.
We want to find: $2^3 * 2^2$. We will calculate this a few ways to understand the rules.

First we will work towards finding the exact answer. Evaluate each exponent below as you normally would with the **order of operations**, and then multiply the numbers together:

Now we will re-examine this result solely from the viewpoint of exponents. Write out each exponent as a multiplication problem, and notice the results you get from this exercise.

What do you notice about the exponents?

<u>Product Rule (formal mathematical definition)</u>
Let x be a real number, and let a and b be integers. Then:

Example 2

Use the product rule to evaluate the following.

a. $x^2 * x^4$

b. $3x^3 * -1x^4 * 2x$

c. $x^3 + x^7$

d. $\frac{2}{3}x^4y^2z^5 * 6x^3y^2 * \frac{1}{2}y^3z^4$

Power Rule

Using the order of operations, first rewrite the expression inside the parentheses without exponents:

$(2^3)^2$ =

Now rewrite this entire result just utilizing what we know about the definition of exponents:

What is the relationship between the exponents in this case?

The Power Rule (formal mathematical definition)

Let x be a real number, and let a and b be integers. Then:

Example 3

Use the power rule to evaluate the following:

a. $(x^3)^5$

c. $(3x^4y^5)^3$

b. $(x^2y^7z)^4$

d. $\left(\frac{2}{3}x^5y^2\right)^3$

Is there a difference between $(x+y)^n$ and $(xy)^n$? Explain.

The Quotient Rule

Using the order of operations, first rewrite the expression inside the parentheses without exponents:

$$\frac{2^5}{2^3}$$

First we will evaluate and find the final answer by using the order of operations below.

Notice the final answer of this question. How can we rewrite this number in its exponent form?

What relationship do you see between the exponents in the original question and the final answer?

The Quotient Rule (formal mathematical definition)

Let x be a real number, and let a and b be integers. Then:

Example 4

Use the quotient rule to simplify the following:

a. $\frac{x^7}{x^3}$

c. $\frac{15x^4y^3z^{10}}{2x^2y^2z^5}$

b. $\frac{x^5y^8z^{10}}{xyz}$

d. $\frac{12x^7y^3}{8x^3z^4}$

Examples: Exponent Rules, Basic Examples

Simplify the following using only the product, power, and quotient rules. Your final answer should not contain negative exponents.

a. $x^2y^7 * 5x^3y^5z$

b. $(4x^7y^3)^2$

c. $\frac{15x^7y^3}{10x^2y}$

d. $\frac{2}{5}x^2y^5 * \frac{1}{3}x^7yz^3$

e. $\left(\frac{5}{3}x^3y^2z^3\right)^3$

f. $(-3x^5)^2$

g. $(-3x^5)^3$

h. $(2x^4y^2z^6)^2 * 3x^2z^3$

i. $\frac{(x^3y^7)^2}{x^4y}$

j. $\frac{(x^4y^2)^3}{(x^2y)^2}$

Examples: Exponent Rules, With Numbers Only

Simplify the following using only the product, power, and quotient rules. Your final answer should be a number.

a. $3^2 * 3$

b. $(-4)^2 * 2$

c. $\frac{2^7}{2^4}$

d. $\left(\frac{2}{3}\right)^3 * 3^2$

e. $\frac{6^2}{2^3}$

f. $\left(\frac{1}{2}\right)^4 * 2^2$

Examples: Exponent Rules, Trickier and Harder Examples

Simplify the following using only the product, power, and quotient rules. Your final answer should not include negative exponents.

a. $(3x^2y^4)^2 * (x^4y^5)^3$

b. $\frac{(2x^5y^2)^3}{(x^2yz^5)^2}$

c. $\left(\frac{2}{3}x^5y^7z^3\right)^2 * 9x^3y^3$

d. $\left(\frac{3}{5}x^5y^2z^7\right)^2 * \frac{xy^3}{z^5}$

e. $\frac{(3x^5y^2z^4)^3}{(6x^2yz^3)^2}$

f. $\frac{(4x^3y^2z^5)^2}{(2xz^2)^4}$

Lesson: Negative Exponents

Simplify: $\frac{25}{125}$

If we wrote these numbers as exponents, this calculation would look like this:

$$\frac{5^2}{5^3} = \frac{1}{5}$$

Consider what would happen if we used the quotient rule on this problem:

$$\frac{5^2}{5^3} = 5^{2-3} - 5^{-1} = \frac{1}{5}$$

This problem shows us that exponents can also be negative.

<u>Three Rules for Negative Exponents</u>
Let x and y be a real numbers, and let n be an integer. Then:

$$x^{-n} = \frac{1}{x^n}$$

$$\frac{1}{x^{-n}} = x^n$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^{n} = \frac{y^n}{x^n}$$

In general your final answer cannot have a negative exponent, so it is essential that you understand what to do with it.

Example 1

Rewrite the following expressions without negative exponents.

a. 4^{-2}

b. $\frac{1}{3^{-3}}$

c. $\left(\frac{2}{5}\right)^{-2}$

d. $\left(\frac{1}{3}\right)^{-2}$

One of the biggest challenges with negative exponents is identifying how to apply them to the proper base. As a quick review, remember how we identified the bases in the last section:

Example 2

Identify the base of the following exponents:

Expression	First Base	First Exponent	Second Base	Second Exponent
3^5				
$-2x^5$				
$5x^3y^7$				

This is extremely important to remember when working with negative exponents. In general, our final answer can't have negative exponents so you need to be used to reformatting your answers properly if negative exponents come up.

Example 3

Simplify the following expressions, and write your final answer without negative exponents.

a. $\frac{a^3b^{-2}}{c^{-4}d^6}$

b. $\frac{3a^{-3}b^4c^{-5}}{d^2e^{-2}f^5}$

c. $\frac{a^5b^3c^4}{4d^{-2}e^{-4}f^{-2}}$

d. $\frac{a^{-3}b^{-2}c^{-6}}{d^4e^7f}$

With this in mind, we should now be able to work on combining all of the rules of exponents we've learned.

Example 4

Simplify the following. Your final answer cannot contain negative exponents.

a. $(3x^{-2}y^{4})^{3}$

b. $\left(\frac{2x^{-4}y^{6}}{x^{5}y^{-3}}\right)^{4}$

c. $(-2x^{3}y^{-2}z)^{-3}$

d. $\left(\frac{2x^{3}y^{-4}}{3x^{-2}y^{-5}}\right)^{-3}$

Examples: Negative Exponents, Basic Examples

Your final answer should not contain negative exponents.

a. $\left(\frac{4}{5}\right)^{-2}$

b. $3x^{-6}y^4z^{-2}$

c. $\frac{4x^{-4}y^7}{z^{-5}}$

d. $\frac{6x^3y^{-5}}{x^7y^2}$

e. $\frac{14x^{-3}y^5z^3}{21x^{-2}y^{-3}z^9}$

f. $(3x^{-4}y^{-2}z^4)^2$

g. $\left(\frac{5x^2y^{-4}}{z^2}\right)^5$

h. $(x^{-3}y^2z^{-4})^{-3}$

Examples: Negative Exponents, Tricky Examples

Your final answer should not contain negative exponents.

a. $(3x^{-4}y^5z^{-2})^{-3}$

b. $(-5x^2y^{-3}z^4)^{-2}$

c. $\left(\frac{4x^{-5}y^2}{x^{-3}y^{-5}}\right)^{-2}$

d. $\left(\frac{3x^3y^{-3}z^4}{2x^{-2}y^4z^{-2}}\right)^{-3}$

e. $\frac{(2x^{-4}y^5)^4}{x^{-3}y^7}$

f. $\frac{(3x^{-3}y^2z^{-3})^{-3}}{(2x^4y^{-5}z^3)^{-4}}$

Examples: Negative Exponents versus Negative Numbers

Examples Simplify:

a. 6^{-2}

b. -6^2

c. $(-6)^2$

d. $(-6)^{-2}$

Examples Simplify:

a. $5x^{-2}$

b. $(5x)^{-2}$

c. $-5x^2$

d. $(-5x)^{-2}$

Examples Simplify

a. -7^2

b. $(-7)^{-2}$

c. 7^{-2}

d. -7^{-2}

Examples Simplify

a. $3x^{-2}$

b. $(-3x)^{-2}$

c. $(-3x)^{-3}$

Lesson: A Review of Exponent Rules for Algebra Students

Power Rule: $x^m * x^n =$

Product Rule: $(x^m)^n =$

Quotient Rule: $\frac{x^m}{x^n} =$

Negative Exponents

$x^{-n} =$

$\frac{1}{x^{-n}} =$

$\left(\frac{x}{y}\right)^{-n} =$

Simplify the following using exponent rules. Your final answer should not contain negative exponents.

a. $3x^7y^{-4} * 5x^{-2}y^9$

b. $\left(\frac{2x^4y^5}{3z^5}\right)^3$

c. $\frac{10x^4y^7}{2x^2y^3z^{-5}}$

d. $\frac{4x^3y^{-2}}{6x^{-4}y^{-5}}$

e. $\frac{8x^{-3}y^4z^{-3}}{10x^5y^{-2}z^{-7}}$

f. $\left(\frac{3x^{-3}y^{-2}}{6x^2y^4}\right)^{-2}$

g. $\left(\frac{2x^{-7}z^3}{3x^{-2}y^{-3}z^5}\right)^{-3}$

h. $\left(\frac{5x^{-2}y^{-3}z^3}{x^3y^4z^{-7}}\right)^{-2}$

Rational Exponents

Video 1: How to work with rules like $x^{\frac{1}{n}}$ or $x^{\frac{m}{n}}$ or $x^{-\frac{m}{n}}$

In this video, we will learn how to evaluate expressions that look like this:

$8^{\frac{1}{3}}$ or $25^{\frac{3}{2}}$

Define $x^{\frac{1}{n}}$ and show how to use this rule to evaluate $25^{\frac{1}{2}}$

<u>Examples</u> Evaluate the following:

a. $125^{\frac{1}{3}}$

b. $32^{\frac{1}{5}}$

c. $9^{\frac{1}{2}}$

d. $(-9)^{\frac{1}{2}}$

e. $-9^{\frac{1}{2}}$

f. $\left(\frac{27}{125}\right)^{\frac{1}{3}}$

Define $x^{\frac{m}{n}}$ and show how to use this rule to evaluate $25^{\frac{3}{2}}$

<u>Examples</u> Evaluate the following:

a. $27^{\frac{2}{3}}$

b. $4^{\frac{3}{2}}$

c. $16^{\frac{5}{4}}$

d. $-8^{\frac{2}{3}}$

e. $(-8)^{\frac{2}{3}}$

f. $(-16)^{\frac{3}{4}}$

g. $-81^{\frac{3}{4}}$

h. $\left(\frac{125}{27}\right)^{\frac{2}{3}}$

Define $x^{-\frac{m}{n}}$ and show how to use this rule to evaluate $25^{-\frac{3}{2}}$

Examples Evaluate the following:

a. $25^{-\frac{1}{2}}$

b. $27^{-\frac{1}{3}}$

c. $16^{-\frac{3}{4}}$

d. $4^{-\frac{5}{2}}$

e. $\left(\frac{25}{4}\right)^{-\frac{3}{2}}$

f. $(-25)^{-\frac{3}{2}}$

g. $(-27)^{-\frac{2}{3}}$

h. $-16^{-\frac{5}{4}}$

Lesson: Using Exponent Rules with Rational Exponents

First, let's review the main exponent rules:

Next, let's do a quick review on operations with fractions. Complete the following operations:

a. $\frac{2}{3} \times \frac{5}{6}$

b. $\frac{2}{5} \times 10$

c. $\frac{2}{3} + \frac{2}{5}$

Now we will combine all of these ideas into the following examples!

Examples Simplify the following. Your final answer should not include any negative exponents.

a. $x^{\frac{2}{3}} * x^{\frac{1}{5}}$

b. $\left(4x^{\frac{2}{3}}y^{6}\right)^{\frac{3}{2}}$

c. $\frac{x^{\frac{2}{3}}y^{-\frac{1}{5}}}{x^{\frac{1}{6}}y^{\frac{2}{5}}}$

d. $\left(\frac{125x^4y^{-\frac{1}{3}}}{x^{-\frac{1}{2}}y^{\frac{2}{5}}}\right)^{-\frac{2}{3}}$

e. $\left(\frac{4x^6y^{-\frac{1}{3}}}{x^{-3}y^{\frac{1}{2}}}\right)^{-\frac{3}{2}}$

Lesson: Simplifying Radicals with Rational Exponents

Convert the following radicals to rational exponents, simplify, and then rewrite in radical form.

a. $\sqrt{7^2}$

b. $\sqrt{9^2}$

c. $\sqrt[3]{x^{15}}$

d. $\sqrt[4]{x^2}$

Rational Exponents
Video 3: Convert a radical expression to exponential form

Examples Convert the following expressions to exponential form, and then simplify if needed.

a. $\sqrt{7}$

b. $\sqrt[3]{8}$

c. $\sqrt{5^2}$

d. $\sqrt[3]{7^6}$

e. $\sqrt[4]{x^{16}}$

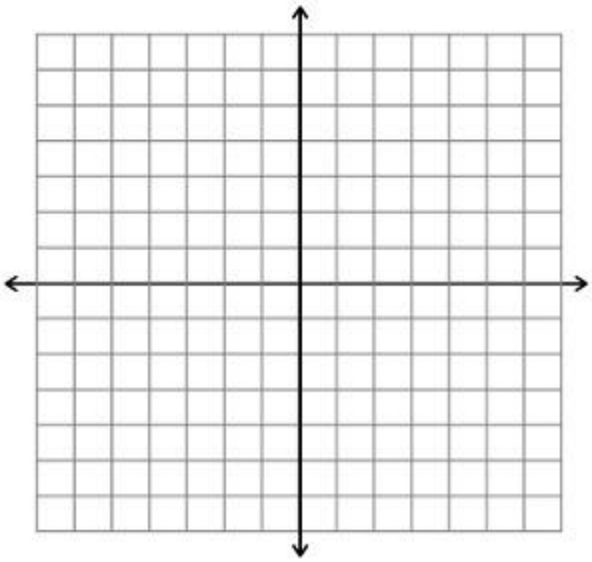
f. $\sqrt[6]{x^2}$

Lesson: Exponential Functions

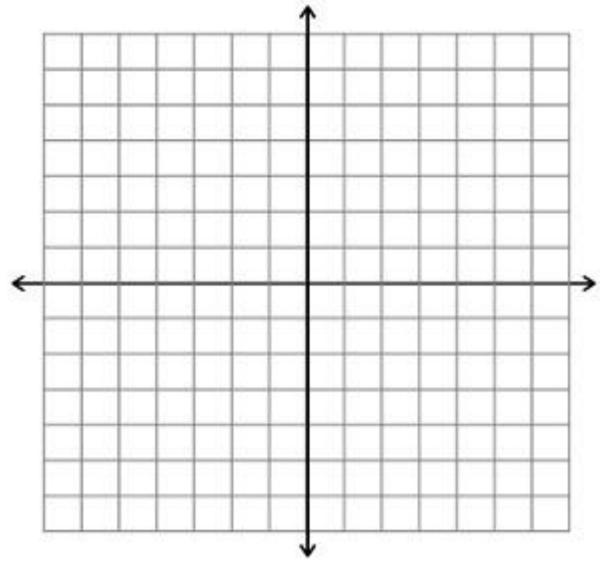
An exponential function is a function of the form ______________________________ where $a > 0, a \neq 1$.

<u>Examples</u> Graph the following functions, then determine their domain and range.

a. $f(x) = 2^x$



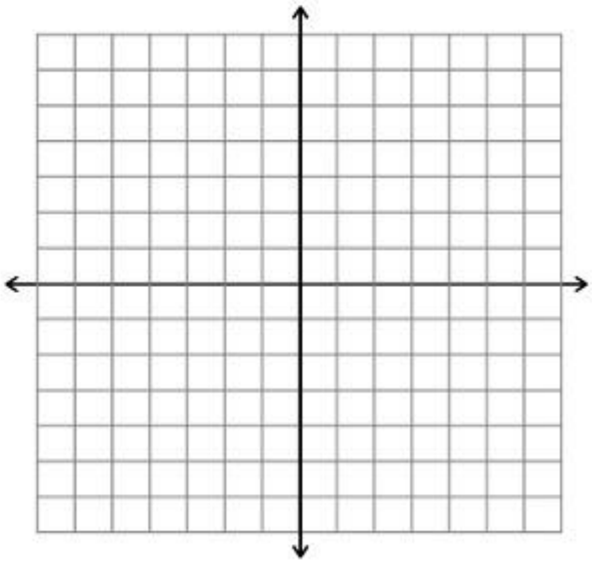
b. $g(x) = \left(\frac{1}{2}\right)^x$



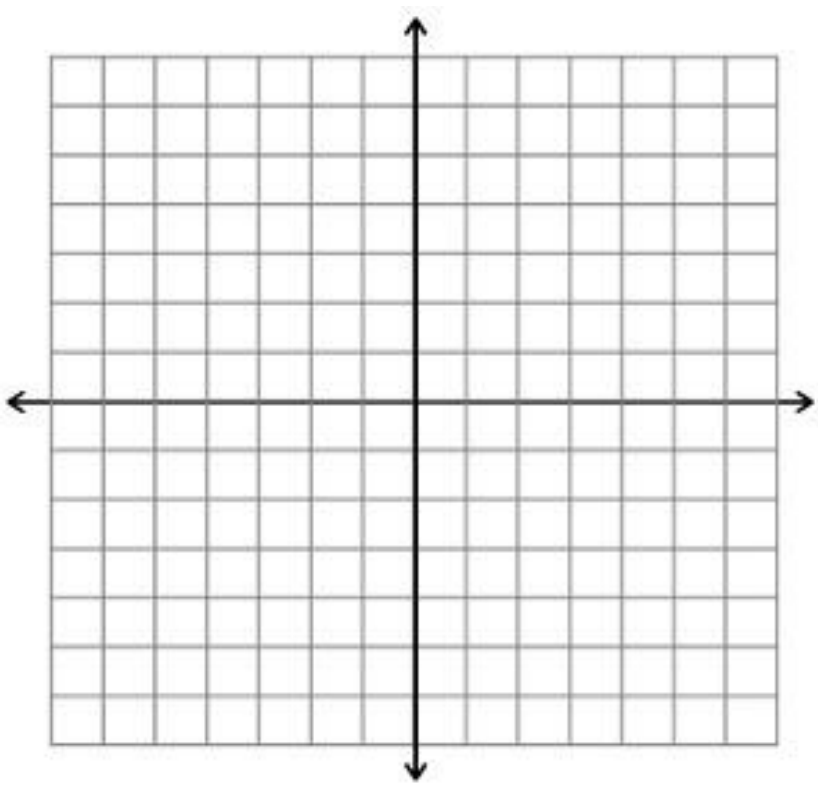
Using the graphs above, summarize some of the characteristics of these graphs:

The number e is an irrational number that is frequently used in math. What is an approximation of e?

Example Graph $f(x) = 2^x, g(x) = e^x, h(x) = 3^x$ all on the same graph below for comparison:



Translations work similarly to other graphs we have discussed. Using the graph of $f(x) = 2^x$ as a "base graph", draw the following graphs together on the same graph: $f(x) = 2^x, g(x) = 2^x + 3, h(x) = 2^{x-1}$



Lesson: Solving Exponential Equations

In this lesson, we will solve equations that look like this:

$$3^x = 27 \qquad \text{or} \qquad 4^{x+3} = 8^{x-2}$$

To do this, we will write directions around the following example: $2^{3x} = 8^{x+1}$

<u>Examples</u> Solve:

a. $49^{x-3} = 7^{2x}$

b. $9^{4x} = 27^{x+4}$

<u>Examples</u> Solve:

a. $5^{x-2} = \frac{1}{25}$

b. $\left(\frac{3}{5}\right)^{2x} = \left(\frac{27}{125}\right)^{x+2}$

c. $27^{3x+1} = 9^{4x+2}$

Lesson: Geometric Sequences and Series

Define a geometric sequence:

<u>Example</u> Determine a_6 and a_n for the geometric sequence 4, 12, 36, 108, ….

<u>Example</u> Determine r and a_1 for the geometric sequence with $a_2 = -18$ and $a_5 = 486$

How do we define the sum of the first n terms of a geometric sequence?

Example Evaluate: $\Sigma_{i=1}^{8} 3 * 2^{i}$

If we start with 2 and use a common ratio of $\frac{1}{2}$, what eventually happens as the series continues infinitely? Summarize the explanation below.

Summarize the Sum of the Terms of an Infinite Geometric Sequence

<u>Examples</u> Evaluate the following infinite sums.

a. The first term is 1 and the common ratio is $\frac{1}{3}$

b. $\Sigma_{i=1}^{\infty} \left(-\frac{1}{2}\right)\left(-\frac{3}{4}\right)^{i-1}$

c. $\Sigma_{i=1}^{\infty} \left(\frac{3}{7}\right)^{i}$

Lesson: Sequences

Define a sequence.

<u>Examples</u> Write the first five terms of each sequence.

a. $a_n = \frac{1}{n}$

b. $a_n = \frac{n \mp 1}{n+3}$

c. $a_n = (-1)^{n+1} * (2n + 1)$

What does it mean for a sequence to converge?

What does it mean for a sequence to diverge?

What is a recursively defined sequence?

Examples Find the first four terms in the sequence.

a. $a_1 = 3; a_n = 2a_{n-1} - 4$

b. $a_1 = 2; a_n = a_{n-1}^2 + n - 2$

Lesson: Series

What is the summation sign and what does it mean? What do we called the “i” in this formula?

Define a finite series.

Define an infinite series.

Examples Evaluate the following sums.

a. $\Sigma_{i=1}^{4}\left(3^{i}+1\right)$

b. $\Sigma_{j=3}^{6}a_j$

c. $\Sigma_{k=1}^{4}(4x_k-1)$ if $x_1=2, x_2=3, x_3=5, x_4=8$

d. $\Sigma_{i=1}^{3}f(x_i)\Delta x$, if $f(x)=3x, x_1=1, x_2=4, x_3=7, \Delta x=3$

What are the summation properties?

What are the summation rules for summing up terms?

Examples Use the summation properties and rules to evaluate each series.

a. $\Sigma_{i=1}^{50} 3$

b. $\Sigma_{k=1}^{25} 3k$

c. $\Sigma_{j=1}^{33} (4j^2 + 1)$

Lesson: An Intro to Polynomials

We can extend our knowledge of exponents to another algebraic topic – polynomials

Before we define what a polynomial is, it is usually easier to see a few examples of what is or is not a polynomial.

The following ARE polynomials:

- $x^2 - 3x + 8$
- $x^7 - 5x^4 + 2x^3 + x$
- 9
- $7x + \pi$
- $x^3 + \frac{2}{3}$

The following ARE NOT polynomials:

- $\sqrt{x} - 4$
- $4x^{-4} - 7x^{-3}$
- $\frac{4}{x} - \frac{2}{x+3}$

Now we can more formally define a polynomial.

Polynomial
An expression with real numbers, variables, and whole number exponents.

Descending Order
Ordering a polynomial from the largest exponent to the smallest exponent, and any freestanding number without a variable is last

Example 1
Put the polynomials in descending order.

a. $x^3 + 4x^5 - 2x^7 + 3x^2 + 5x^8$

b. $x^2 - x^3 + x^4 - x^5 - x^6 + 3$

Term
Any part in a polynomial that is separated by a "+" or "-" sign is considered to be its own term

Variables
The letters in a polynomial that represent numbers

Coefficients
The numbers attached to the variables

Constant Term
The freestanding number within a polynomial that is not attached to any variables

Example 1
Examine the polynomials, then determine the variables, coefficients, and constant terms

Polynomial	Terms	Variable(s)	Coefficient(s)	Constant
$2x+3$				
$3x^2-x+5$				
5				
$2x^2y+3y-x$				

Degree of a Term
The degree of a term is the sum of the exponents of that term.

Example 2
Identify the degree of each term

Term	Degree	Term	Degree
x^4		x^4y^3	
$3y^6$		x^7y^9	
x		$x^4y^2z^5$	
6		xy^4	

Degree of a Polynomial

The degree of a polynomial is the same as the degree of the term of highest degree within that polynomial

Leading Coefficient
The leading coefficient is the coefficient attached to the term of highest degree in a polynomial

Trinomial
A polynomial with 3 terms

Binomial
A polynomial with 2 terms

Monomial
A polynomial with 1 term

Example 3
Identify the degree of each polynomial and the type of polynomial it is.

Polynomial	Degree of Each Term, Respectively	Deg of Polynom	Leading Coefficient	Type of Polynomial
$x^5 + 4x^3 - 2x$				
$-5x^2 + 18x$				
$6x + 2x^4 - 3x^2$				
$x^5 + 2x^7 - 4$				
8				

Evaluating Polynomials

Evaluating polynomials is not unlike evaluating other expressions we've seen in this course.

Example 4

Evaluate the polynomials for $x = -2$.

a. $x^2 + 3x + 1$

b. $x^3 - x^2 - 3x + 4$

c. $\frac{1}{4}x^3 - \frac{1}{2}x^2 + 5x$

Lesson: How to Add and Subtract Polynomials

To add or subtract polynomials, combine like terms.

Examples

Perform the following operations.

a. $(2x + 4) + (5x^2 - 3x + 1)$

b. $(5x^2 - 3x + 1) - (x^3 - 2x^2 - x + 1)$

c. $(3x^3 + 2x^2 - 5x) + (4x^3 + 5x^2 + 2x)$

d. $(2x^2 + 5x - 4) - (x^3 - 2x^2 + 3x - 5)$

Lesson: How to Multiply Polynomials

To begin, let's first focus on the process of distributing a monomial with a polynomial.

<u>Examples</u> Multiply the following:

a. $3x(4x^2 + 5x - 7)$

b. $x^2y^4(3x^2 + 2xy - 8y^4)$

c. $\frac{1}{2}x^3y^4(4xy^3 - 2x^3y + 8xy)$

Now write down the process for how to multiply $(x + 2)(x^2 + 3x)$

<u>Examples</u> Multiply the following:

a. $(x+3)(x+2)$

b. $(3x+1)(x-6)$

c. $(x^2+4)(x-2y)$

d. $(x+4)(x-4)$

e. $(x+3)(x^2+4x+5)$

f. $(2x^2+3x+1)(3x^2-4x+2)$

How do you multiply $(x+5)^2$?

<u>Examples</u> Multiply the following:

a. $(x+2)^2$

b. $(2x^2+1)^2$

c. $(3x^2+x+5)$

How do you multiply 3 polynomials together, such as $(x+2)(x+1)(x-3)$?

Examples Multiply the following together:

a. $(x-3)(x+4)(x-1)$

b. $(2x+1)(x-2)(x-1)$

Lesson: How to Solve Quadratic Equations with Factoring

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, and a, b, and c are real numbers.

There are several ways that you can solve quadratic equations, but in this lesson we will focus on how to use factoring as a way to solve them.

Solving a quadratic equation by factoring requires a rule called the **Zero Product Rule.**

Write down the Zero Product Rule:

Examples Solve the following equations using the zero product rule:

a. $3x = 0$

b. $3(x + 1) = 0$

c. $(x + 3)(x + 1) = 0$

d. $x(2x + 1)(3x + 5) = 0$

Show how to use factoring to solve $x^2 + 5x + 6 = 0$.

Examples Solve the following by using factoring:

a. $x^2 + 12x - 45 = 0$

b. $x^2 - 12x + 11 = 0$

c. $2x^2 + 5x + 3 = 0$

d. $55x = -20x^2 - 30$

e. $x^2 = 25$

f. $x^2 = 9x$

Lesson: Tips and Tricks to Help with Harder Quadratics Solved by Factoring

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, and a, b, and c are real numbers.

In the previous lesson, we talked about the basics of using factoring to solve quadratics.

In this lesson, we will look at some trickier examples. First, let's warm up by solving the following:

a. $4x = 14x^2 - 48$

b. $3x^3 = 12x^2$

Sometimes we will have to do further work to simplify the problem before we factor.

<u>Examples</u> Use factoring to solve.

a. $(x + 3)(x + 4) = 6$

b. $5x(x + 2) - 4x = 4(x^2 - 2)$

c. $(x + 4)^2 - 2x = 11$

What can be done to solve $\frac{1}{3}(x+1)^2 - \frac{1}{4}x = \frac{1}{6}(x-2)^2 - \frac{1}{3}$?

<u>Examples</u> Use factoring to solve.

a. $\frac{1}{4}(2x-3)^2 + \frac{1}{4}x = \frac{1}{4}(x-5)^2 - \frac{3}{2}$

b. $\frac{1}{4}x(2-x) - \frac{3}{4} = \frac{1}{5}x(x+1) - \frac{7}{10}$

Lesson: The Basics of Factoring Trinomials of the Form $x^2 + bx + c$

Factoring trinomials will require us to multiply polynomials, so first we will review this skill.

Examples

Multiply.

a. $(x + 3)(x - 5)$

b. $(2x - y)(5x + 2y)$

Definition of a Trinomial

A trinomial is a polynomial with three terms.

How to Factor a Trinomial Using Trial and Error

Show how to factor $x^2 + 3x + 2$

Show how to factor $x^2 + 7x + 12$

Examples

Factor the following.

a. $x^2 + 6x + 5$

b. $x^2 + 12x + 11$

c. $x^2 - 8x + 15$

d. $x^2 - 13x + 30$

e. $x^2 + 10x - 24$

f. $x^2 - 5x - 14$

g. $2x^2 + 16x + 30$

h. $5x^4 + 55x^3 - 45x^2$

Lesson: The Basics of Factoring Trinomials of the Form $-x^2 + bx + c$

How can you factor $-x^2 - 5x - 6$?

What about $-x^2 - 15x - 36$?

Examples

Factor the following.

a. $-x^2 + 13x - 30$

b. $-x^2 + 16x - 55$

c. $-x^2 - 2x + 3$

Lesson: The Basics of Factoring Trinomials of the Form $\boldsymbol{x^2 + bxy + cy^2}$

How do you factor $x^2 + 5x + 6$?

How do you factor $x^2 + 5xy + 6y^2$?

<u>Examples</u> Factor the following.

a. $x^2 + xy - 12y^2$

b. $x^2 - 2xy + y^2$

c. $3x^2 + 21xy + 18y^2$

Lesson: The Basics of Factoring Trinomials of the Form $ax^2 + bx + c$

In this lesson we will focus on trinomials that do not have a leading coefficient.

To begin, first let's get warmed up by factoring the following using trial and error:

a. $x^2 + 7x + 12$

b. $x^2 - 3xy + 10y^2$

How do you factor $2x^2 + 5x + 3$ using trial and error?

How do you factor $3x^2 + 16x + 5$ using trial and error?

Another way to factor $3x^2 + 16x + 5$ is by grouping. Show how to use this technique below:

Examples Factor the following:

a. $7x^2 - 11x + 4$

b. $3x^2 + 10x + 8$

c. $5x^2 - 11x - 36$

d. $3x^2 - 17xy - 6y^2$

e. $-10x^2 + 19x - 6$

Lesson: Factoring Special Trinomials

How do you factor $x^2 + 6x + 9$?

What's another way to write this factorization?

What do you call this type of trinomials?

<u>Examples</u> Factor the following:

a. $x^2 + 12x + 36$

b. $9x^2 - 12x + 16$

c. $x^2 - x + \frac{1}{4}$

d. $36x^2 - 60xy + 25y^2$

How do you factor $x^2 - 4$?

What is this type of factoring called?

Examples Factor the following:

a. $x^2 - 25$

b. $x^2 - 16$

c. $x^2 - 1$

d. $25x^2 - 36$

e. $16x^2 - 81y^2$

f. $16 - x^2$

g. $25y^2 - 64x^2$

h. $\frac{49}{64} - x^2$

Lesson: How to Factor The Sum and Difference of Cubes

Write down the formula for the sum of cubes

$a^3 + b^3 =$

Write down the formula for the difference of cubes

$a^3 - b^3 =$

Show how to factor $x^3 + 8$

Show how to factor $8x^3 - 27$

Examples Factor:

a. $x^3 - 1$

b. $x^3 - \frac{1}{8}$

c. $x^3 + 64$

Review: An Overview of All the Types of Factoring

We will review all of the different types of factoring by presenting an example of each type.

Greatest Common Factor

1. $4x^2 + 8xy$

2. $6x^3y^2 - 8x^2y^5 + 10x^5y$

Grouping

3. $3x + 6y - xz^2 - 2yz^2$

4. $12x^2 - 2y + 3x - 8xy$

Basic Trinomials of the Form $\boldsymbol{x^2 + bx + c}$

5. $x^2 + 7x + 10$

6. $x^2 - 2x - 15$

Trickier or Special Forms of Trinomials of the Form $x^2 + bx + c$

7. $-x^2 - 3x + 28$

8. $6x^2 + 54x + 120$

9. $x^2 - 6xy + 8y^2$

10. $x^2 - 8x + 16$

11. $x^2 - 25$

12. $x^2 - \frac{9}{4}$

Trinomials of the Form $ax^2 + bx + c$

13. $2x^2 + 11x + 5$

14. $7x^2 - 15x + 8$

15. $-3x^2 - 15x^2 + 198x$

16. $16x^2 - 24xy + 9y^2$

More Practice with the Difference of Squares

17. $x^2 - 81$

18. $25x^2 - 4$

19. $\frac{1}{4}x^2 - 9y^2$

Sum and Difference of Cubes

20. $x^3 + 8$

21. $x^3 - 27$

22. $27x^3 - 1$

23. $8x^3 + 64y^3$

Lesson: The Greatest Common Factor

The Greatest Common Factor (GCF)

The Greatest Common Factor is the biggest common factor between a set of terms.

Examples

Find the greatest common factor in the following polynomials, then factor the polynomial.

a. Polynomial: $3x + 3y$

   GCF:

   Factor out the GCF:

b. Polynomial: $3xy + 6xz$

   GCF:

   Factor out the GCF:

c. Polynomial: $10x^2 - 25xy + 15xz$

   GCF:

   Factor out the GCF:

d. Polynomial: $3x^7y + 8x^3z^5 - 5y^4z^3$

   GCF:

   Factor out the GCF:

Examples

Factor the following polynomials.

a. $12xy^2 + 10x^2y^4 - 14xy^5$

b. $5xy - 10yz + 12xz$

c. $3x + 6x^2 - 12x^3$

d. $5x^4y^2z^5 + 10x^2y^4z^6 - 25x^5yz^3$

Lesson: Factoring by Grouping

Grouping can best be understood with the following example.

Factor $ax + ay + bx + by$ by grouping:

Examples

Finish the following examples where the grouping had been started.

a. $3(x - y) + 2z(x - y)$

b. $2a(3b + 2c) - 5(2c + 3b)$

c. $4x(y - z) - 3(z + y)$

d. $4z(4x - 7) - (4x - 7)$

Examples

Factor the following.

a. $5x + 10y + xz + 2yz$

b. $10x^2 + 15x^3y - 2 - 3xy$

c. $3xy + 4y - 27x - 36$

d. $3xy + 6x + 21y + 42$

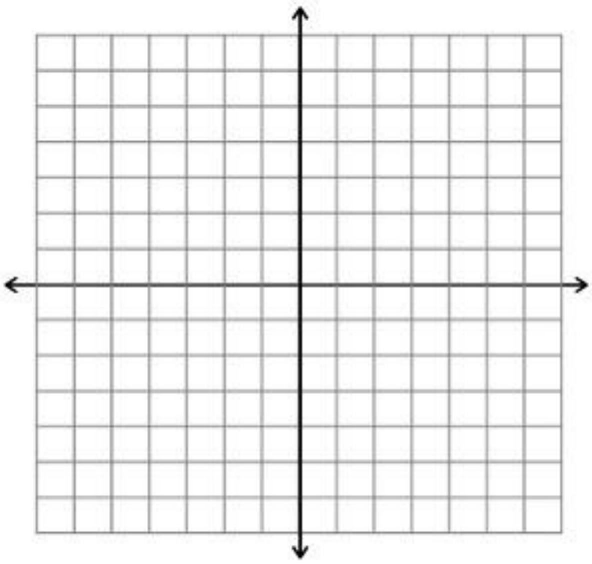
e. $3x^2 - 48y + 8x - 18xy$

Lesson: Understanding the Graphs of Quadratic Functions

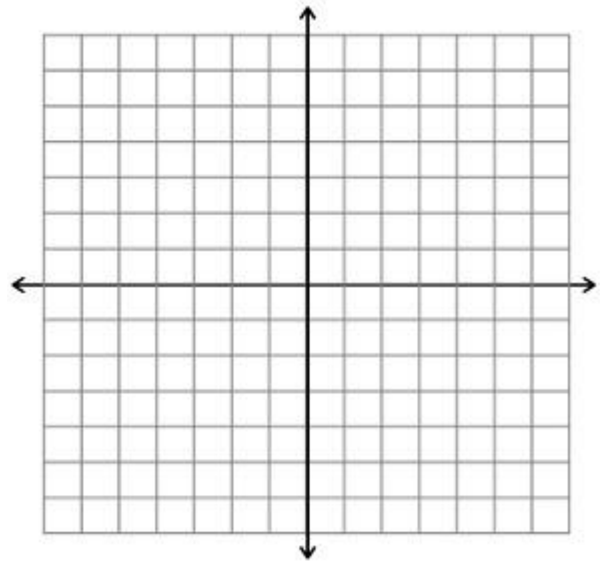
Quadratic functions are functions of the form _________________________________, where $a \neq 0$. But in previous lessons we worked on graphing quadratic functions, ie parabolas in another way. Let's review that first.

<u>Examples</u> Graph the following:

a. $f(x) = (x+2)^2 - 3$



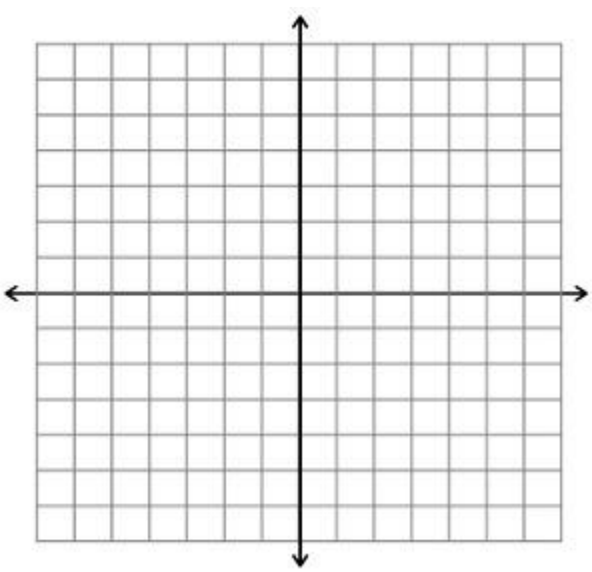
b. $g(x) = -(x-3)^2 + 1$



To begin, we will add on one more way to manipulate the graph of a parabola. Explain how to graph the follow:

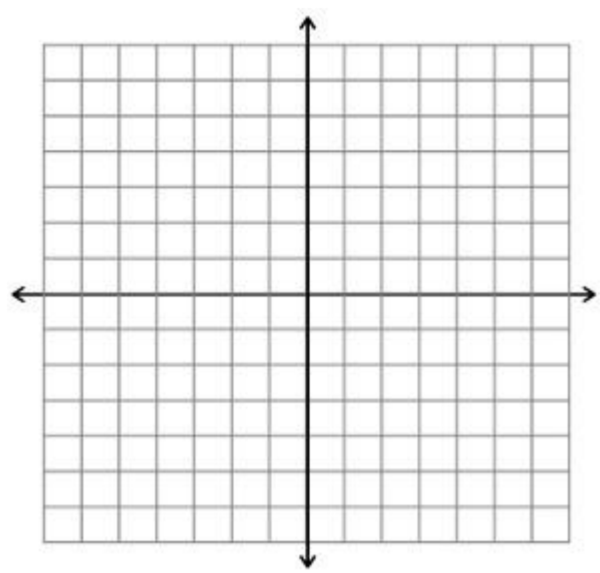
a. $f(x) = 3x^2$

How do you interpret the 3?



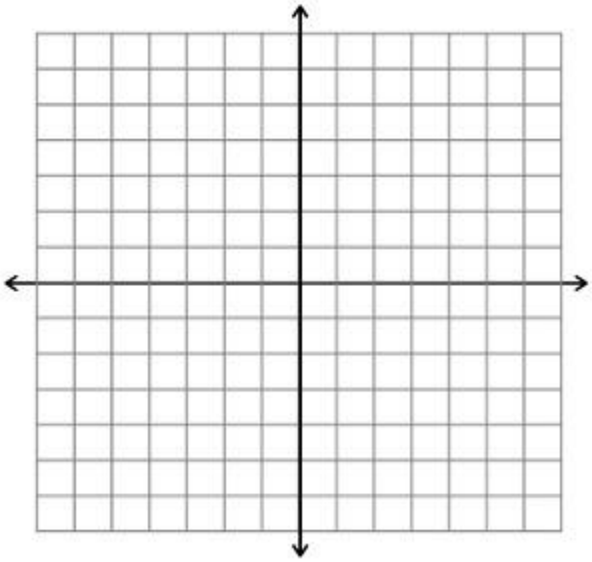
b. $g(x) = -\frac{1}{2}(x-2)^2 + 1$

How do you interpret the $-\frac{1}{2}$?

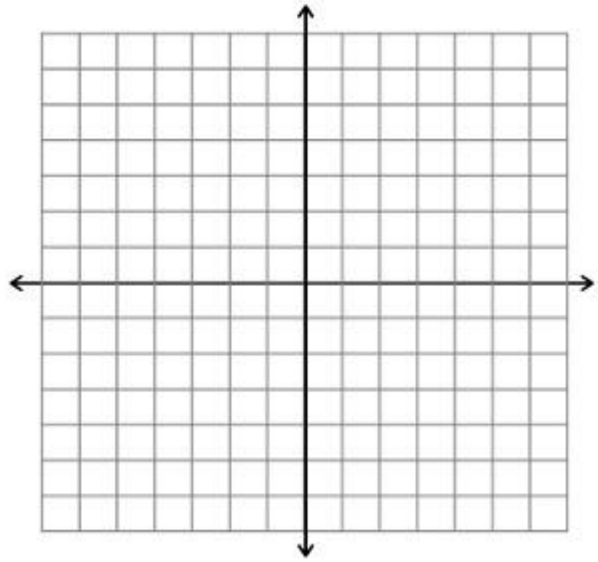


Examples Graph the following:

a. $f(x) = -2(x+1)^2 + 3$



b. $g(x) = \frac{1}{3}(x-2)^2 - 2$

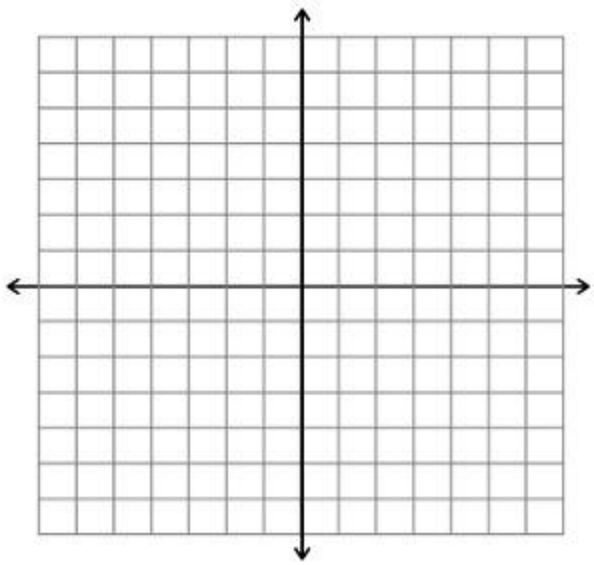


Using all of the examples we have discussed, what is another equation form for the parabola?

Explain some of the characteristics of a parabola that we get from this format:

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x-3)^2 - 4$



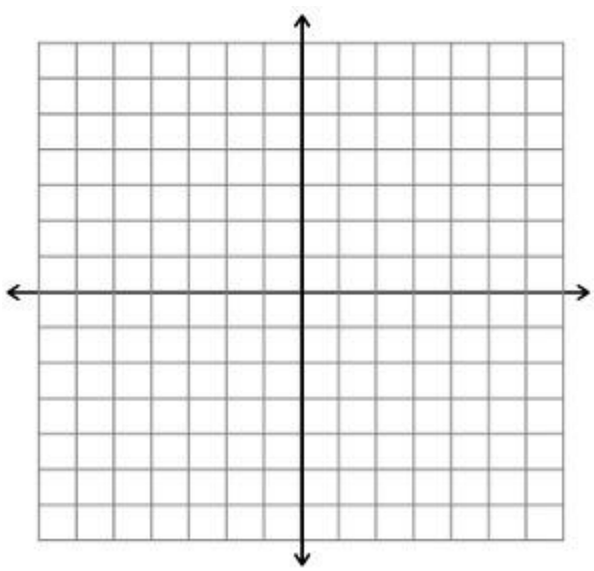
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2 + 8$



Vertex:

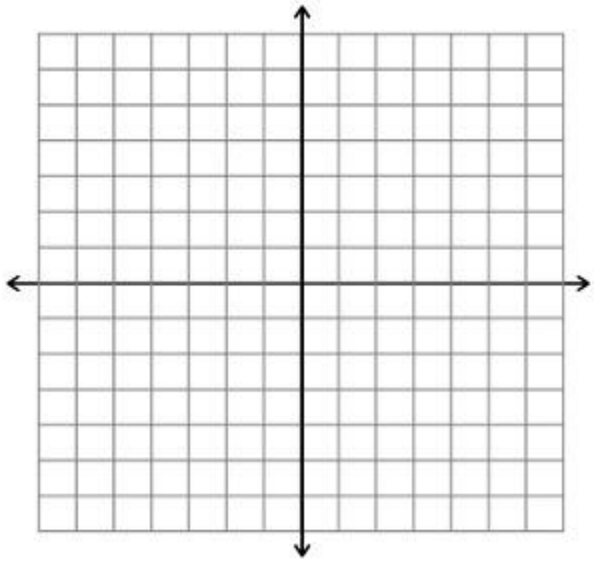
Axis of Symmetry:

X-intercepts:

Y-intercepts:

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{4}(x+2)^2 - 3$



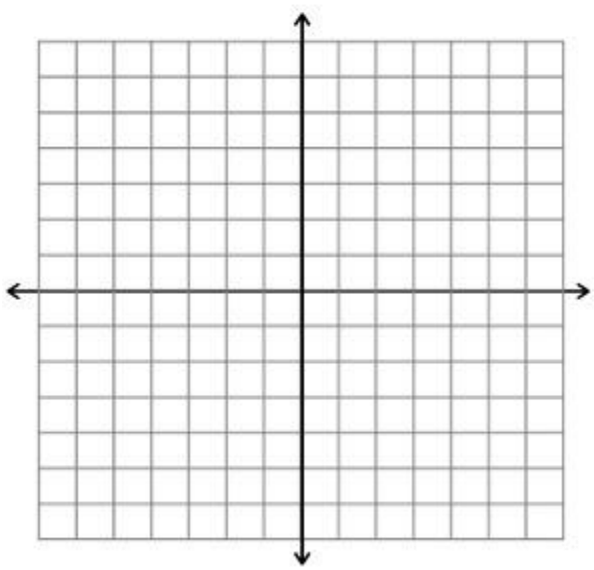
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -\frac{1}{3}(x-1)^2 + 3$



Vertex:

Axis of Symmetry:

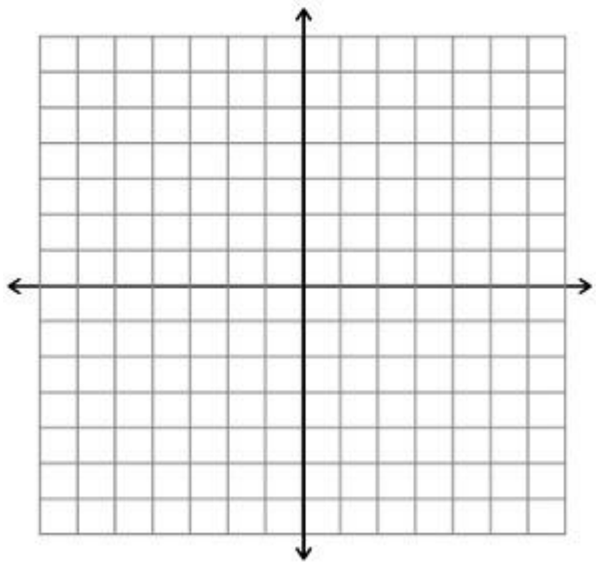
X-intercepts:

Y-intercepts:

Examples: Understanding How Many X-Intercepts There Are

__**Examples**__ For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{2}(x+1)^2 - 8$



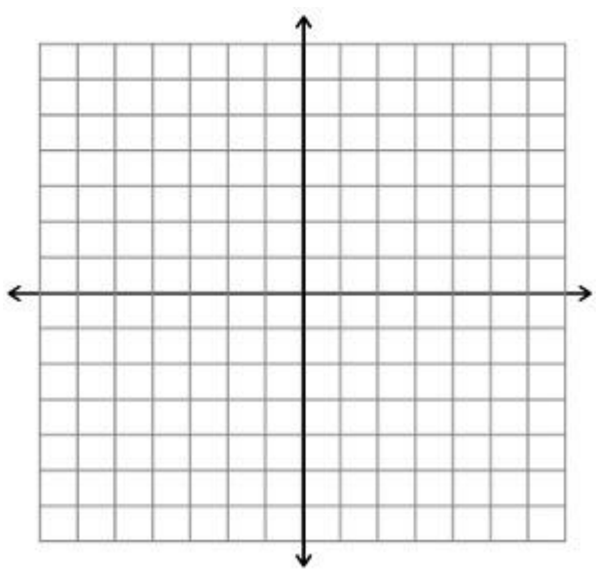
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2$



Vertex:

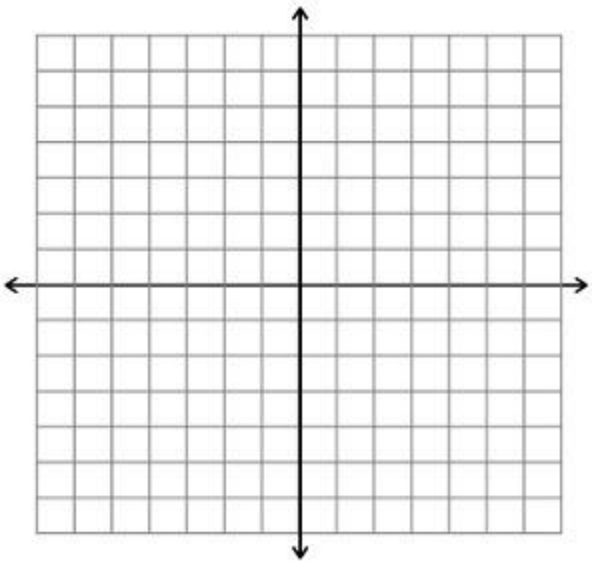
Axis of Symmetry:

X-intercepts:

Y-intercepts:

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x-2)^2 + 1$



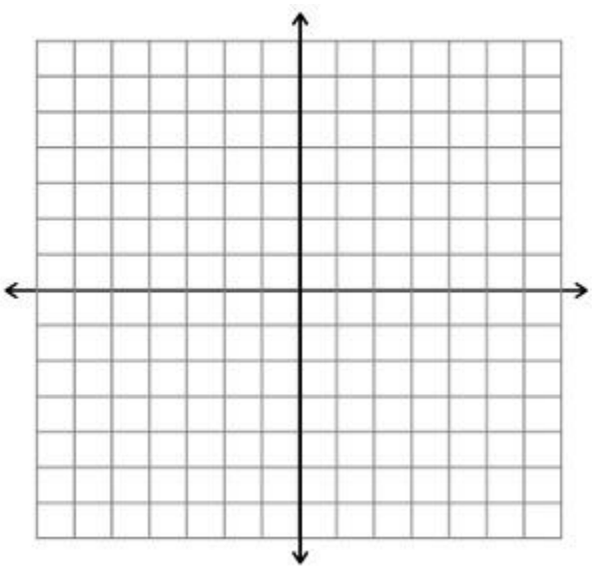
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -(x+3)^2 - 1$



Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

Lesson: Finding the Vertex of Quadratic Equations in the form $\boldsymbol{f(x) = ax^2 + bx + c}$

What is the vertex formula?

Show how to use the vertex formula for $f(x) = 3x^2 + 6x - 2$

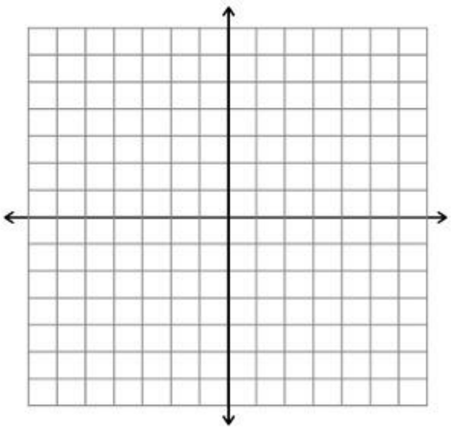
<u>Examples</u> Find the vertex of the following:

a. $f(x) = -x^2 + 4x - 2$

b. $f(x) = -\frac{1}{2}x^2 + 3x - 1$

Lesson: Putting Quadratics into the $\boldsymbol{f(x) = a(x-h)^2 + k}$ Form

Before you begin this lesson, you should already feel comfortable graphing an equation like $f(x) = 2(x-1)^2 + 3$. Graph this below:



Now show all the steps to put $f(x) = 2x^2 - 8x + 2$ into $f(x) = a(x-h)^2 + k$ form:

Examples Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = x^2 + 4x - 2$

b. $g(x) = x^2 + 6x + 10$

c. $h(x) = -x^2 + 2x - 4$

d. $y = 3x^2 + 6x - 12$

Examples: Putting Quadratics into the $\boldsymbol{f(x) = a(x-h)^2 + k}$ Form

Examples Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = \frac{1}{2}x^2 + x - 3$

b. $g(x) = -\frac{1}{3}x^2 + 4x - 2$

c. $y = x^2 + 3x + 2$

Lesson: Determining Whether a Quadratic Function will have a Minimum or a Maximum

Using the form $f(x) = ax^2 + bx + c$, explain what determines whether the parabola opens up or down:

<u>Examples</u> Determine whether the following quadratics open upwards or downwards, and then determine their vertex:

a. $f(x) = -2x^2 + 4x - 2$ upwards downwards

b. $g(x) = -\frac{1}{2}x^2 - 2x + 2$ upwards downwards

c. $h(x) = x^2 + 6x + 1$ upwards downwards

Lesson: Applications of Quadratic Functions 1

A ball is thrown upwards from a cliff 24 feet about the ground. The height h of the ball, in feet, t seconds after it is thrown is given by:

$$h(t) = -16t^2 + 32t + 24$$

How long does it take the ball to reach its maximum height?

What is the maximum height attained by the ball?

Simplifying Radicals
The Basics

We will look at how to simplify radicals like this:

$$\sqrt{50} = 5\sqrt{2} \quad \text{or} \quad \sqrt[3]{250} = 5\sqrt[3]{3}$$

You might want to have the list of squares / cubes / quartics ready for this exercise. If you don't, pause the video and write them all down on a separate sheet of paper so that you can have it handy for this entire lesson.

First we look at **the multiplication property of radicals:**

<u>**Examples**</u> Multiply the following:

a. $\sqrt[3]{2} * \sqrt[3]{3}$

b. $\sqrt{7} * \sqrt{5}$

c. $\sqrt[3]{3} * \sqrt[3]{9}$

d. $\sqrt[3]{6} * \sqrt[3]{9}$

How are examples c and d related?

Being able to multiply radicals together is powerful, because it also allows us to break apart radicals and simplify them. Using the example $\sqrt{12}$, write down how to simplify:

Examples Simplify the following:

a. $\sqrt{50}$

b. $\sqrt{20}$

c. $\sqrt[3]{54}$

d. $\sqrt[4]{32}$

e. $\sqrt[3]{24}$

At this point, we should probably define when a radical is actually simplified.

A simplified radical will ……

<u>Examples</u> Explain why the following radicals are not simplified:

a. $\sqrt{24}$

b. $\frac{5}{\sqrt{7}}$

c. $\sqrt[4]{\frac{3}{5}}$

d. $\sqrt[3]{8x^8y^2}$

Examples Simplify the following:

a. $\sqrt{24}$

b. $\sqrt[3]{32}$

c. $\sqrt[4]{64}$

d. $\sqrt[5]{64}$

e. $\sqrt{250}$

f. $\sqrt[3]{250}$

Lesson: Simplifying Radicals with Variables

We are going to rely on a skill we've already learned to help with simplifying radicals.

<u>Examples</u> Show you can simplify the following utilizing rational exponents.

a. $\sqrt{x^6}$

b. $\sqrt[3]{x^{15}}$

c. $\sqrt{x^6y^8}$

d. $\sqrt[4]{x^{12}y^{20}}$

How can we simplify $\sqrt{x^5}$? Watch the 2 explanations in the video and summarize the explanation that makes more sense to you.

Example Simplify the following.

a. $\sqrt[3]{x^{13}}$

b. $\sqrt[4]{x^{22}}$

c. $\sqrt{12x^6y^7}$

d. $\sqrt[3]{250x^8y^{16}}$

e. $\sqrt[4]{\frac{32x^7}{y^8}}$

Lesson: Multiplying Radicals

Write a one-sentence explanation of how to multiply $\sqrt[3]{3} * \sqrt[3]{4}$

<u>Examples</u> Multiply the following. Simplify if possible.

a. $\sqrt[3]{6} * \sqrt[3]{4}$

b. $\sqrt[4]{4x^3} * \sqrt[4]{12x^2}$

c. $\sqrt{5x} * \sqrt{10xy}$

Simplifying Radicals
Multiplying and Dividing Radicals with Different Indices

We will look at how to perform operations like:

$$\sqrt{x} * \sqrt[3]{x} \qquad \text{or} \qquad \frac{\sqrt[4]{x}}{\sqrt[3]{x}}$$

Before we begin, you should be comfortable with doing problems like this:

$8^{\frac{1}{3}}$

$25^{\frac{3}{2}}$

Write down the approach for performing this operation: $\sqrt{x} * \sqrt[3]{x}$

<u>Examples</u> Perform the following operations:

a. $\sqrt[3]{x} * \sqrt[4]{x}$

b. $\frac{\sqrt[5]{x^2}}{\sqrt[3]{x}}$

<u>Examples</u> Perform the following operations:

a. $\sqrt[4]{x^3} * \sqrt[5]{x^3}$

b. $\frac{\sqrt{x}}{\sqrt[3]{x^2}}$

c. $\sqrt[3]{x^2} * \sqrt{x}$

Adding and Subtracting Radicals

In this video we will look at how to perform operations like:

$$\sqrt{3} + \sqrt[3]{5} - 5\sqrt{3} + 7\sqrt[3]{3} \qquad \text{or} \qquad \sqrt{24x^4y^3} - 3x^2y\sqrt{6y}$$

Before you begin, you should already be able to simplify radicals like this:

a. $\sqrt{24x^6y^9}$

b. $\sqrt[4]{16x^7y^{11}}$

What are like terms?

Add the like terms in this problem: $3x + 7y - 9x + 7y^2 - 2x - 2y$

What are like radicals?

Add the like radicals in this problem: $5\sqrt{3} - 2\sqrt[3]{3} + 8\sqrt[3]{3} - 2\sqrt{5} + 8\sqrt{3}$

Examples Perform the following operations:

a. $4\sqrt{7}-5\sqrt[3]{3}+2\sqrt{3}-3\sqrt[3]{7}-9\sqrt{3}+\sqrt[3]{3}$

b. $2x\sqrt{7}-3\sqrt{7}+8x\sqrt{7}-2\sqrt{7}$

c. $3x\sqrt[3]{5}+2x\sqrt[3]{4}-3\sqrt[3]{4}+5x\sqrt[3]{4}-7x\sqrt[3]{5}+5\sqrt[3]{4}$

It is possible that you might have to simplify a radical before you can add it. Note how this works with the example:

$$\sqrt{24}+2\sqrt{54}$$

Examples Perform the following operations:

a. $5\sqrt{8}-\sqrt{18}$

b. $4\sqrt[3]{54}-7\sqrt[3]{16}$

Examples Perform the following operations:

a. $\sqrt{8x^2y^4} - 9xy^2\sqrt{2}$

b. $4x\sqrt[3]{16x^6y^7} + 9x^3y\sqrt[3]{250y^4}$

c. $5x^2y^3\sqrt[4]{32x^7z^{10}} - 2x^3z^2\sqrt[4]{2x^3y^{12}z^2}$

d. $3x\sqrt[3]{24x^5y^7} + 3xy\sqrt[3]{3x^2y} + y^2\sqrt[3]{375x^8y}$

Lesson: Multiplying Radicals Expressions

First, let's review how to multiply radicals in general. Show how to multiply:

a. $\sqrt[3]{4x^2y} * \sqrt[3]{6x^2y}$

b. $\sqrt{6xy} * \sqrt{10y}$

Now we will apply that logic to larger rational expressions.

Write a one-sentence explanation of how to multiply: $\sqrt{2}(\sqrt{3} + \sqrt{10})$, then multiply.

Example Multiply $2\sqrt[3]{3}(\sqrt[3]{9} + 5\sqrt[3]{6})$

Example Multiply $3x\sqrt{5}\left(\sqrt{10x}-5x\sqrt{2}\right)$

Write out an explanation of how to multiply: $\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{6}+\sqrt{3}\right)$

Example Multiply: $\left(3\sqrt{5}-\sqrt{2}\right)\left(\sqrt{10}+4\sqrt{2}\right)$

<u>Example</u> Multiply: $\left(3\sqrt[3]{4}-\sqrt[3]{6}\right)\left(5\sqrt[3]{2}+\sqrt[3]{6}\right)$

How do you multiply $\left(\sqrt{2}+\sqrt{3}\right)^2$

<u>Examples</u> Multiply: $\left(\sqrt[3]{9}+\sqrt[3]{6}\right)^2$

Lesson: Rationalizing a Denominator with One Square Root

What does it mean to rationalize the denominator?

How would you rationalize $\frac{2}{\sqrt{3}}$? Explain *why* we can do this.

Can every fraction with a radical in the denominator be rationalized like this? Circle: Yes No

What types of problems can be rationalized this way?

Examples Rationalize the following.

a. $\frac{20}{\sqrt{8}}$

b. $\sqrt{\frac{40}{60}}$

c. $\sqrt{\frac{81x^7}{2y}}$

d. $\frac{\sqrt{22}}{\sqrt{x^9}}$

Dividing Radicals/Rationalizing the Denominator
Part 2: Higher Roots

In this video we will look at how to simplify expressions like:

$$\frac{5}{\sqrt[3]{3}} \quad \text{or} \quad \frac{\sqrt[4]{2}}{\sqrt[4]{9}}$$

Before you begin, you should already be able to simplify problems like this:

a. $\frac{5}{\sqrt{3}}$

b. $\frac{\sqrt{6}}{\sqrt{18}}$

First, let's observe why higher roots are a different situation. Write down what happens if we use the same methods as above to rationalize the following: $\frac{5}{\sqrt[3]{3}}$

We have to pivot our thinking to "count powers". Note how this works with this examples: $\sqrt[3]{3} * \_\_\_ = 3$

Examples Determine what to fill in the blank to create a true statement:

a. $\sqrt[3]{25} * ? = 5$

b. $\sqrt[4]{2} * ? = 2$

c. $\sqrt[4]{27} * ? = 3$

d. $\sqrt[5]{4} * ? = 2$

e. $\sqrt[3]{7} * ? = 7$

f. $\sqrt[4]{25} * ? = 5$

g. $\sqrt[5]{3} * ? = 3$

h. $\sqrt[3]{36} * ? = 6$

Now you should have a sense of what number you want this to equal. This is a very strange exercise, but using the pattern above, determine what to multiply by to get to the desired root:

a. $\sqrt[3]{4} * ? =$

b. $\sqrt[4]{5} * ? =$

c. $\sqrt[5]{9} * ? =$

d. $\sqrt[4]{36} * ? =$

Now we should have a decent sense of how to rationalize and what the answer should equal. Explain how to rationalize the following:

a. $\frac{5}{\sqrt[3]{2}}$

b. $\frac{7}{\sqrt[4]{5}}$

Examples Rationalize the following denominators:

a. $\frac{6}{\sqrt[3]{5}}$

b. $\frac{2}{\sqrt[4]{9}}$

c. $\frac{2}{\sqrt[5]{125}}$

d. $\frac{7}{\sqrt[4]{8}}$

Examples Rationalize the following denominators:

a. $\frac{\sqrt[3]{5}}{\sqrt[3]{4}}$

b. $\frac{2\sqrt[3]{3}}{\sqrt[3]{4}}$

c. $\frac{5\sqrt[4]{2}}{\sqrt[4]{27}}$

d. $\frac{6\sqrt[3]{5}}{\sqrt[3]{3}}$

e. $\frac{3}{\sqrt[5]{x^3}}$

f. $\frac{\sqrt[5]{25}}{\sqrt[5]{16}}$

Lesson: Rationalizing a Denominator with Two Square Roots

How do you find the conjugate of $4+\sqrt{2}$? Write one sentence to explain what you need to do.

<u>Examples</u> What is the conjugate of the following?

a. $3-\sqrt{5}$

b. $-4-\sqrt{2}$

Show how to rationalize: $\frac{5}{\sqrt{2}-\sqrt{3}}$

<u>Examples</u> Rationalize the following.

a. $\frac{\sqrt{2}}{\sqrt{3}-\sqrt{6}}$

b. $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}+\sqrt{5}}$

c. $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

Solving Quadratic Equations using the Square Root Property

In this video, we will look at how to solve equations by using the square root property, which looks like this:

$$(x+2)^2 = 9 \quad \rightarrow \quad x+2 = \pm 3$$

Quadratic equations are equations of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

What are the solutions to $x^2 = 25$?

In general, how do you solve equations of the form $Q^2 = k$, where Q is an expression and k is a constant? In other words, what is the **square root property**?

<u>Examples</u> Solve the following equations using the square root property:

a. $(x+3)^2 = 25$

b. $(x+3)^2 = 12$

Examples Solve the following equations using the square root property:

a. $(5x-2)^2+5=29$

b. $(6x+7)^2+4=-41$

Solving Quadratic Equations by Completing the Square

In this video, we will look at how to solve equations by factoring, which looks like this:

$x^2 + 6x = 7 \qquad b = 6, \quad \frac{1}{2}b = 3, \quad \left(\frac{1}{2}b\right)^2 = 9$

Quadratic equations are equations of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

Before we begin, you should be familiar with the square root property. Solve $(2x+1)^2 = 49$ using the square root property.

First let's talk about how to complete the square. Show how to complete the square on $x^2 + 6x$ below:

Examples Complete the square on the following:

a. $x^2 + 8x$

b. $x^2 - 10x$

c. $x^2 + 3x$

d. $x^2 - \frac{1}{2}x$

e. $x^2 + \frac{2}{3}x$

Show how to solve $2x^2 + 12x - 16 = 0$ by completing the square:

Examples Solve the following by completing the square:

a. $x^2 + 4x - 6 = 0$

b. $x^2 + 5 = 2x$

Examples Solve the following by completing the square:

a. $3x^2 + 15x = -6$

b. $\frac{1}{2}x^2 + 3x - 2 = 0$

Examples: The Distance Formula

Find the distance between the following sets of points.

a. (6, 2) and (2, 11)

b. (3, -1) and (-4, -2)

Solving Quadratic Equations by the Quadratic Formula

Complete the square on: $ax^2 + bx + c = 0$

State the quadratic formula:

<u>Examples</u> Use the quadratic formula to solve the following:
a. $x^2 + 3x + 2 = 0$

b. $2x^2 + 3x = 5$

c. $\frac{1}{2}x^2 + \frac{1}{3}x + \frac{5}{6} = 0$

Lesson: The Discriminant

Restate the Quadratic Formula, and then label the discriminant.

Value of the Discriminant	Interpretation

Examples Find the values of the discriminant for the following equations. Then solve the equations (using the most efficient method) to determine if this makes sense.

a. $x^2 - 4x + 4 = 0$

b. $x^2 + 6x + 5 = 0$

c. $3x^2 + 2x + 5 = 0$

Choosing the Best Method to Solve a Quadratic

There are 4 ways we can solve quadratics:

1.	3.
2.	4.

Using each of these methods only once, determine the best way to solve these quadratics:

a. $x^2 + 6x = 2$
b. $x^2 + 3 = 2x$
c. $(2x + 1)^2 = 49$
d. $3x^2 + 2 = 3x$

Examples Using each of these methods only once, determine the best way to solve these quadratics:
a. $(3x-2)^2=-24$
b. $2x^2+5x-1=0$
c. $x^2-12=x$
d. $x^2-2x+5=0$

Solving Equations that End Up Having Quadratics

In this lesson, we will review some techniques from previous lessons.

Solve $\frac{2}{x} - \frac{1}{x+1} = \frac{2}{3}$

Solve $\sqrt{2x+8} = x + 4$

Examples Solve the following:

a. $\frac{4}{x-2}+\frac{x}{x-3}=\frac{2}{x-3}$

b. $\sqrt{3x+5}=x-3$

Solving Equations that Are Similar to Quadratics

In this lesson, we will look at how to solve equations that look like this:

$$x^{\frac{2}{3}} - x^{\frac{1}{3}} - 12 = 0 \qquad \text{or} \qquad (2x+3)^2 + 5(2x+3) + 6 = 0$$

Before we begin, what do the following equations have in common if you **look at their exponents**?

a. $x^2 + 5x + 6 = 0$

b. $x^{\frac{2}{3}} + 5x^{\frac{1}{3}} + 6 = 0$

c. $x + 5x^{\frac{1}{2}} + 6 = 0$

d. $x^4 + 5x^2 + 6 = 0$

Explain how to solve $x^4 - 13x^2 + 36 = 0$

Examples Solve the following:

a. $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 15 = 0$

b. $x - 2x^{\frac{1}{2}} - 8 = 0$

d. $x^{-2} + 8x^{-1} - 9 = 0$

Examples Solve the following:
a. $(x+3)^2-4(x+3)-5=0$

b. $(2x+1)^2-4(2x+1)-21=0$

Solving Equations with Radicals
Part 1: One Square Root in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt{3x+1} = 2 \qquad \text{or} \qquad \sqrt{4x-2} = 5 - x$$

Use the example $\sqrt{3x+1} = 4$ to show the general procedure for solving these types of equations:

Why must we check the solutions?

<u>Example</u> Solve $\sqrt{3x+1} = 8$

<u>Examples</u> Solve

a. $\sqrt{5x-4}+2=8$

b. $\sqrt{1+3x}+5=2$

Explain what is different about solving $\sqrt{5x+4}=x+2$

Examples Solve the following.
a. $\sqrt{3x-6} = x-2$

b. $\sqrt{2x+4} = x-2$

c. $\sqrt{5x+1} + 5 = 3x$

Solving Equations with Radicals
Part 2: Two Square Roots in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt{4x-2} = 3\sqrt{2x+6} \qquad \text{or} \qquad \sqrt{3x-1} + 2 = \sqrt{4x-2}$$

Let's begin by explaining how to solve: $\sqrt{5x+2} = \sqrt{3x+6}$

Examples Solve:
a. $\sqrt{3-2x} = \sqrt{2x+7}$

b. $\sqrt{5+3x} = \sqrt{3+2x}$

c. $4\sqrt{3-2x} = 2\sqrt{1+3x}$

Example Solve: $6\sqrt{x} = 4\sqrt{2x+1}$

Now write out the detailed explanation of how to solve $\sqrt{4x+1} - 2 = \sqrt{2x+1}$

Examples Solve:

a. $\sqrt{3x+6} = 2 + \sqrt{2x-4}$

b. $\sqrt{3x+1} = 2 + \sqrt{2x-2}$

Examples Solve:

a. $\sqrt{2x+4}+\sqrt{1-2x}=3$

b. $\sqrt{2x+4}=2+\sqrt{2x+3}$

Solving Equations with Radicals
Part 3: Two Cube Roots of Higher in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt[3]{3x+1} = 2 \qquad \text{or} \qquad \sqrt[4]{2x-1} = 5$$

Explain how to solve: $\sqrt[3]{3x+1} = 2$

Explain how to solve: $\sqrt[4]{3x+4} = 2$

When is it more likely a solution will not work?

Examples Solve the following:

a. $\sqrt[3]{4x+7}=3$

b. $\sqrt[3]{3-5x}=2$

c. $\sqrt[4]{3-2x}=3$

d. $\sqrt[4]{4x-1}=-7$

Lesson: Determining if Functions are One-to-One

Before we begin, you should already be familiar with the concept of functions and the vertical line test. First let's do some review.

<u>Review</u> Determine whether the following relations are functions, then determine the domain and range.

a. $\{(2,3),(5,7),(2,4)\}$

b.

Now we will discuss a specific type of function: a one-to-one function. Define this below:

How can we visualize this?

Example Determine if any of the following are i) functions and ii) one-to-one functions

a. $\{(2,4),(-1,7),(3,7)\}$

b. $\{(3,6),(-1,2),(-3,-4)\}$

Similar to how we have a test to determine if a graph represents a function, we have another test to determine if a function is one-to-one.

Describe the horizontal line test below:

Examples Determine whether the following graphs represent i) functions and ii) one-to-one functions (draw your own):

a.

b.

c.

Lesson: Finding the Inverse of Functions

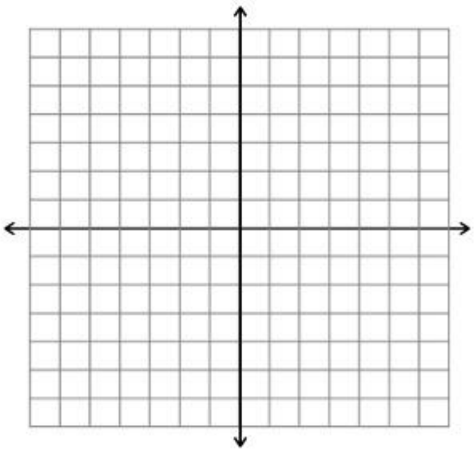
One-to-one functions have inverses. Describe what the inverse of a function is:

Example Find the inverse of the following function: $\{(1, 4), (7, 8), (3, -2)\}$

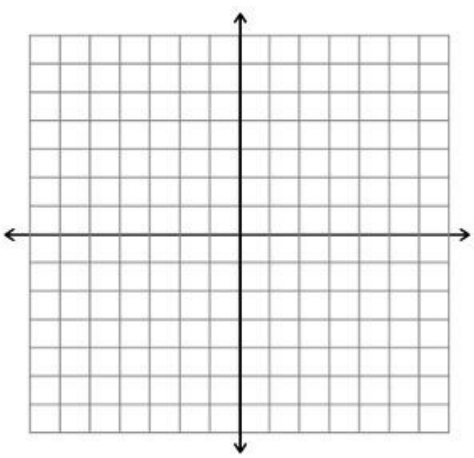
Describe the steps to find the inverse of a function:

Examples Find the inverse of the following one-to-one functions. Then plot the original function on the graph with its inverse.

a. $f(x) = 3x + 1$



b. $g(x) = -\frac{1}{2}x + 4$



What is the relationship of a graph with its inverse?

<u>Examples</u> Find the inverse of the following one-to-one functions.

a. $h(x) = \sqrt[3]{x}$

b. $f(x) = x^2, x \geq 0$

Not included (no notes file):

037 (Examples of Solving Absolute Value Equations)

042 (Examples on Absolute Value Inequalities)