Lesson: An Intro to Functions

Define a relation:

Define a domain:

Define a range:

<u>Example</u> In the set of ordered pairs $\{(2,3), (4, -1), (7, 2)\}$, define the domain and range of the relation.

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Define a function:

What is the visual explanation that explains what a function is or is not?

<u>Example</u> In the set of ordered pairs $\{(2,3), (4, -1), (7, 2)\}$, is this a function?

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Example Are the following functions?

a. $\{(4,-1),(3,2),(7,-1)\}$

b. $\{(2,4),(3,5),(2,-1)\}$

What is the notation for a function? How do we describe/use this notation?

Example In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, what is $f(4)$?

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<u>Example</u> Suppose $f(x) = 2x + 1$ and $g(x) = x^2 - 3$. Find the value of the following:

a. $f(2)$

b. $g(1)$

c. $f(a)$

d. $g(h + k)$

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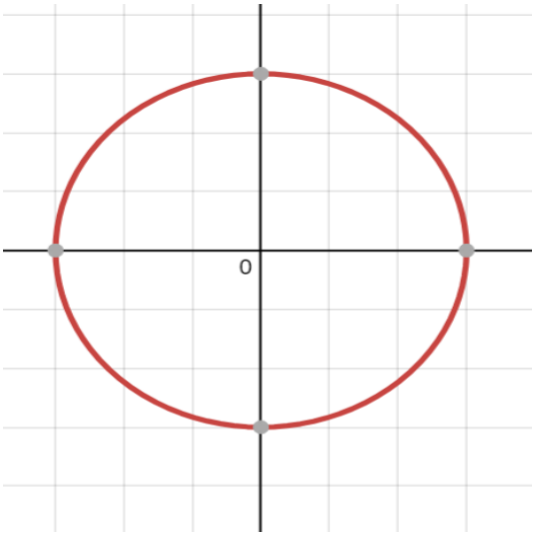
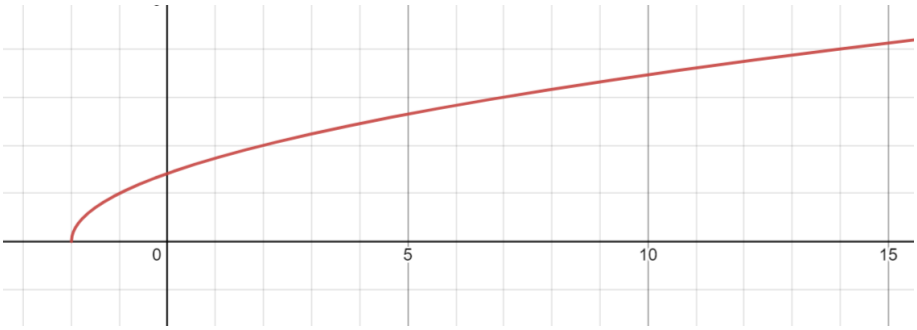
What is the vertical line test?

<u>Example</u> Do the following graphs represent a function?

a.



b.



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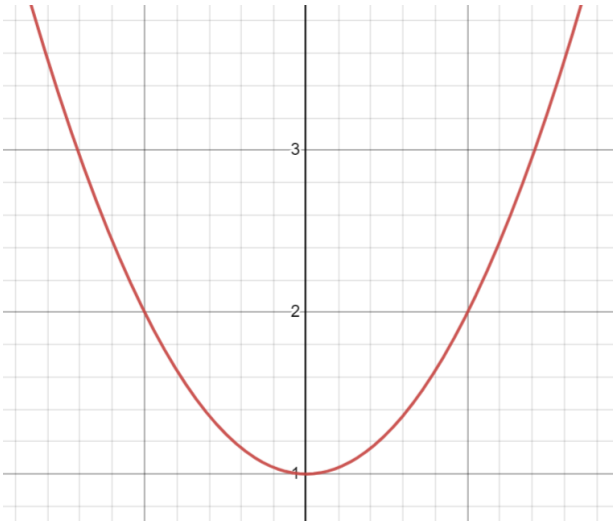
Lesson: Determining the Domain and Range of Functions In General

How do you determine the domain and range of a set of points like $\{(2,4), (5,8), (4, 6)\}$?

What is the domain and range of that set of points?

In general, it is easiest to determine the domain and range when we have the graph in front of us.

For the following graph, write out how you should determine the domain and range.



Write a sentence on how you should find the domain:

State the domain:

Write a sentence on how you should find the range:

State the range:

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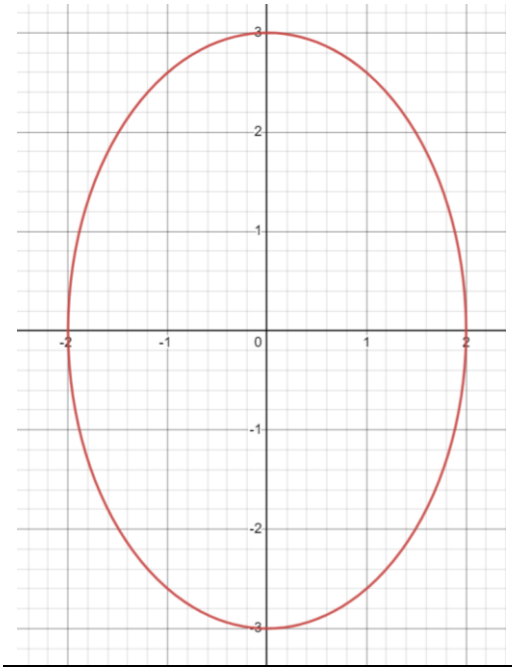
Examples Determine if the following graphs represent functions, and then state the domain and range.

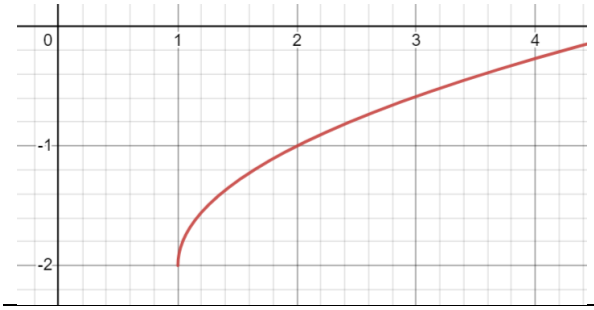
a.



Domain:

Range:

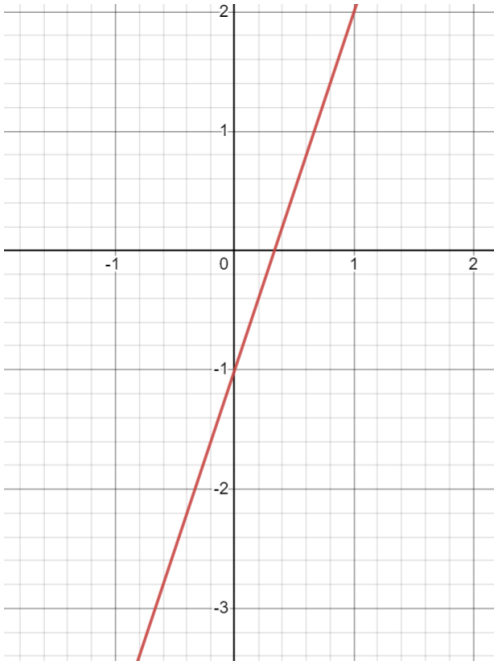
b.



Domain:

Range:

c.



Domain:

Range

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Lesson: The Domain of Some Specific Types of Functions

What is a polynomial? Write some examples.

What is the domain of any polynomial function?

What is a rational function?

What is the domain of any rational function?

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Examples Determine the domain of the following.

a. $f(x) = 3x - 1$

b. $g(x) = \frac{4}{x+2}$

c. $h(x) = 3x^3 - 2x^2 + 4x - 1$

d. $f(x) = \frac{2x+1}{x^2-9}$

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What is a square root function?

What is the domain of a square root function?

<u>Examples</u> Find the domain of the following:

a. $f(x) = \sqrt{x - 2}$

b. $g(x) = \sqrt{4x - 1}$

c. $h(x) = \sqrt{4 - x}$

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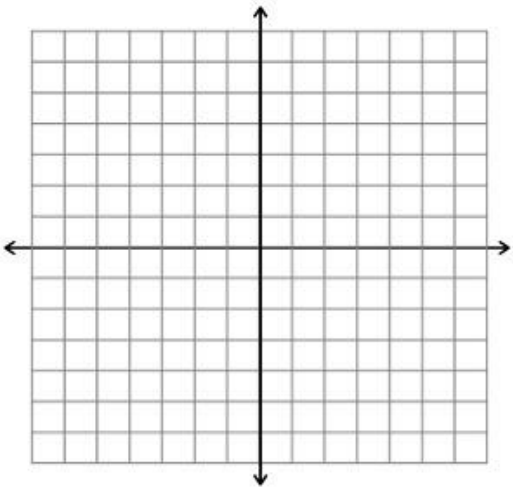
Lesson: How to Graph a Function

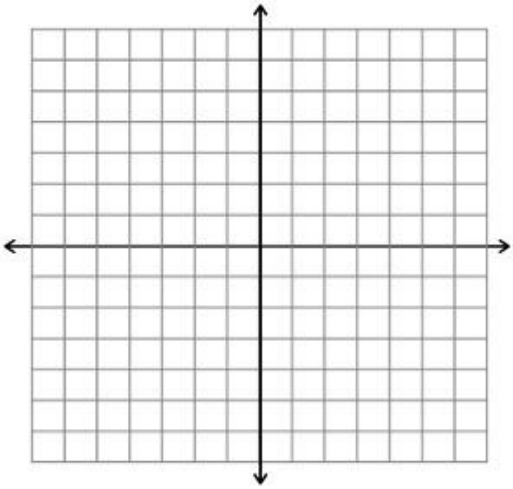
How do you graph a function?

<u>Examples</u> Graph the following functions.

a. $f(x) = 3x + 1$



b. $g(x) = 4$



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Lesson: Review of Functions

How do you represent a function?

What is a function often interchangeable with? How can you rewrite $f(x) = 3x + 8$?

So what IS a function? What does it actually represent?

What's another handy feature of functions?

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What are the domain and range?

Does domain and range only work for functions?

<u>Examples</u> Are the following functions, and what is the domain and range?

a. $\{(2,3), (5, -1), (-3, 7)\}$

b. $\{(-1, 5), (-8, 3), (-1, 6)\}$

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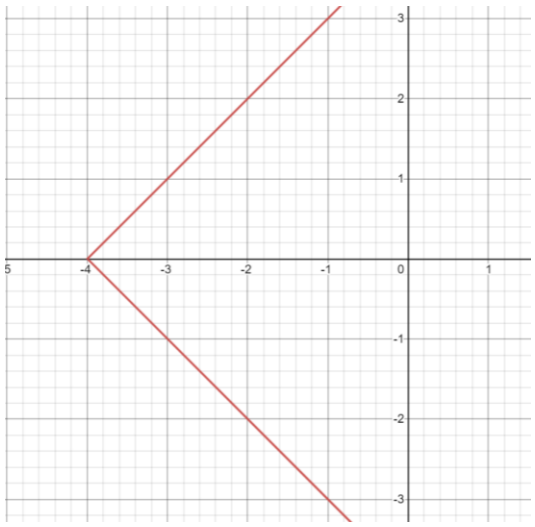
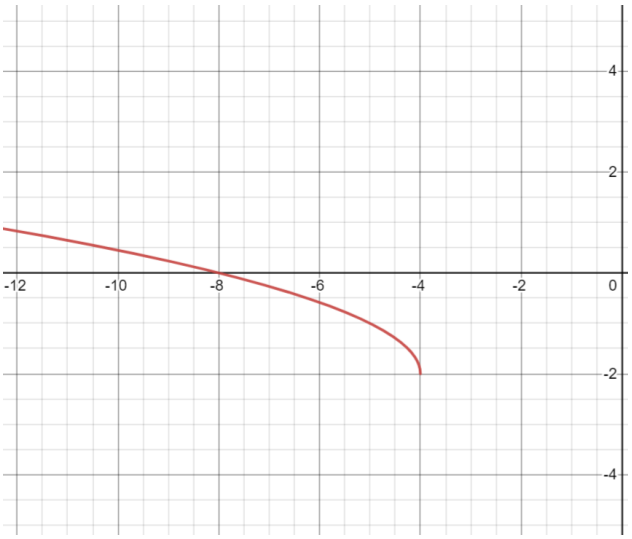
What is the Vertical Line Test, and how does it help you determine if you have a function?

<u>Example</u> State the domain and range of the following, and use the vertical line test to determine if the graph represents a function.

a.



b.



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Examples Determine if the following represent functions.

a. $y = 3x^2$

b. $x = y^2$

c. $y = \frac{5}{x^2}$

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Example Write the following in function notation: $3x + 2y = 6$

Example Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x - 1$ and $h(x) = \{(2,5),(6,-3),(-1,-4)\}$. Find the following:

a. $f(7) =$

b. $g(3) =$

c. $f(k) =$

d. $g(x + h) =$

e. $h(6) =$

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Lesson: Increasing, Decreasing, and Constant Functions

Below are the "formal math" definitions of whether a function is increasing, decreasing, or constant. Let I be an open interval and let x_1 and x_2 be two points in that interval.

- A function is **increasing** over I if when $x_1 < x_2, f(x)_1 < f(x_2)$
- A function is **decreasing** over I if when $x_1 < x_2, f(x)_1 > f(x_2)$
- A function is **constant** over I if when $x_1 < x_2, f(x)_1 = f(x_2)$

While these are the formal definitions, what we really want to know are the intuitive meanings of these definition.

Sketch a graph that is increasing:

Decreasing:

And constant:

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Example Sketch the graph in the video, and then identify where the graph is increasing, decreasing, or constant.

a.

b.

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Lesson: Graphs of Basic Functions

In this section, we will review some basic graphs that are vital to know for this class and calculus. Knowing the "basic shapes" of certain graphs will make your life much easier and give you greater intuition when graphing.

Show hot to graph the following and highlight the **key points** and **key patterns.** This will come in handy for the next lesson. Take your time to sketch your own NEAT CLEAN CLEAR graphs.

$f(x) = x$

$f(x) = x^2$

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$f(x) = x^3$

$f(x) = \sqrt{x}$

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$f(x) = \sqrt[3]{x}$

$f(x) = |x|$

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Lesson: Piecewise Functions

What is a piecewise function? Draw an example of what they look like.

<u>Example</u> Given the function: $f(x) = \begin{cases} 3x - 1, & x \geq 2 \\ \frac{1}{2}x - 4, & x < 2 \end{cases}$, evaluate the following:

a. $f(3) =$

b. $f(-1) =$

c. $f(2) =$

<u>Example</u> Given the function: $f(x) = \begin{cases} 4, & x > 3 \\ 4x - 2, & -2 < x \leq 3 \\ x^2 - 5, & x \leq -2 \end{cases}$ evaluate the following:

a. $f(4) =$

b. $f(-5) =$

c. $f(-2) =$

d. $f(3) =$

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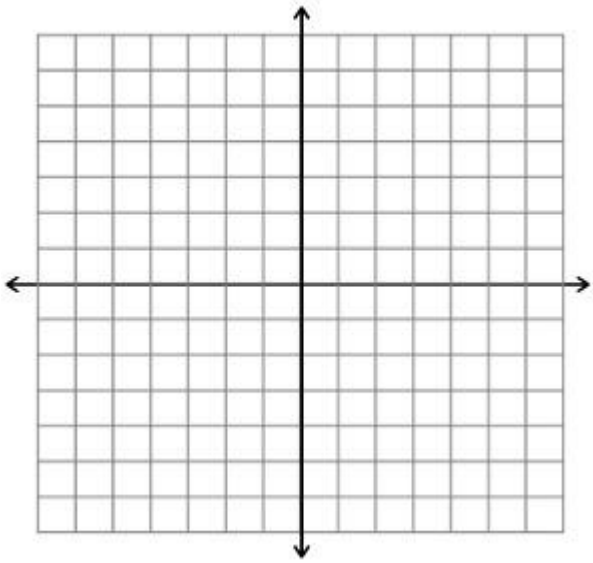
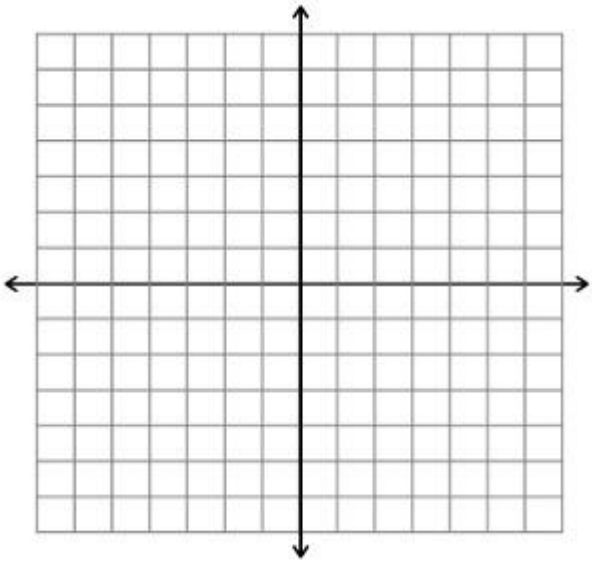
Example Show how to graph the following piecewise functions.

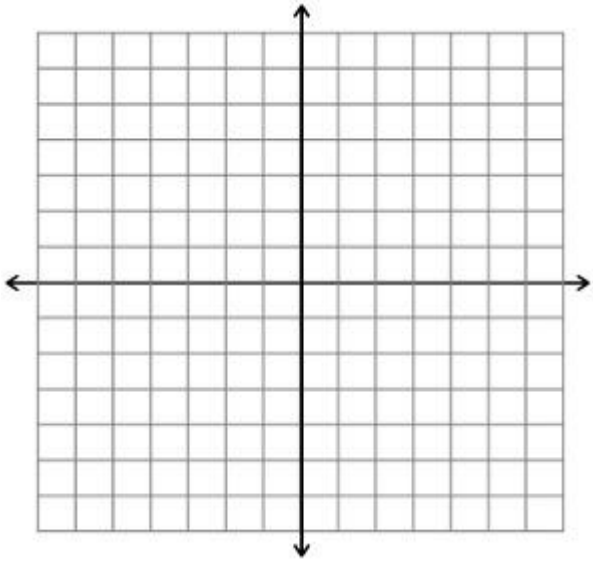
a. $f(x) = \begin{cases} 4, & x > 5 \\ -1, & x \leq 5 \end{cases}$



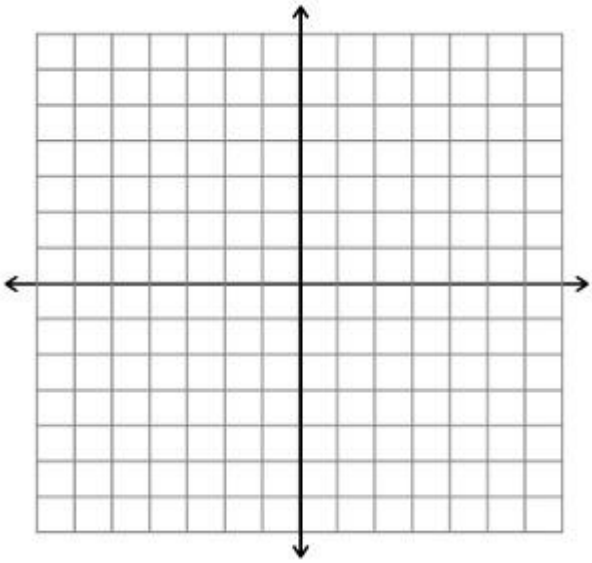
b. $f(x) = \begin{cases} 3x + 1, & x > -3 \\ -2x, & x \leq -3 \end{cases}$



c. $f(x) = \begin{cases} 3, & x > 2 \\ 2x, & -1 < x \leq 2 \\ -x + 3, & x \leq -1 \end{cases}$



d. $f(x) = \begin{cases} 3x + 1, & x \geq 1 \\ 6, & -1 \leq x < 1 \\ x^2, & x < -1 \end{cases}$



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Lesson: The Greatest Integer Function

Show how to graph the greatest integer function $f(x) = [\![x]\!]$

<u>Example</u> Graph $f(x) = \left[\!\left[\frac{1}{2}x - 2\right]\!\right]$

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Examples: Finding Average Rates of Change

Calculate the average rate of change of the functions over the given intervals.

a. $f(x) = x^2 + 3x; [0, 3]$

b. $f(x) = 2 + \sin x; \left[\frac{\pi}{2},\ \pi\right]$

c. $f(x) = \sqrt{4x + 1}; [0, 2]$

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Examples: Determining the Slope at a Point and the Tangent Line

Find the slope of the curve at the given point and find the equation of the tangent line at that point. Do this only using the formula for the slope of the secant line.

a. $f(x) = x^2 - 1, (2, 3)$

b. $f(x) = x^2 - 4x; (2, -4)$

c. $y = \frac{1}{x}; (2, \frac{1}{2})$

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Lesson: An Intro to Limits

Graph $f(x) = \frac{(x^2-4)}{x-2}$ below. What does it look like this graph should equal when x=2?

Explain this previous question now in mathematical terms

<u>Example</u> Find $\lim_{x \to 1} f(x)$ when $f(x) = \begin{cases} x^2 + 1 & x \neq 1 \\ -2 & x = 1 \end{cases}$. Draw a picture of this situation.

What is the value of $f(1)$?

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Example What is $\lim_{x \to 2} f(x)$ when $f(x) = x$? Draw a graph to accompany your answer.

What do we call the function $f(x) = x$?

What can we say about the limit of these types of functions?

Example What is $\lim_{x \to -3} f(x)$ when $f(x) = 4$? Draw a graph to accompany your answer.

What do we call the function $f(x) = c$, where c is a constant?

What can we say about the limit of these types of functions?

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Write out all the limit laws on this space:

Write out the other 2 handy theorems:

Theorem 1: Limits of Polynomials

Theorem 2: Limits of Rational Functions

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Example Using the limit laws, calculate the following limits:

a. $\lim_{x \to 3}(2x + 1)$

b. $\lim_{x \to 1} \frac{5x+2}{x-4}$

c. $\lim_{x \to 2} \sqrt{5x + 4}$

d. $\lim_{x \to -3} 5\sqrt{2 - 3x}$

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Examples: Intro to Calculating Limits

Calculate the following limits:

a. $\lim_{x \to 3}(x^2 + 2x + 1)$

b. $\lim_{x \to 0}(8 - 2x)^{\frac{2}{3}}$

c. $\lim_{x \to \frac{2}{5}}(5x + 2)(3x - 1)$

d. $\lim_{x \to \frac{1}{3}} \sqrt[3]{3x + 7}$

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Lesson: Review of Function Operations

Given that f and g are functions, how do we define the function operations?

$(f+g)(x) =$

$(f-g)(x) =$

$(fg)(x) =$

$\left(\frac{f}{g}\right)(x) =$

<u>Example</u> Let $f(x) = 3x+1, g(x) = x^2-4, h(x) = 3x^2+7x+2$, and $k(x) = x^2+6x+9$. Find each of the following:

a. $(f+g)(x) =$

b. $(f+g)(3) =$

c. $(g-k)(1) =$

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<u>Example</u> Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = 3x^2 + 7x + 2$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $\left(\frac{h}{f}\right)(x) =$

b. $(gh)(x) =$

c. $\left(\frac{k}{h}\right)(1) =$

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Lesson: Review of Composition of Functions

For two functions f and g, what is the composition $(f \circ g)(x)$?

<u>Example</u> Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = \sqrt{x}$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $(f \circ g)(x) =$

b. $(g \circ f)(x) =$

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<u>Example</u> Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = \sqrt{x}$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $(k \circ f)(1) =$

b. $(g \circ h)(4) =$

c. $(h \circ k)(x) =$

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Lesson: Translations and Reflections of Functions

Using the graph $f(x) = |x|$ as the "base function," sketch how vertical translations work (summarize what was discussed in the video).

How do you describe vertical translations in function notation?

Using the graph $f(x) = x^2$ as the "base function," sketch how horizontal translations work (summarize what was discussed in the video).

How do you describe horizontal translations in function notation?

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Use the graph $f(x) = \sqrt{x}$ to show how reflections over the x- and y-axis work (summarize what was discussed in the video).

How do you describe reflections over the x-axis and y-axis using function notation.

Use the graph $f(x) = |x|$ to show how stretching and shrinking works (summarize what was discussed in the video).

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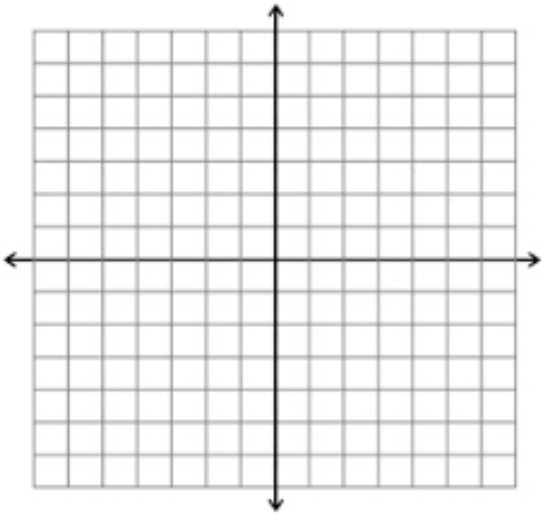
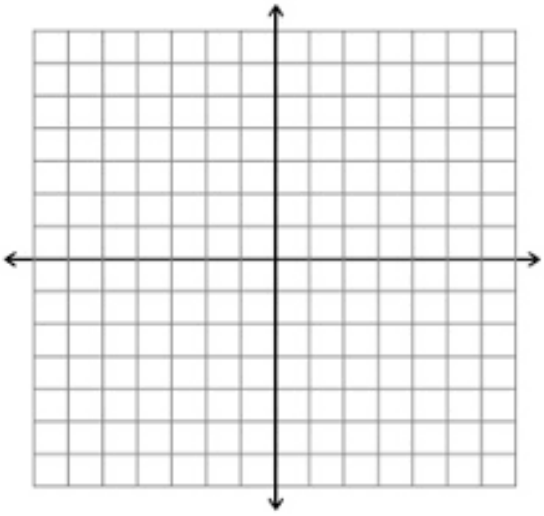
Explain how stretching and shrinking works with function notation.

<u>Example</u> Describe any translations / reflections / stretching / shrinking and sketch a graph of each of the following.

a. $f(x) = -x^2 + 3$



b. $g(x) = (x + 2)^3 - 3$



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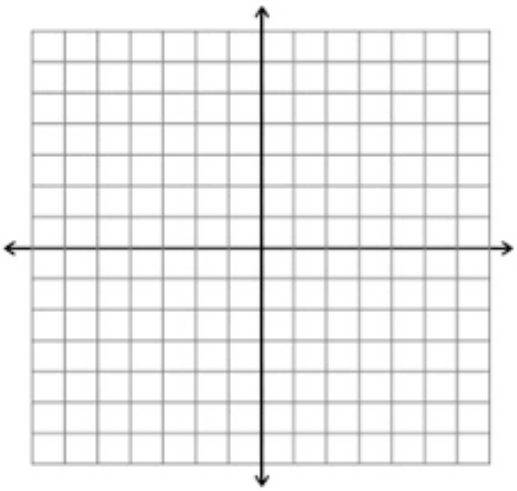
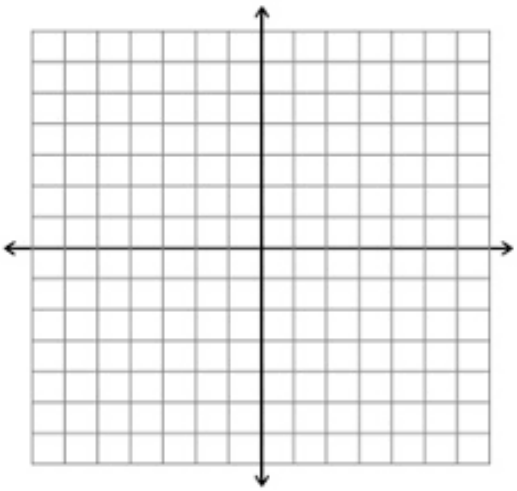
Example Describe any translations / reflections / stretching / shrinking and sketch a graph of each of the following.

a. $g(x) = -2(x+1)^2 - 5$



b. $h(x) = 2\sqrt{-x} + 1$



Examples Write the equation of the given functions with the following characteristics.

a. The function $f(x) = x^2$ moved 3 units left and 5 units down.

b. The function $g(x) = \sqrt[3]{x}$ reflected over the y-axis, translated 2.5 units right and 1.75 units up.

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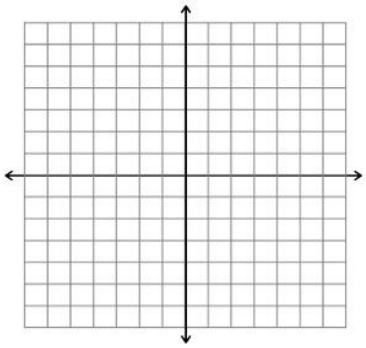
Transformations of Functions

These are looking at graphs of things like:

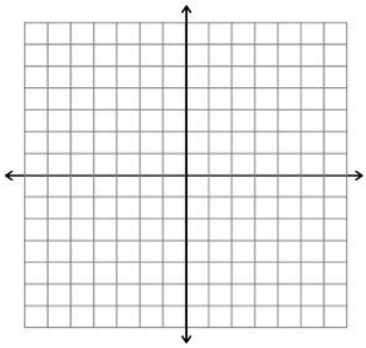
$$f(x) = x^2 - 5 \quad \text{or} \quad f(x) = |x+3| - 2 \quad \text{or} \quad f(x) = \sqrt{x-3}$$

First we need to discuss the shape of a few basic functions. Write these down and note the important points.

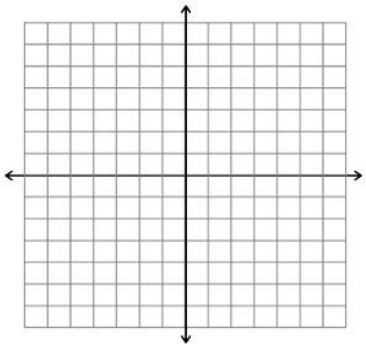
a. $f(x) = x^2$



b. $f(x) = |x|$



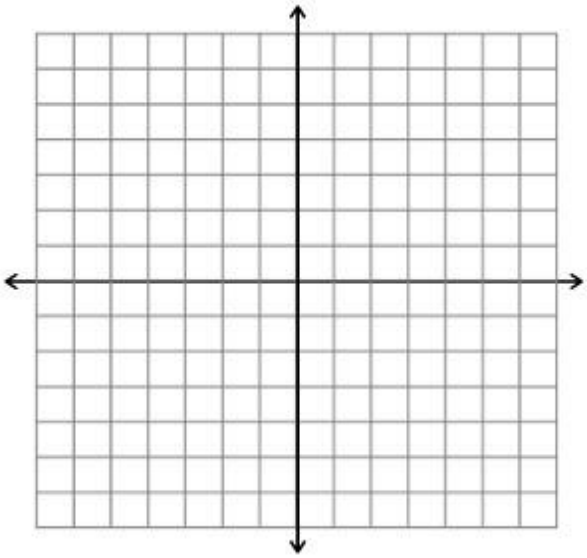
c. $f(x) = \sqrt{x}$



Using a few different colors, sketch out what the graphs of

a. $f(x) = x^2$ versus b. $g(x) = x^2 + 3$ versus c. $h(x) = x^2 - 1$

look like all on the same graph below:



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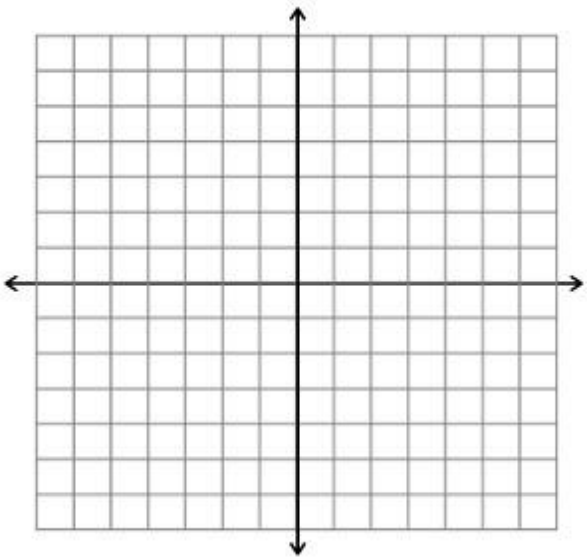
This is an example of a vertical shift. Explain what this means in mathematical terms:

Using a few different colors, sketch out what the graphs of

a. $f(x) = |x|$ versus b. $g(x) = |x - 1|$ versus c. $h(x) = |x + 2|$

look like all on the same graph below:



This is an example of a horizontal shift. Explain what this means in mathematical terms:

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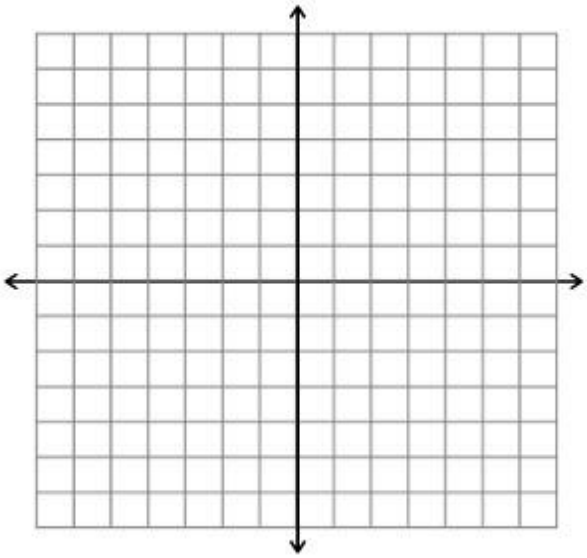
Using a few different colors, sketch out what the graphs of

a. $f(x) = |x|$ versus b. $g(x) = -|x|$

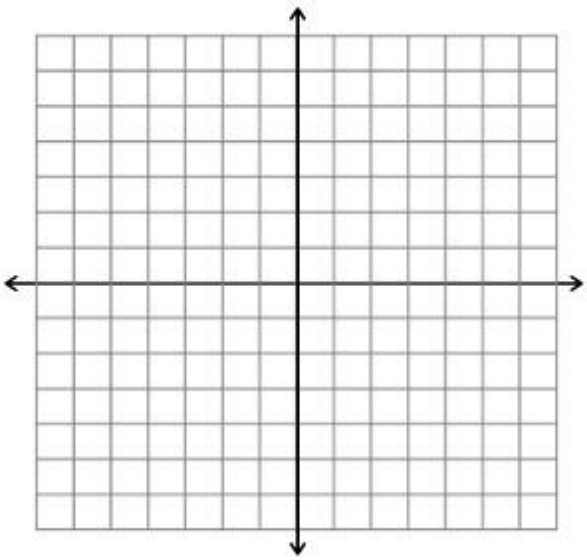
look like all on the same graph below:



This is an example of a reflection over the x-axis. Explain what this means in mathematical terms:

<u>Example</u> Explain in plain language what types of shifts/reflections are occurring, and then graph the function.

$$f(x) = |x| + 3$$



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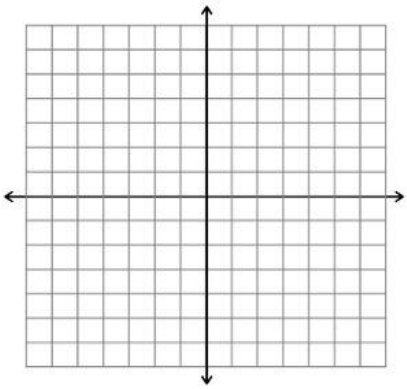
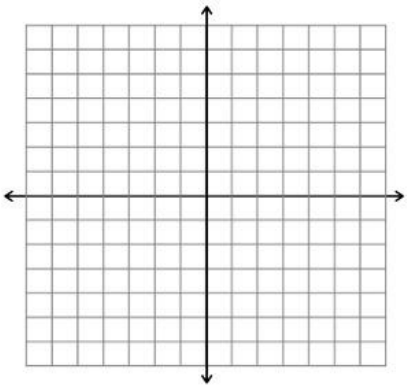
Examples Explain in plain language what types of shifts/reflections are occurring, and then graph the function.

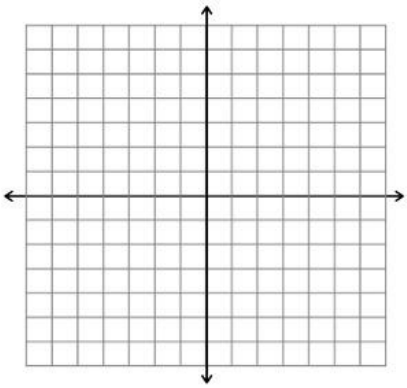
a. $f(x) = (x-1)^2$



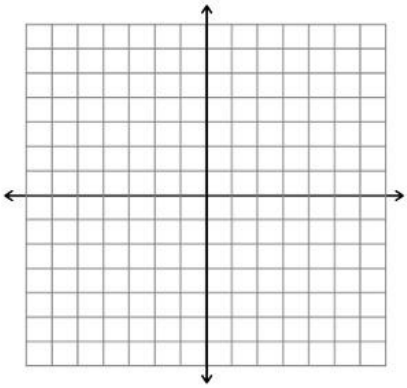
b. $g(x) = \sqrt{x} - 2$



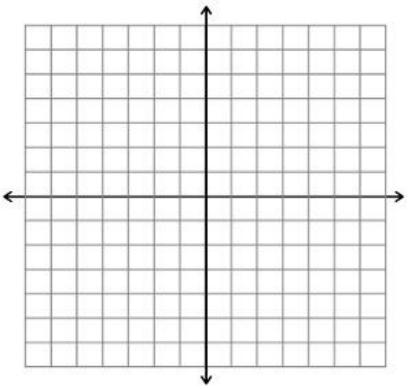
c. $h(x) = |x+1| - 2$



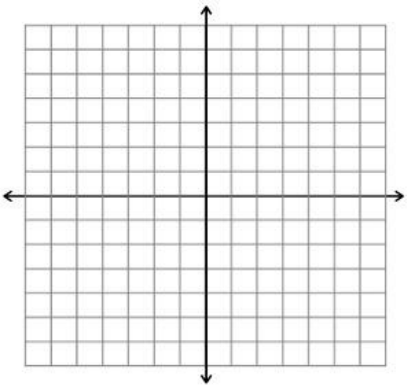
d. $k(x) = (x-2)^2 + 4$



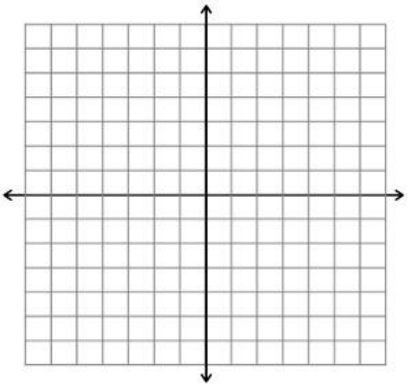
e. $f(x) = -(x+3)^2 - 2$



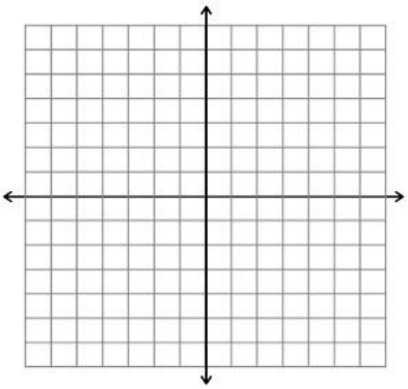
f. $g(x) = -|x+3| - 1$



g. $h(x) = -\sqrt{x-2}$



h. $k(x) = -\sqrt{x+1} - 3$



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Examples Write the equations that represent the listed transformations.

a. A function $f(x) = \sqrt{x}$ is shifted 7 units down.

b. A function $g(x) = x^3$ is shifted 3 units left and 2 units up.

c. A function $h(x) = |x|$ is shifted 2.5 units right and reflected over the x-axis.

d. A function $f(x) = e^x$ is shifted 4 units up and reflected over the x-axis.

e. A function $g(x) = \sqrt[3]{x}$ is shifted 3.2 units right, 5.7 units up, and reflected over the x-axis

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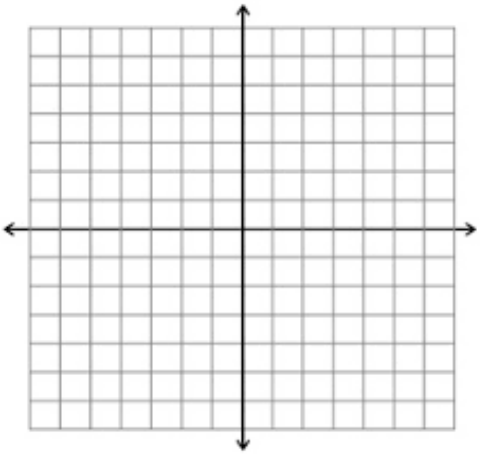
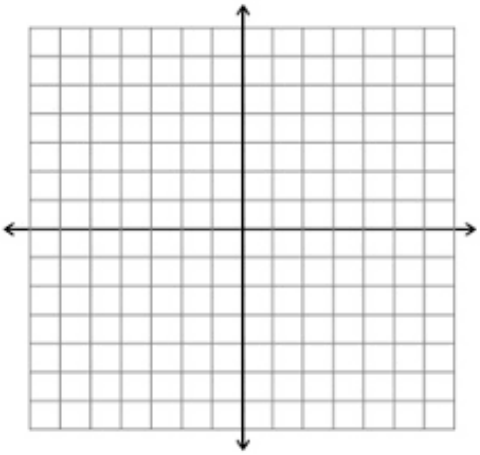
Examples: Translations and Reflections of Functions

Describe any translations / reflections / stretching / shrinking and sketch a graph of each of the following.

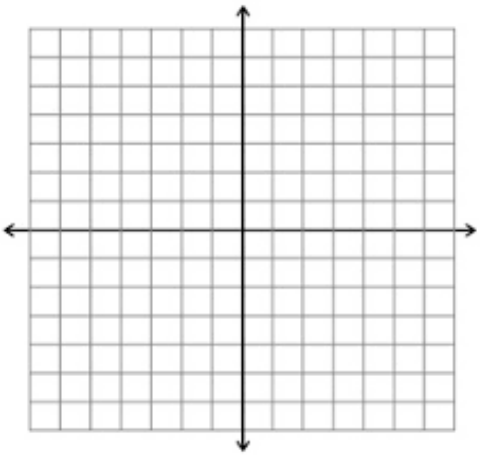
a. $f(x) = -(x+2)^2 - 3$



b. $g(x) = \frac{1}{2}(x-1)^3 + 2$



c. $h(x) = -\frac{1}{3}\sqrt{x+1} + 1$



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Lesson: Symmetry and Even or Odd Functions

Explain when a graph is symmetric with respect to the y-axis:

Explain when a graph is symmetric with respect to the x-axis:

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Explain when a graph is symmetric with respect to the origin:

What makes a function odd?

What makes a function even?

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So what is the test for even or odd functions?

<u>Examples</u> Determine what type of symmetry the following functions have:

a. $y = x^2 + 4$

b. $x^2 + y^2 = 25$

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Examples Determine what type of symmetry the following functions have:

a. $y = x^3 - 5$

b. $y = 7x^2 + 2x - 4$

c. $y = 2x^{\frac{1}{3}} - 3x$

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Lesson: The Difference Quotient

Explain what the difference quotient is and draw some sketches to explain how it works. In other words, watch the video explanation and then explain it in your own words on this page.

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State the formula for the difference quotient here:

<u>Example</u> Calculate the different quotient for the following.

a. $f(x) = 4x + 5$

b. $f(x) = x^2 - 1$

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Lesson: The Difference Quotient

Find the difference quotient.

a. $f(x) = \frac{1}{x+3}$

b. $f(x) = x^2 - 4x + 1$

c. $f(x) = -x^2 + 3$

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Lesson: A Review of Inverse Functions

What is a one-to-one function?

<u>Examples</u> Are the following one-to-one functions? If no, why not?

a. $f(x) = x^2$

b. $g(x) = 2x + 1$

What is the horizontal line test?

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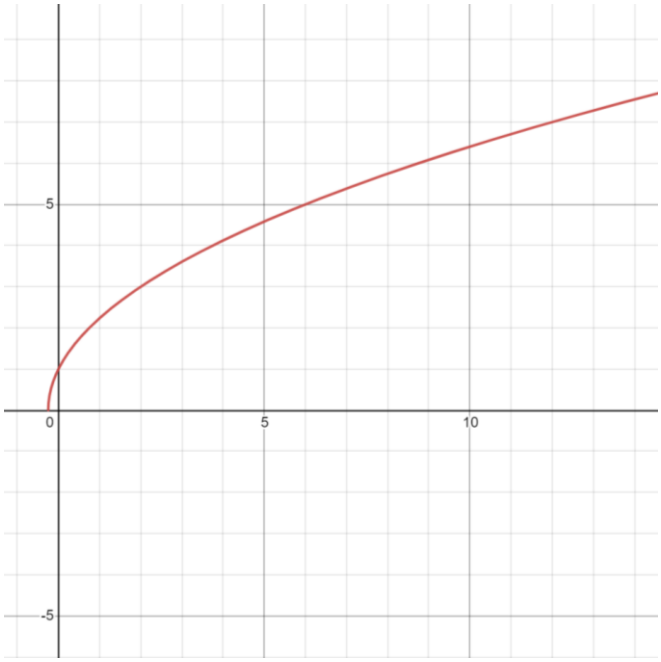
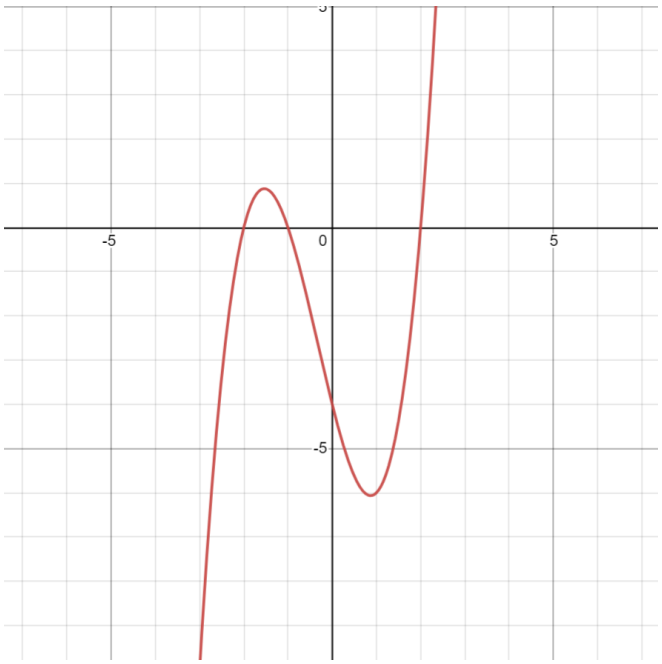
Example Use the horizontal line test to determine if the functions are one-to-one:

a.



b.



What is an inverse function?

What does an inverse function have to do with one-to-one functions?

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Describe how to use composition to determine if 2 functions are inverses of one another.

Example Use composition to determine if the functions $f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x - 2$ are inverses.

Example Use composition to determine if the functions $f(x) = \frac{2}{x+6}$ and $g(x) = \frac{6+x}{2}$ are inverses.

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Using $f(x) = 4x + 2$, outline the steps for how to find the inverse of a function.

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Example Find the inverse of the following functions.

a. $f(x) = \sqrt[3]{3x+1}$

b. $f(x) = \frac{x+2}{x-1}$

c. $f(x) = \sqrt{x-5}$

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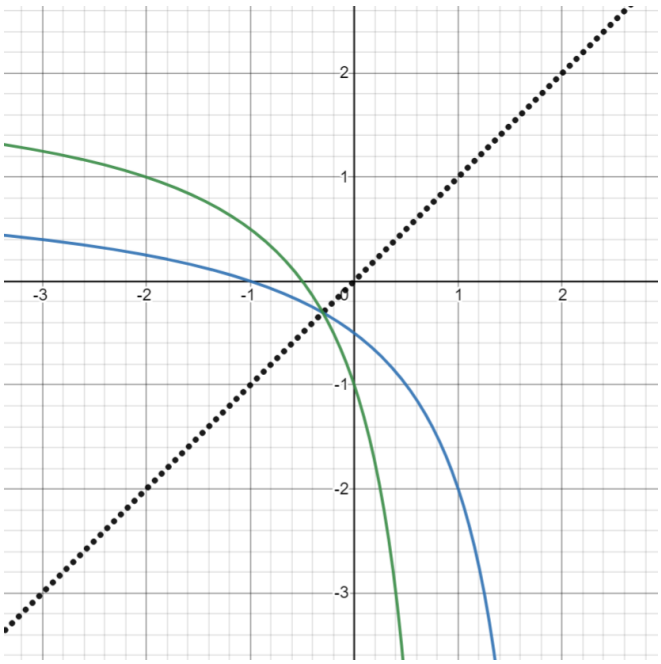
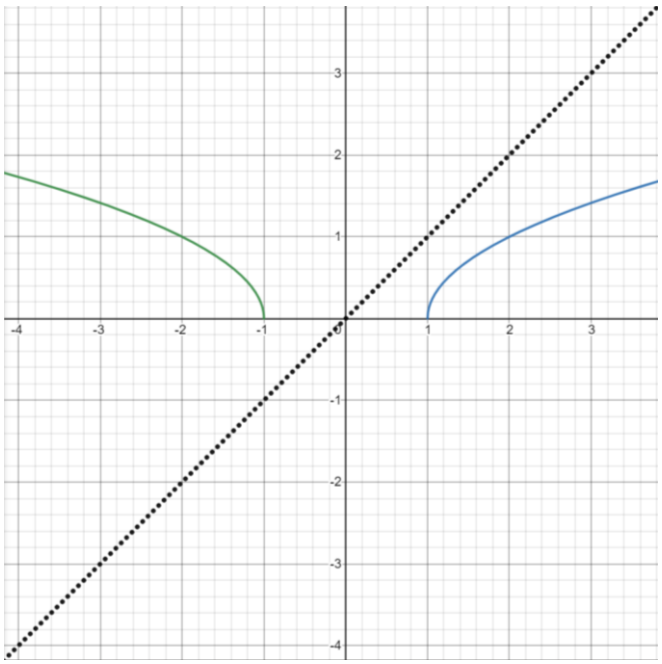
How can we tell on a graph if 2 functions are inverses of one another?

<u>Examples</u> Are the following graphs inverses of one another?

a.



b.



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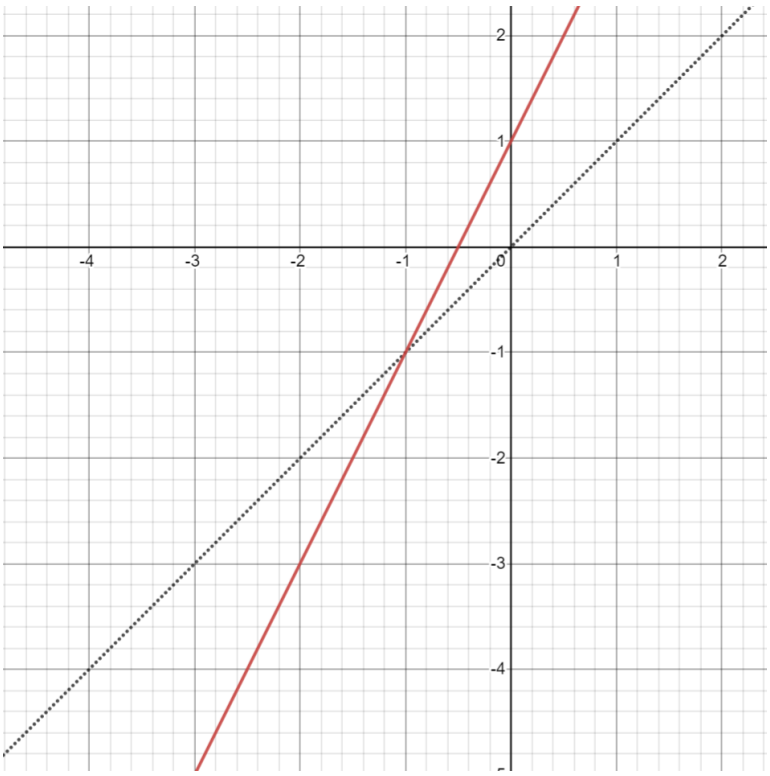
Example Draw the inverse of the following graph.



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Lesson: Solving Rational Equations

Rational equations are equations that look like:

$\frac{2x}{x+1} - \frac{3}{x} = \frac{5}{x+1}$ or $\frac{3}{x^2-4} + \frac{2}{x^2+4x+4} = \frac{5}{x-2}$

What is the general strategy for solving a rational equation?

What do you have to watch out for with rational equations?

<u>Examples</u> Solve each equation.

a. $\frac{x}{x-4} - 4 = \frac{4}{x-4}$

b. $\frac{3}{x+2} + \frac{10}{x^2-6x-16} = \frac{5}{x-8}$

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Examples Solve each equation.

a. $-\frac{14}{3x^2+15x-18}=\frac{x}{x-1}+\frac{2}{3x+18}$

b. $\frac{x}{x^2+4x-32}+\frac{2}{x^2-14x+40}=\frac{6}{x^2-2x-80}$

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Lesson: A Review of Solving Radical Equations and Equations with Rational Exponents

What is the general strategy with solving radical equations?

What do you have to watch out for when solving radical equations?

<u>Examples</u> Solve each equation.

a. $x - 3\sqrt{x+2} = 2$

b. $\sqrt{6x+7} - 9 = x - 7$

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<u>Examples</u> Solve: $\sqrt{3x+4}-5=\sqrt{3x-11}$

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Example Solve each equation.

a. $\sqrt[3]{2x+1} = \sqrt[3]{3x+5}$

b. $\sqrt[4]{x+3} = 2$

Example Solve each equation.

a. $x^{\frac{5}{4}} = 32$

b. $(2x+1)^{\frac{2}{3}} = 36$

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<u>Example</u> Solve for x: $y = k\sqrt{x}$

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Lesson: A Review of Solving Equations Quadratic in Form (Using Substitution)

<u>Examples</u> Solve each equation.

a. $x^4 - 10x^2 + 9 = 0$

b. $(x+1)^{\frac{2}{5}} - 3(x+1)^{\frac{1}{5}} + 2 = 0$

c. $x^{-\frac{2}{3}} - x^{-\frac{1}{3}} - 6 = 0$

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Lesson: A Review of Variation

Describe the 3 types of variation and outline any patterns that might help you remember.

Direct Variation

Inverse Variation

Joint Variation

Combined Variation

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Example If y varies directly with x, and y=15 when x=3, find y when x=6

Example If a varies jointly with b and c, and a=40 when b=2 and c=5, find a when b=3 and c=2.

Example If p varies inversely with m and directly with the square of t, and p=32 when t=2 and m=4, find p when t=3 and m=9

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Example Rewrite the following situations as variation.

a. $C = 2\pi r$, when C is the circumference of a circle and r is the radius.

b. $r = \frac{d}{t}$, where r is the speed when traveling d miles over t hours.

c. $f = \frac{mv^2}{r}$, where f is the centripetal force of an object of mass ma moving along a circle with radius r at velocity v.

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Examples: Applications of Variation

The circumference of a circle varies directly as the radius. A circle of radius 8in has a circumference of 50.27in. Find the circumference of the circle if the radius is 5in.

The current in a simple electrical circuit varies inversely as the resistance. If the current is 30 amps when the resistance is 10ohms, find the current if the resistance if 5ohms.

The volume of a gas varies inversely as the pressure and directly as the temperature. If a gas occupies a 2.6L volume at 600 kelvins (kelvin is a temperature unit) and a pressure of 24 newtons, find the volume at 240 kelvins and a pressure of 32 newtons.

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Lesson: The Factor Theorem

What does the factor theorem say?

<u>Example</u> Determine whether $(x-1)$ is a factor of each polynomial:

a. $f(x) = 9x^4 - 13x^2 + 4$

b. $f(x) = 4x^3 + 10x^2 + 2x - 4$

Given the above, what can we say is also a zero for example a?

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Example Factor $f(x) = 3x^3 + 4x^2 - 17x - 6$ into linear factors given that -3 is a zero.

Example Factor $f(x) = 2x^3 - x^2 - 8x + 4$ into linear factors given that $\frac{1}{2}$ is a zero.

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Lesson: Graphs of Basic Polynomial Functions and Translations of the Basic Graphs

Explain the graphs of $f(x) = x^2, f(x) = x^4, f(x) = x^6$, etc.

Explain the graphs of $f(x) = x^3, f(x) = x^5, f(x) = x^7$, etc.

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If we multiply the polynomial like so: $f(x) = ax^n$, what happens to the graph?

What happens if we have $f(x) = a(x-h)^n + k$?

<u>Example</u> Sketch the graphs:

a. $f(x) = (x-5)^5 + 3$

b. $f(x) = -2(x+1)^6 - 4$

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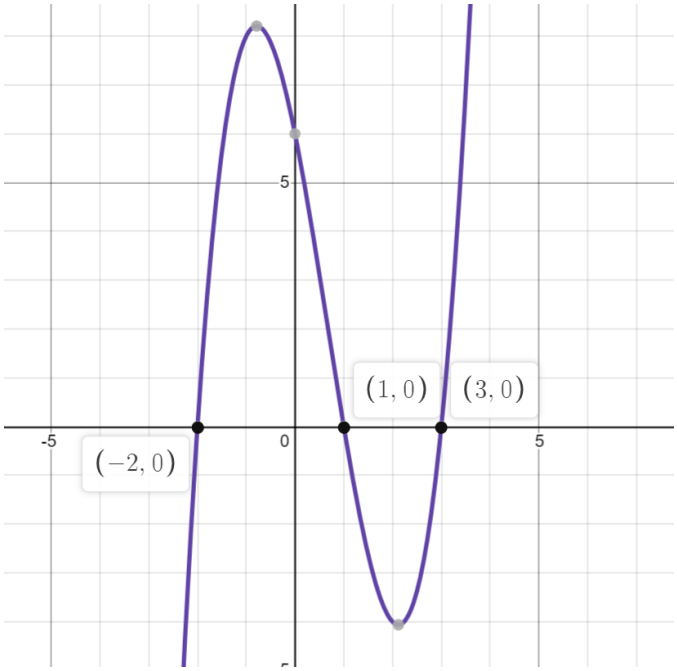
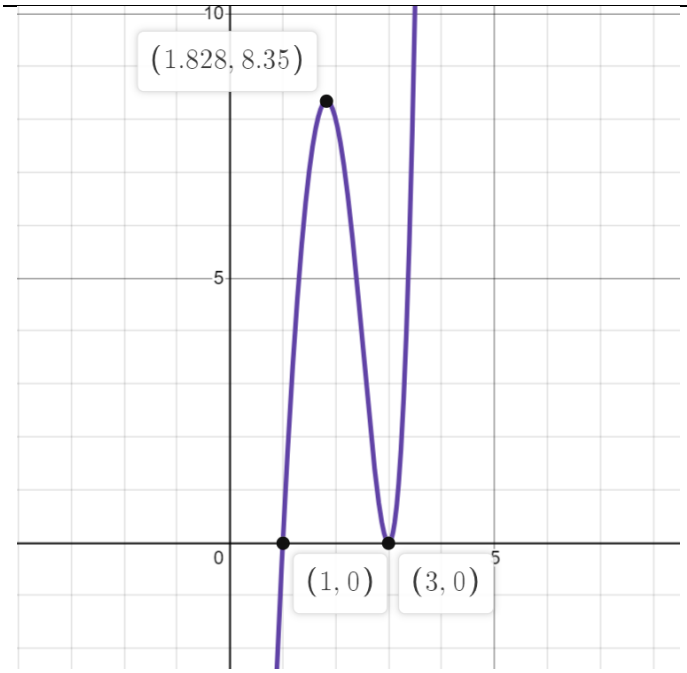
Lesson: Graphs of Polynomial Functions, Turning Points, and End Behavior

In the last video, we discuss the graphs of basic polynomials and translations. In this video, we will look at more complicated possibilities.

Below are the graphs of a few polynomials. What patterns do you notice?



$$f(x) = x^3 - 2x^2 - 5x + 6 = (x-1)(x+2)(x-3)$$



$$f(x) = x^5 - 7x^4 + 19x^3 - 37x^2 + 60x - 36$$
$$= (x-3)^2(x-1)(x-2i)(x+2i)$$

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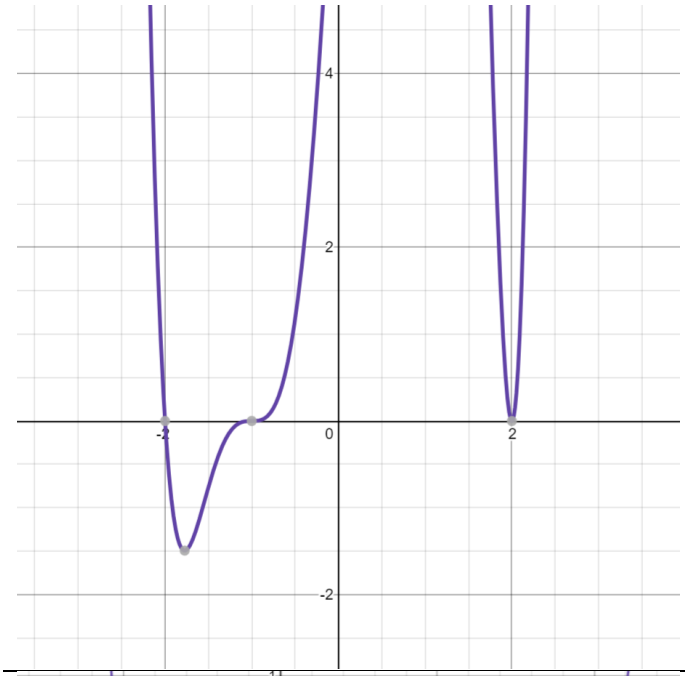
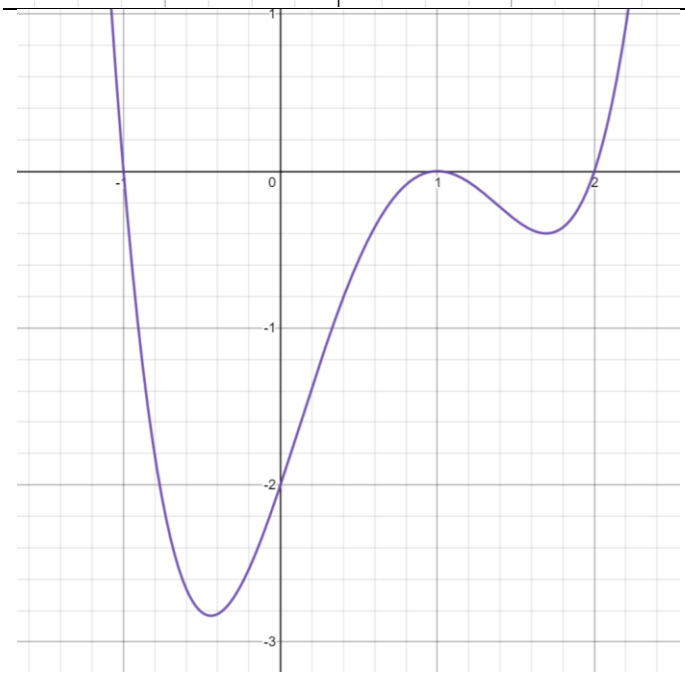


$f(x) = (x-2)^2(x+1)^3(x+2)$



$f(x) = (x-1)^2(x+1)(x-2)$

What do you notice about the behavior of these graphs at zeros? Summarize this in your own words, it was discussed as a general pattern in the video.

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What are turning points and how many will a degree n polynomial have?

What is the end behavior of a polynomial and how do you determine it?

<u>Example</u> What are the number of turning points of the following and what is the end behavior of each?

a. $f(x) = x^2 + 3x + 2$

b. $f(x) = (x+1)^2(x-2)$

c. $f(x) = -\frac{1}{2}x(x-3)^4(x+1)^3(x+2)$

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List the steps required to graph a polynomial:

<u>Example</u> Graph $f(x) = x^3 - x^2 - 4x + 4$

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<u>Example</u> Graph $f(x) = -(x+1)(x-2)^2(x-3)$

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What is the relationship between x-intercepts, zeros, solutions, and factors of polynomials????

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Examples: Graphs of Polynomial Functions, Turning Points, and End Behavior

Graph the following polynomials:

a. $f(x) = x^3 + 5x^2 + 2x - 8$

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b. $f(x) = 3x^4 + 5x^3 - 2x^2$

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c. $f(x) = (x-1)^3(x+2)^2$

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Lesson: The Intermediate Value Theorem and the Boundedness Theorem

State the Intermediate Value Theorem

<u>Example</u> Using synthetic division to show $f(x) = -x^3 + 2x^2 - 4x + 5$ has a real zero between 1 and 2.

<u>Example</u> Using synthetic division to show $f(x) = x^4 - 4x^3 - x + 3$has a real zero between 0.5 and 1.

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State the Boundedness Theorem

<u>Example</u> Show that $f(x) = x^4 + 5x^2 + 3x - 7$ has no real zero less than -3.

<u>Example</u> Show that $f(x) = 2x^5 - x^4 + 2x^3 - 2x^2 + 4x - 4$ has no real zero greater than 2

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Lesson: Synthetic Division and the Remainder Theorem

The outcome of division can be restated using the division algorithm.

State the division algorithm:

Show how we use polynomial long division with $(x^2 - 3x - 1) \div (x + 2)$, and then restate the result using the format of the division algorithm.

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Now show how to do the operation $(x^2 - 3x - 1) \div (x + 2)$ with synthetic division:

Restate this result using the division algorithm:

<u>Example</u> Use synthetic division to perform: $\frac{4x^3-3x^2-7}{x+1}$. Rewrite the result using the division algorithm.

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So what are 3 keys about using synthetic division to remember?

What is the remainder theorem?

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<u>Example</u> Use the remainder theorem to find the desired functional values: $f(x) = -3x^4 + 5x^3 - 30x + 20, f(2)$. What is the equivalent division problem associated with this? Write the result of your division using the division algorithm.

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Example Use the remainder theorem to find the desired functional values: $f(x) = 3x^2 + 5, k = 3 - i$

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Lesson: Synthetic Division and Potential Zeros of Polynomials

What is a “zero” of a polynomial?

How does the remainder theorem help us to determine if we have a zero?

What are some other terms that we use for “zero” of a polynomial?

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<u>Example</u> Determine if the given k is a zero of the polynomial. If it is not, then what is the value of $f(k)$?

a. $f(x) = x^2 - 4x - 5; k = 5$

b. $f(x) = 2x^3 - 3x^2 + 5; k = 1$

c. $f(x) = x^2 - 2x + 2; k = 1 + i$

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Lesson: The Rational Zeros Theorem

The rational zeros theorem is a way to determine *possible* candidates for *rational* zeros of a polynomial function.

State the Rational Zeros Theorem:

<u>Example</u> Find all the possible rational zeros for $f(x) = 2x^3 - x^2 - 8x + 4$ and then factor into linear factors.

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<u>Example</u> Find all the possible rational zeros for $f(x) = 5x^4 - 19x + x^2 + 31x + 6$ and then factor into linear factors.

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Lesson: The Fundamental Theorem of Algebra and the Number of Zeros Theorem

What does the Fundamental Theorem of Algebra state?

What is the number of zeros theorem?

<u>Example</u> What is the maximum number of zeros that each polynomial could possibly have?

a. $f(x) = x^2 - 3x + 1$

b. $f(x) = x^{17} - 3x + 1$

c. $f(x) = 9$

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Example Find a polynomial $f(x)$ that has zeros $-3, 3$, and 1, where $f(2) = -10$

Example Find a polynomial $f(x)$ that has zeros $6, -3$, and -2, where $f(1) = 20$

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Lesson: Conjugate Zeros Theorem

Before we can discuss the theorem, let's review a few properties of conjugates.

What is the conjugate of the following:

a. $3 + 2i$ b. $5 - 3i$ c. $-7 + i$

What is another way that we denote conjugates? Summarize the explanation in the video. Give an example of how this notation works.

What are some properties of conjugates?

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What is the conjugate zeroes theorem?

<u>Example</u> Find a polynomial function of least degree with only real coefficients and zeros 2 and $(3 - i)$.

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Example Suppose $k = -i$ is one zero of $f(x) = x^4 + 26x^2 + 25$. What are the other zeros?

Example Factor $f(x) = 2x^3 + (3 - 2i)x^2 + (-8 - 5i)x + (3 + 3i)$ into linear factors given $k = 1 + i$ is a factor.

Before you begin, what is unique about this problem?

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Lesson: Finding Zeros of a Polynomial Function

In previous videos we discussed:

- The factor theorem
- The rational zeros theorem
- The fundamental theorem of algebra
- The number of zeros theorem
- The conjugate zeros theorem

Using all of these theorems, we can combine them to find the zeros of *any* polynomial.

What can we say about a polynomial function of odd degree with real coefficients?

What can we say about a polynomial function of even degree with real coefficients?

What if a polynomial doesn't have only real coefficients?

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Example Find all zeroes of the following polynomials.

a. $f(x) = x^3 - 21x - 20$

b. $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$

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Examples: Finding Zeros of a Polynomial Function

Find all the zeros of the polynomials.

a. $f(x) = x^4 + 13x^2 + 36$

b. $f(x) = 2x^4 - 11x^3 + 4x^2 + 29x + 12$

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c. $f(x) = x^4 - 7x^3 + 17x^2 - x - 26$

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Lesson: Descartes' Rule of Signs

Let $f(x)$ be a polynomial function with real coefficients and a nonzero constant term, with terms in descending powers of x. Explain how to use Descartes' Rule of Signs:

<u>Example</u> Determine the number of different possibilities for the number of positive, negative, and nonreal complex zeroes:

a. $f(x) = 3x^5 + 4x^4 - x^3 + 2x - 1$

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b. $f(x) = x^6 + x^5 - x^4 + x^3 + x^2 - x + 1$

c. $f(x) = 3x^5 - 7x^3 + 6x + 8$

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Lesson: The Basic Graphs of Rational Functions

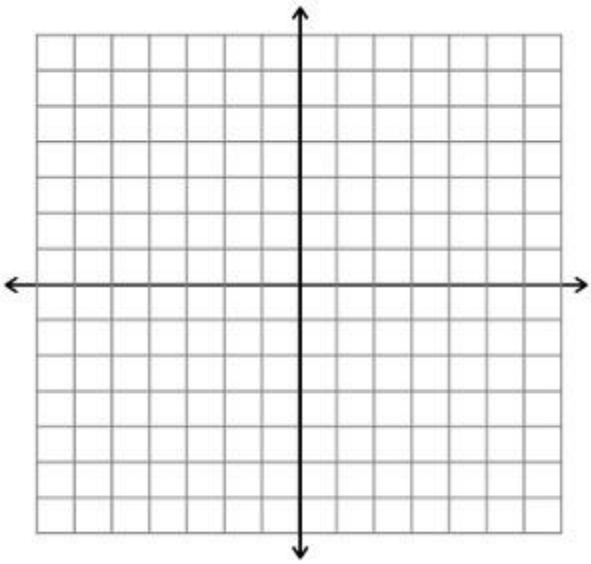
A rational function is a function of the form:

$\frac{f(x)}{g(x)}, g(x) \neq 0$

Where $f(x)$ and $g(x)$ are polynomials.

There are 2 graphs that we view as the most basic version of rational functions.

Make a table and graph $f(x) = \frac{1}{x}$



What is the domain and range?

Where does the graph not exist (ie where is it discontinuous)

What are the y-axis and x-axis for this graph?

What kind of symmetry does this graph have?

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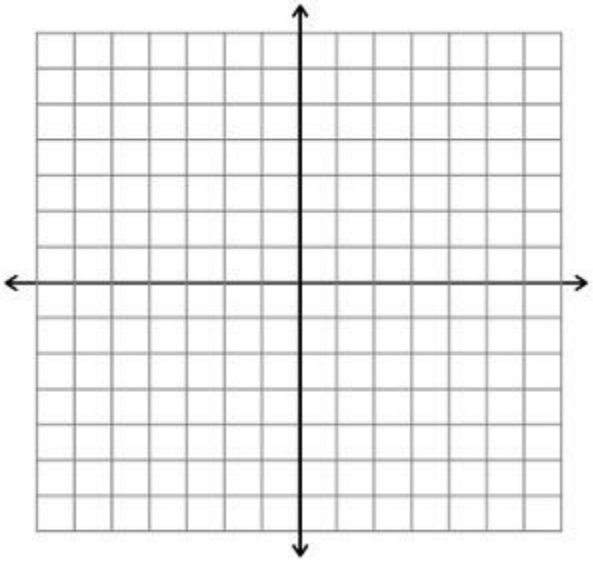
Make a table and graph $f(x) = \frac{1}{x^2}$



What is the domain and range?

Where does the graph not exist (ie where is it discontinuous)

What are the y-axis and x-axis for this graph?

What kind of symmetry does this graph have?

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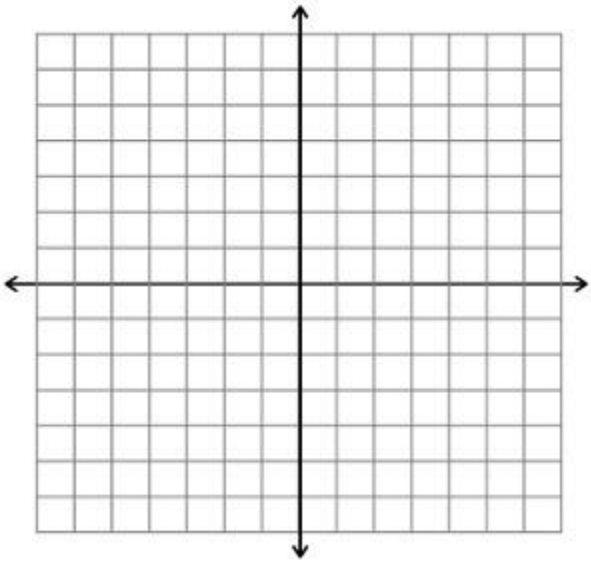
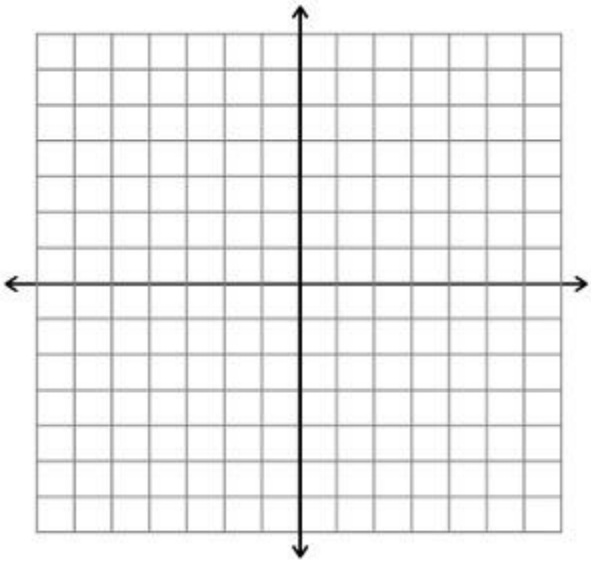
Example Describe the translations for the following graphs, and then draw an approximate sketch of the graph.

a. $f(x) = \frac{1}{(x+1)^2} - 2$



b. $g(x) = -\frac{2}{x} + 1$



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Sketch what happens for odd powers of n for: $f(x) = \frac{1}{x^n}$, ie just draw a sketch of $y = \frac{1}{x^3}, y = \frac{1}{x^5}, y = \frac{1}{x^7}$

Sketch what happens for higher even powers of n for: $f(x) = \frac{1}{x^n}$, ie just draw a sketch of $y = \frac{1}{x^4}, y = \frac{1}{x^6}, y = \frac{1}{x^8}$

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Lesson: Asymptotes of Rational Functions

What is an informal definition of asymptotes?

Is it possible that a graph would graph an asymptote?

Here are the formal definitions of asymptotes:

A graph $f(x) = \frac{p(x)}{q(x)}$ $(q(x) \neq 0)$ has a

- **Vertical asymptote** if $|f(x)| \to \infty$ as $x \to a$. The asymptote is x=a.
- **Horizontal asymptote** if $f(x) \to b$ as $|x| \to \infty$. The asymptote is y=b.

What is super important about asymptote?

Where do you typically find vertical asymptotes?

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How can you determine horizontal asymptotes? Describe the two ways, and also explain how oblique asymptotes. (In other words, explain what we talked about in the video).

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<u>Examples</u> Find any vertical, horizontal, or oblique asymptotes.

a. $f(x) = \frac{x+2}{x^2+7x+12}$

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b. $f(x) = \frac{3x+1}{4x-2}$

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c. $f(x) = \frac{x^2+3x+2}{x^2-9}$

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d. $f(x) = \frac{x^2+x}{x+3}$

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Examples: Finding Oblique Asymptotes

Find all the asymptotes for the following functions.

a. $f(x) = \frac{x^2+2x}{2x+1}$

b. $f(x) = \frac{x^2+2}{x+1}$

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Lesson: Graphing Rational Functions

To learn how to graph rational functions, we will outline the steps and then graph. **Clearly write directions for each step as we go through this process!!!**

Graph $f(x) = \frac{x+3}{x^2+7x+10}$. Outline all the relevant steps discussed in the video.

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Graph $f(x) = \frac{x+1}{x-3}$. Outline all the relevant steps discussed in the video.

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Graph $f(x) = \frac{x^2+1}{x-3}$. Outline all the relevant steps discussed in the video.

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Examples: Graphing Rational Functions

Graph $f(x) = \frac{(x+2)^2}{(x-1)(x+3)}$. Outline all the relevant steps discussed in the video.

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Graph $f(x) = \frac{2x^2-3x+1}{x-2}$. Outline all the relevant steps discussed in the video.

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Examples: Equations of Rational Functions

Copy the graphs from the video and then describe how to find the equations of those graphs.

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Lesson: A Review of Basic Inequalities

Please check out the examples video on how to graph on a number line, use interval notation, or use set notation in case you forgot!

The Inequality Signs

$<$ Less than

$\leq$ Less than or equal to

$>$ Greater than

$\geq$ Greater than or equal to

The Properties of Inequalities

If a, b, and c are real numbers then:

1. If $a < b$, then $a + c < b + c$

2. If $a < b$ and $c > 0$ then $ac < bc$

3. If $a < b$ and $c < 0$, then $ac > bc$

These properties also hold if you switch to $>, \leq, \geq$

<u>Examples</u> Solve the following. Put your final answer in interval notation.

$-2x + 8 < 10$

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<u>Examples</u> Solve the following. Put your final answer in interval notation.

a. $-\frac{1}{3}(x+3)+\frac{1}{3}x+\frac{2}{5}x \leq \frac{1}{10}$

b. $-5 \leq 1+2x < 13$

c. $2 \leq -6x-3 < 9$

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Lesson: Quadratic Inequalities

A quality inequality is an inequality in the form: $ax^2 + bx + c < 0$ where a, b, and c are real numbers. You can interchange $<$ with $\leq, >, \geq$.

There are 3 steps to solving a quadratic inequality. Write out each step as you solve the next example.

<u>Example</u> Write out the steps that you use to solve the following: $x^2 - x - 6 > 0$. Write your answer in interval notation.

Explain Step 1:

Explain Step 2:

Explain Step 3:

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Example Solve: $3x^2 - 4 \leq x$. Write your answer in interval notation.

Example Solve $x^2 \geq 25$. Write your answer in interval notation.

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Lesson: Rational Inequalities

Rational inequalities look like this:

$\frac{5}{x+3} > 3$ or $\frac{2x+5}{4x-1} \leq 2$

We will out the steps to solve a rational inequality by using the following example.

<u>Example</u> Solve $\frac{4}{x+2} \geq 6$ Put your final answer in interval notation.

Explain Step 1 and show how to do it:

Explain Step 2 and show how to do it:

Explain Step 3 and show how to do it:

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What do you have to watch out for with rational inequalities?

<u>Example</u> Solve $\frac{3}{x+6} \leq 2$ Put your final answer in interval notation.

<u>Example</u> Solve $\frac{x+3}{x-5} \geq 1$ Put your final answer in interval notation.

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Lesson: A Review of Exponential Functions and their Graphs

What is the form of a general exponential equation?

<u>Example</u> Let $f(x) = 3^x$ and $g(x) = \left(\frac{1}{2}\right)^x$. Find the following.

a. $f(-2)$

b. $f\left(\frac{3}{2}\right)$

c. $g(-3)$

d. $g(3.2)$

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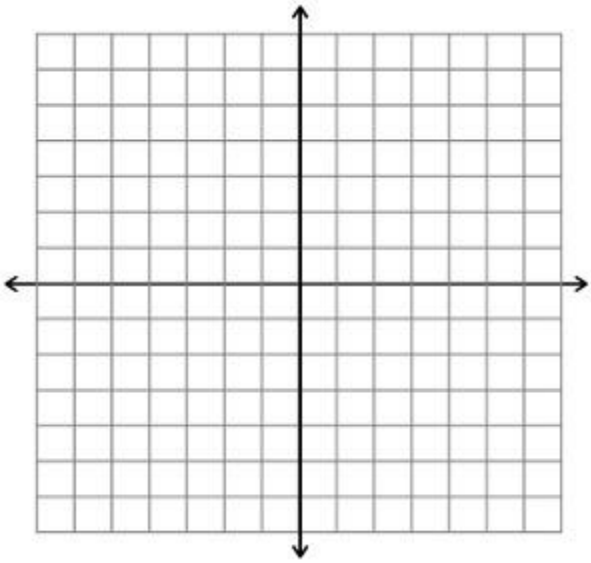
Sketch the graph of $f(x) = 2^x$ and create a table to go with it.



When will an exponential function $f(x) = a^x$ take this shape?

What is the x-axis for this graph?

What are 3 standard points to note here?

What is the domain and range?

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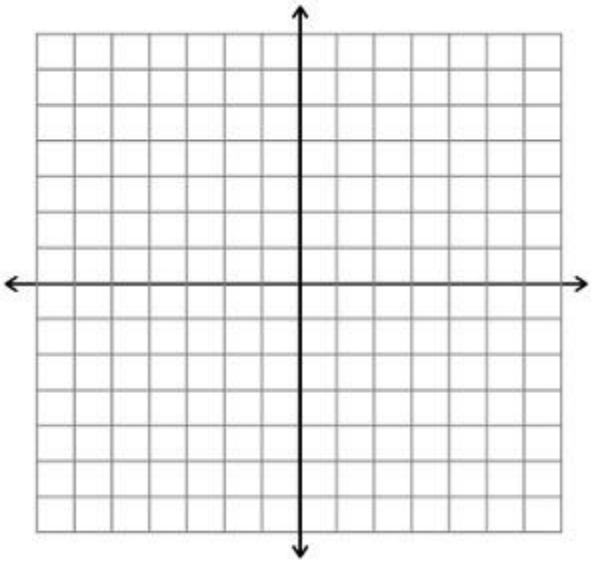
Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^x$ and create a table to go with it.



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What is the x-axis for this graph?

What are 3 standard points to note here?

What is the domain and range?

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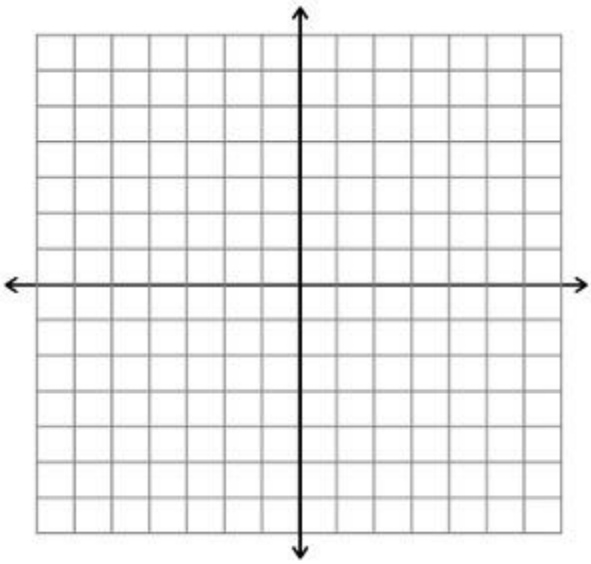
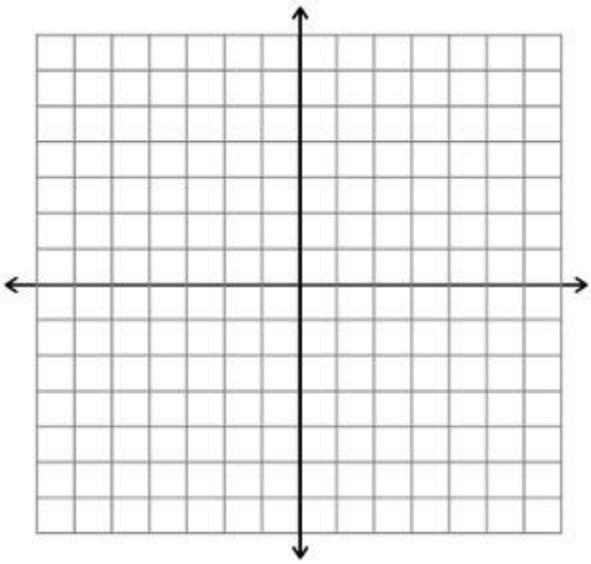
Examples Describe the translations and sketch the following graphs.

a. $f(x) = -3^x + 1$



b. $f(x) = \left(\frac{1}{2}\right)^{x+1} - 2$



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Lesson: Solving Basic Exponential Equations

Outline the general steps to solve $3^{x+1} = 81$.

Outline the general steps to solve $4^{x-2} = 8^{x+1}$.

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<u>Examples</u> Solve the following.

a. $27^{2x} = 9^{x-1}$

b. $e^{3-x} = (e^2)^{-x}$

c. $\left(\sqrt{3}\right)^{x+3} = 9^x$

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Examples Solve the following.

a. $x^{\frac{3}{2}} = 8$

b. $x^{\frac{5}{2}} = 4$

c. $x^{-4} = \frac{1}{16}$

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Lesson: Compound Interest

What is the formula for compound interest and what does it mean?

<u>Example</u> Suppose $2000 is invested into an account paying 2.5% interest per year compounded monthly. How much is in the account after 10 years, and how much interest is earned in that time?

What else do we call the A from the compound interest formula?

What else do we call the P from the compound interest formula?

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Example In 5 years, you want to have $8000 in cash to buy a car (including taxes). Let's use savings to help us reach this goal.

a. You find an account with a 3.25% interest rate compounded biannually (2x a year). What amount would you have to deposit today to ensure you have $8000 in 5 years?

b. Let's pretend that as of today, you have $5000. What interested rate would you need to ensure that this will grow to $8000 in 5 years?

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What is the number e?

What is the formula for continuous compounding?

<u>Example</u> Suppose $2000 is invested into an account paying 2.5% interest per year compounded continuously. How much is in the account after 10 years, and how much interest is earned in that time?

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Lesson: Review of the Basics of Logarithmic Functions

Explain how to find the inverse of $f(x) = 3^x$, and explain how this relates to logarithms.

Define a logarithm:

<u>Example</u> Write the following in logarithmic form.

a. $3^2 = 9$ b. $5^0 = 1$ c. $4^{-2} = \frac{1}{16}$

<u>Example</u> Write the following in exponential form.

a. $\log_3 1 = 0$ b. $\log_5 125 = 3$ c. $\log_3 81 = 4$

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<u>Example</u> Show how to solve the following basic logarithmic equations.

a. $\log_x 8 = 3$

b. $\log_3 x = -2$

c. $\log_{49} 7 = x$

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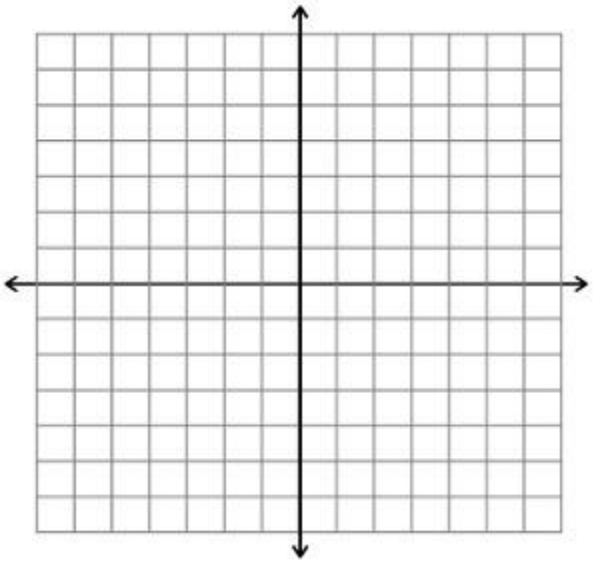
Create a table and show how to graph $f(x) = \log_2 x$



What 3 points can you easily find on the graph?

When does the graph take a shape like this for $f(x) = \log_a x$?

What is the y-axis for this graph?

What are the domain and range?

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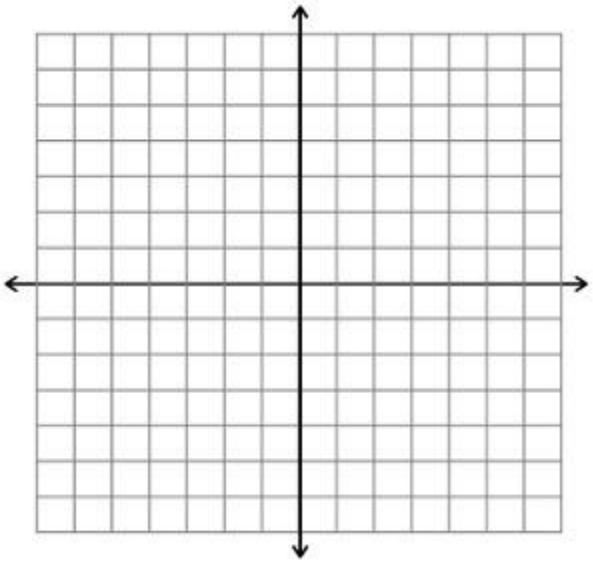
Create a table and show how to graph $f(x) = \log_{\frac{1}{2}} x$



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What is the y-axis for this graph?

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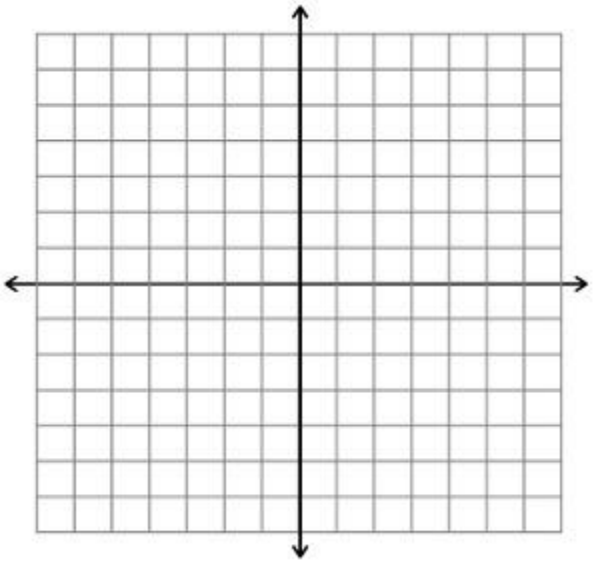
<u>Examples</u> Graph the following: $g(x) = \log_2(x-1) + 1$



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Lesson: Review of the Basics of Properties of Logarithsm

List the major properties of logarithms.

Explain what the common logarithms mean:

$\log x =$

$\ln x =$

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Examples Expand the following as much as possible.

a. $\log_3\left(\frac{3x^3y^7}{\sqrt{z}}\right)$

b. $\ln\left(\sqrt[3]{x^2y}\right)$

c. $\log_3\sqrt{6}$

d. $\log_5\frac{25x^3y^2}{p^7r^3}$

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Example Write as a single logarithm.

a. $\log(x+2)+\log(x-2)-\log(x+3)$

b. $3\log_2 3+2\log_2 y-5\log_2 x$

c. $\frac{1}{2}\log_5 16+3\log_5 y^2-\log_5 x^7-\log_5 y$

d. $4\ln 2+\ln x^6-2\ln y^3-3\ln x^2$

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Lesson: Change of Base Theorem

State the change of base theorem:

<u>Example</u> Show how to evaluate the following:

a. $\log_5 17$

b. $\log_2 3$

c. $\log_{\frac{1}{2}} 5$

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Lesson: Solving Exponential Equations

Outline the steps to solve: $2^{x+3} = 7^{3x}$

<u>Examples</u> Solve the following.

a. $e^{x^2} = 100$

b. $e^{2x-3} * e^{3x} = 5e$

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Examples Solve.

a. $3(2)^{x+2} + 6 = 105$

b. $e^{2x} + 8e^{x} + 15 = 0$

c. $3^{2x} + 2(3^{x}) = 24$

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Lesson: Solving Logarithmic Equations

Outline the steps to solve: $\log(3 - x) = 0.75$

Compare these steps to: $\log(x + 20) = \log(x + 10) + \log(6)$. How are these problems different?

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Examples Solve the following.

a. $\log_4(x^3 + 37) = 3$

b. $\log x + \log x^3 = 4$

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<u>Examples</u> Solve the following.

a. $\log(x^2 - 4) - \log(x - 2) = \log(8)$

b. $\log(5 + 4x) - \log(3 + x) = \log 3$

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Lesson: Solving Exponential and Logarithmic Formulas

<u>Examples</u> Solve for the desired variable.

a. for b: $p = a + \frac{c}{\ln b}$

b. for n: $A = \frac{Pr}{1-(1+r)^{-n}}$

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c. for A: $\log A = \log B - C \log D$

d. for x: $D = 100 + 5 \log x$

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Lesson: Models of Exponential Growth and Decay

Explain the equation for the general model:

Some examples of growth can come from populations or investments. Some examples of decay can come from half-life/radioactive decay, or carbon dating.

Define the following:

Doubling time:

Half-life:

What is the formula for continuous compounding?

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Example A sample of 400g of radioactive lead decays to polonium according to the function: $A(t) = 400e^{-0.03t}$.

a. Find the amount of lead remaining after 5 years.

b. Find the half-life.

Example The amount of carbon-14 present in a paint after t years if given by $y = y_0 e^{-0.0002t}$. The paint contains 37% of its carbon-14. How old are the paintings?

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Example Find the doubling time of an investment earning 5% interest if interest is compounded continuously.

Example During an epidemic, the number of people who have never had a particular virus and are not immune decreases exponential, where this decline is modeled by the function: $f(x) = 20{,}000e^{-0.025t}$, where t is time in days.

a. How many people were susceptible at the beginning of the epidemic?

b. How many people are susceptible after 2 weeks?

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Lesson: Right-Triangle Based Definitions of Trigonometric Functions and Cofunctions

Instead of using standard position to describe trig functions, we can alternatively use right triangles.

Draw a right triangle to orient yourself.

What does A represent?

Show how to define the 6 trig functions with this right triangle.

1.

2.

3.

4.

5.

6.

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What mnemonic device can help you remember the trig functions?

Example Draw the triangle in the video, and then show how to find the values of sine, cosine, and tangent.

Example Using the same example above, show how to find the values of cosecant, secant, and cotangent.

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Draw a new right triangle, and show an alternative angle measure we can find from this picture:

These relationships are called the **cofunction identities,** summarize these below:

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Examples Write each function in terms of its cofunction

a. $\sin 10°$

b. $\cot 65°$

c. $\csc 34°$

Example Find one solution for each equation. Assume all angles involved are acute angles.

a. $\cos\theta = \sin(2\theta - 30°)$

b. $\sec(\theta + 15°) = \csc(2\theta + 25°)$

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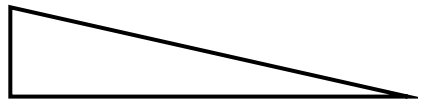
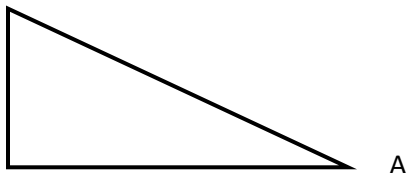
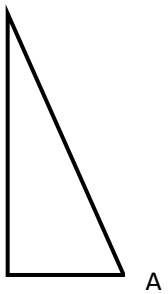
Lesson: More on Trig Function Values as Angles Change and Special Angles







Explain how angle A changes in these pictures:

<u>Examples</u> Determine whether each statement is true or false.

a. $\tan 25° < \tan 22°$

b. $\cos 25° < \sin 25°$

There are a few angles that occur frequently in trig and have some *nice* properties that we like to call them out. Draw out a triangle where each angle is $60°$ and describe any other properties:

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If you bisect this triangle, what happens? Draw a picture to illustrate, and then describe the trig functions associated with it:

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Finally, draw an isosceles right triangle and describe any special characteristics with it:

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We have now examined some *extremely* common trig measures that will come up all the time in trig. They are so important that we will create a separate reference sheet for them.

Read this! I highly recommend that you remove the last page from this packet, and instead put it at the very front of your notes so that you can easily reference it. If you are moving on to calculus, KEEP THIS PAGE IN GOOD CONDITION FOR THE NEXT FEW YEARS. You will thank yourself later that you have this reference as these angles will come up over and over again, and students frequently forget them. I will occasionally create these handouts and make these suggestions to you as we go along in trig – you will thank yourself (and me) later if you keep these. It is extremely common for students to forget some of KEY parts of trig and then struggle in calculus – I don't want this to happen to you!

Some Function Values of Special (Common) Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$0°$						
$30°$						
$45°$						
$60°$						
$90°$						

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Lesson: Reference Angles and Non-Acute Angles

Review: what is a quadrantal angle?

What is associated with every non-quadrantal angle?

Show how we can find reference angles by drawing the reference graphs in the video:

<u>Example</u> Find the reference angle for each angle.

a. $274°$

b. $822°$

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Draw how to find the reference angle based on the quadrant (follow along with the video and draw the diagram on your own):

Now let's talk about using $30°$, $45°$, and $60°$ as reference angles.

<u>Example</u> Find the values of the 6 trig functions for $240°$. Draw a reference!!! (This class is all about pictures!!)

Don't forget the helpful device: All Students Take Calculus 😊

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Let's outline the steps for finding trig function values for any non-quadrantal angle:

1.

2.

3.

4.

How can you avoid making an error with finding a trig function value?

<u>Examples</u> Find the exact values:

a. $\sin(-150°)$

b. $\cos(-510°)$

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Example Evaluate: $\sin^2 120° + \cos^2 120°$

Example Find all values of θ, if θ is in the interval $[0°, 360°)$ and has the given function value: $\cos\theta = \frac{\sqrt{3}}{2}$

Example Is the following true or false? $\cos 60° = 2\cos 30°$

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Examples: Reference Angles and Finding Trig Value Functions

Find the exact values of all 6 trig functions of each angle. Rationalize when applicable.

a. $315°$

b. $-855°$

Evaluate

a. $\cot^2 120° + \tan^2 60° - \cos^2 330°$

b. $\cot^2 135° - \tan^4 45° - \sin^2 150°$

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Examples: Finding the Value of Theta

Find all values of θ if θ is in the interval $[0°, 360°)$ and has the given value.

a. $\sec\theta = -\sqrt{2}$

b. $\cot\theta = -\frac{\sqrt{3}}{3}$

c. $\cos\theta = \frac{1}{2}$

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Lesson: A Review of Angles and Degrees

Draw, label, and define how to make an angle:

Show how to create an angle of positive versus negative measure:

Degree is generally how we measure angles.

Draw the complete rotation of a ray. How many degrees is this?

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Define:

- Acute angle

- Right angle

- Obtuse angle

- Straight angle

Define:

- Complementary angles

- Supplementary angles

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Example Find the complement and supplement of a $55°$ angle.

Example Find the measures of the marked angles shown in the video.

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Examples: Converting Between Minutes and Seconds

<u>Examples</u> Convert the following to degrees.

a. $23°15'32''$

b. $38°27'15''$

<u>Examples</u> Convert the following to degrees, minutes, and seconds.

a. $32.713°$

b. $72.17°$

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Examples: Adding and Subtracting Degrees, Minutes, and Seconds

Add or subtract.

a. $45°15' + 23°57'$

b. $23°18'29'' + 14°19'47''$

c. $94°16' - 27°29'$

d. $72°18'15'' - 15°20'30''$

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Examples: Applications of Minutes and Seconds

Example A turntable makes 45 revolutions in a minute. How many revolutions is that per second?

Example A windmill makes 80 revolutions per minute. How many revolutions is that per second?

Example Wind power is caused by a turbine spinning 10 to 20 revolutions per minute. How many revolutions does it make per second?

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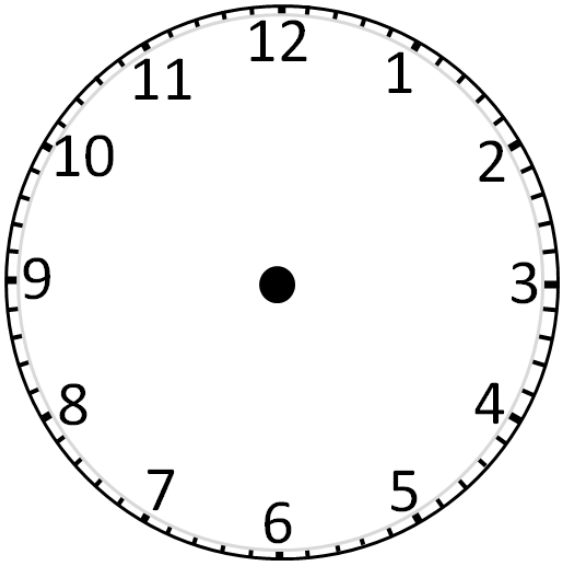
Lesson: Angle Measure with a Clock

Explain the basics of angle measure, ie where is $90°, 180°$?

Each number represents how many degrees? SHOW how to actually determine this?

<u>Example</u> Find in the clock below, and show how to find the angle measure for 5:00. Draw pictures to help illustrate how this works, pictures are key to remembering!!!



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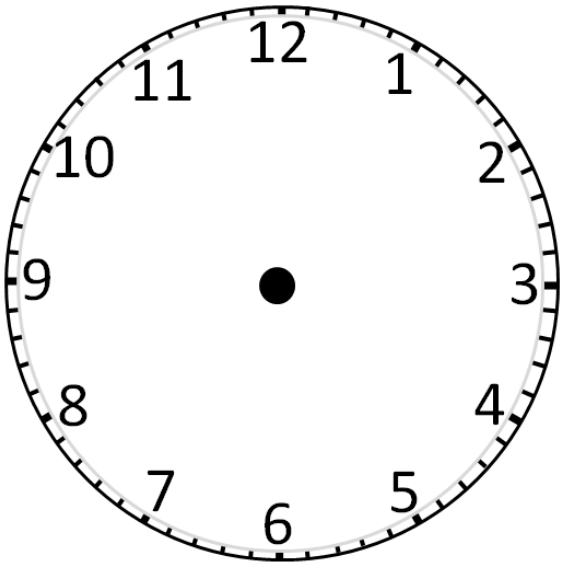
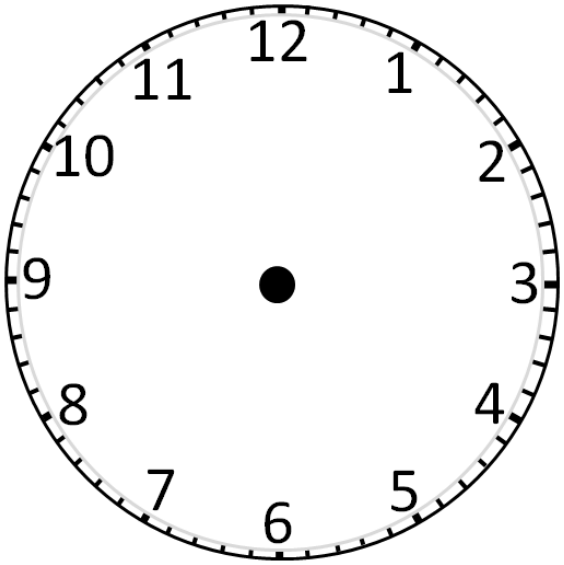
Example Find in the clock below, and show how to find the angle measure for 8:20. Draw pictures to help illustrate how this works, pictures are key to remembering!!!



Example Find in the clock below, and show how to find the angle measure for 10:05. Draw pictures to help illustrate how this works, pictures are key to remembering!!!



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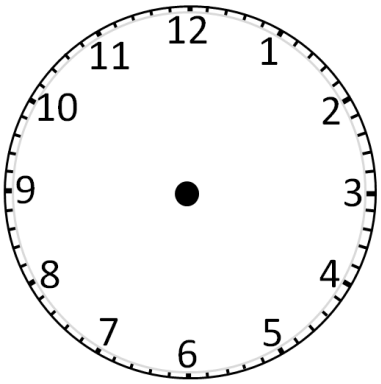
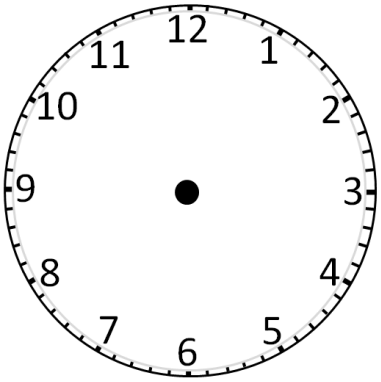
Examples: Angle Measure from a Clock

Find the angles associated with each time.

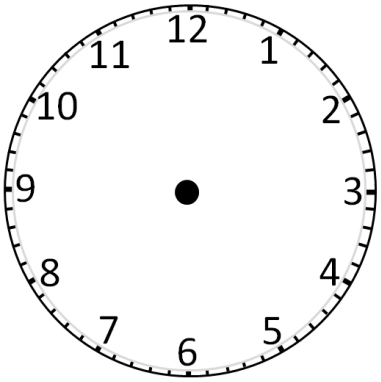
a. 6:25



b. 3:15



c. 2:38



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Lesson: Standard Position and Coterminal Angles

When is an angle in standard position? Draw an example.

Show some examples of how we can determine angle measure by quadrant.

What are quadrantal angles? Draw a picture to illustrate.

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What makes two angles coterminal? Draw a picture to explain.

Example Find the angle of least positive measure that is coterminal with each angle:

a. $715°$

b. $-80°$

Example How can you find all angles coterminal with $45°$? Prove it by filling in the following table.

Value of n	Angle coterminal with $45°$
2	
0	
-1	

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Examples Fill in the table below for which angles are coterminal with the quadrantal angles

Quadrantal Angle	Coterminal Angles
$0°$	
$90°$	
$180°$	
$270°$	

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Lesson: Radian Measure

Radian measure is another way to describe angle measure, as opposed to degrees.

What does radian measure allow us to do?

What is the definition of a radian?

Are radians a unit of measure like feet or meters? Circle: Yes No

What is the formula for the distance around a circle?

What does this formula show?

Write a way to relate radians to degrees:

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So what is π radians?

Show a conversion rate between radians and degrees:

<u>Examples</u> Convert each degree to radians:

a. $35°$

b. $-175°$

c. $27.8°$

Convert each radian measure to degrees:

a. $\frac{7}{4}\pi$

b. $-\frac{2\pi}{3}$

c. 7

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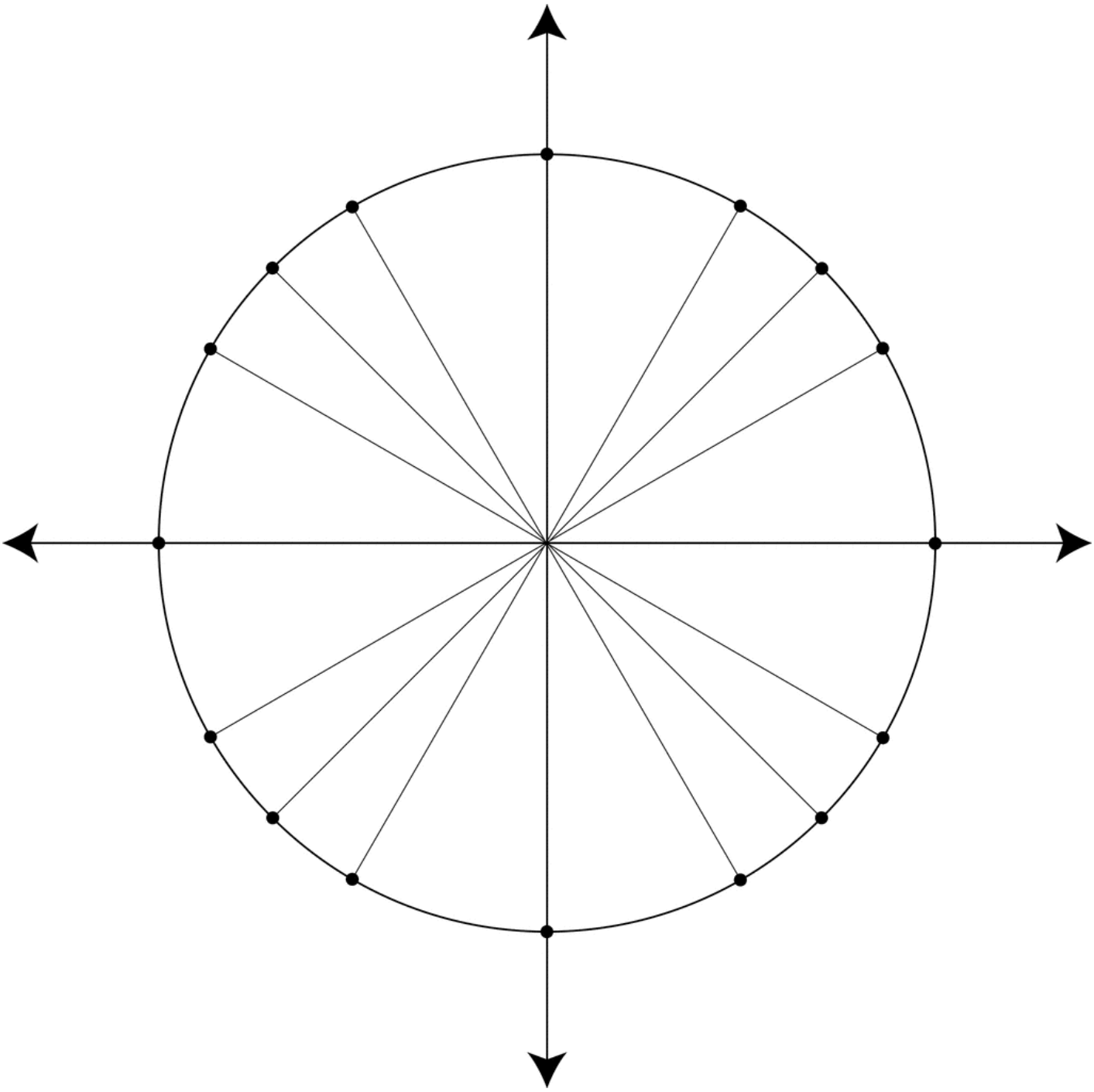
If no unit of angle measure is specified, what should you assume?

Below we will construct an illustration of all the common degree measures we have discussed, and then write their radian equivalent. Take your time to copy this perfectly!



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You will need to know all of those common degrees AND radian measures for the exam!

You may want to reference cheat sheets to help you with the next few problems.

<u>Examples</u> Find the value of each function.

a. $\sin\frac{5\pi}{6}$

b. $\tan\frac{3\pi}{4}$

c. $\csc\frac{5\pi}{3}$

d. $\cos\left(-\frac{2\pi}{3}\right)$

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Lesson: Arc Length on a Circle and Some Applications

We can determine the formula for the arc length of a circle from the definition of radians.

Regarding radians, what does θ equal? Show how this relates to arc length.

What is key about the arc length formula?

<u>Examples</u> A circle has radius 22.30 cm. Find the length of the arc intercepted by a central angle having each of the following measures.

a. $\frac{5\pi}{6}$ radians

b. $66°$

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Examples A circle has radius 15.22 cm. Find the length of the arc intercepted by a central angle having each of the following measures.

a. $\frac{3\pi}{8}$

b. $125°$

What is the definition of latitude?

Example The latitude of Panama City, Panama is $9°$ N and the latitude of Pittsburgh, PA is $40°$ N. Find the north-south distance of these 2 cities. The radius of the Earth is 6400 km.

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Example A rope is being wound around a drum with a radius of 0.7525 ft. How much rope will be wound around the drum if the drum is rotated through an angle of $42.25°$?

Example Two gears are adjusted so that the smaller gear drives the larger one. If the smaller gear rotates through an angle of $150°$, through how many degrees will the larger gear rotate? Refer to the photo in the video.

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Examples: Applications using Arc Length

Charleston, SC and Toronto, Canada, are 1100 km apart and lie on the same north-south line. If the latitude of Charleston is $33°$ N, what is the latitude of Toronto?

Find the radius of the larger wheel in the figure if the small wheel rotates $75.0°$ when the large wheel rotates $45°$.

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How many inches will a small weight rise if a pulley is rotated through an angle of $65°15'$? Through what angle, to the nearest minute, must the pulley be rotated to raise the weight 4 inches?

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Lesson: Area of a Sector of a Circle and Some Applications

What do we define as a sector of a circle?

What is the formula for the area of a section of a circle with radius r?

<u>Examples</u> Find the area of a sector of a circle having the given radius and central angle θ. Answers should be to the nearest tenth.

a. $r = 11.5$ ft, $\theta = \frac{5\pi}{6}$ radians

b. $r = 13.5$ m, $\theta = 125°$

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Example Find the measure (in radians) of a central angle of a sector of a circle of area $25\ in^2$ in a circle of radius 2.5 in.

Example A center-pivot irrigation system provides water to a sector-shaped field as shown in the figure in the video. Find the area of the field if $\theta = 35.0°$ and $r = 155$ yd.

Example A circular sector has an area of $60\ in^2$. The radius of the circle is 6 inches. What is the arc length of the sector?

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Lesson: Linear Speed and Angular Speed and Applications

What is the formula for linear speed?

When are some situations where you might use linear speed?

What is the formula for angular speed?

When are some situations where you might use angular speed?

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Examples Use the formula $\omega = \frac{\theta}{t}$ to find the value of the missing variable.

a. $\omega = \frac{\pi}{3}$ radians per second, $t = 6$ seconds

b. $\theta = 3.721$ radians, $t = 18.34$ seconds

Examples Use the formula $v = r\omega$ to find the value of the missing variable.

a. $r = 10\ m$, $\omega = \frac{9\pi}{5}$ radians per second

b. $v = 10$ m per second, $\omega = \frac{3\pi}{2}$ radians per second

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<u>Example</u> Suppose a point P is on a circle with a radius of 5 cm, and the ray OP Is rotating with an angular speed of $\frac{\pi}{10}$ radian per second.

a. Find the angle generated by P in 3 seconds.

b. Find the distance traveled by P along the circle in 3 seconds.

c. Find the linear speed of P in cm per second.

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The formula $\omega = \frac{\theta}{t}$ can be rewritten as $\theta = \omega t$. We can then use the formula $s = r\theta$ to create a different formula. Show the new formula we can create:

<u>Examples</u> Use the formula above to find the value of the missing variable.

a. $r = 5\ cm, \omega = \frac{2\pi}{3}$ radians per second, $t = 8$ seconds

b. $s = \frac{3\pi}{2}$ km, $r = 4$ km, $t = 3$ seconds

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Example The orbit of the earth around the sun is almost circular. Suppose that the orbit has a radius of 93 million miles.

a. We know that a year is 365 days. Find the angle formed by the Earth's movement in one day.

b. Give the angular speed in radians per hour.

C. Find the linear speed of the Earth in miles per hour.

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Examples: Applications of Linear Speed and Angular Speed

Example Find the angular speed of the minute hand of a clock.

Example Find the linear speed of the tip of the second hand of a clock, if the hand is 25 mm long.

Example A bicycle has tires with a 12-inch radius. The tires are turning at the rate of 200 revolutions per minute. How fast if the bike traveling in miles per hour?

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Example A pulley has a radius of 13.23 cm. It took 15 seconds for 45 cm of belt to go around the pulley.

a. Find the linear speed of the belt in cm per sec.

b. Find the angular speed of the pulley in radians per second.

Example A thread is being pulled off a spool at a rate of 55.4 cm per second. Find the radius of the spool if it makes 155 revolutions per minute.

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Lesson: The Pythagorean Theorem, The Distance Formula, and Intro to Trig Functions

State the Pythagorean theorem:

State the distance formula:

How does the Pythagorean theorem relate to the distance formula?

Now we will introduce **trigonometric functions!** Explain what makes trig functions unique:

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How do trig functions relate to the distance formula? Draw the picture associated with this.

Explain the 6 trig functions and how they relate to what we discussed above.

Sometimes you will see expressions such as $\sin^2 90°$. What does this mean?

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Example The terminal side of an angle θ in standard position passed through the point (-12, -5). Find the values of the 6 trig functions of angle θ.

Example Find the 6 trig function values of an angle θ in standard position if the terminal side of θ is defined by

$3x + 5y = 0, x \geq 0$

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Example Suppose the point (x, y) is in the indicated quadrant. Determine whether given ratio is positive or negative.

$$III, \frac{y}{r}$$

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Examples: Finding Values of Trig Functions

For each given point, sketch the angle of θ in standard position such that θ has the least positive measure and the given point is on the terminal side. The find the values of the 6 trig functions.

a. $(-3, -4)$

b. $(7, 4)$

c. $\left(-\sqrt{2}, \sqrt{2}\right)$

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Examples: Finding the 6 Trig Functions Given an Equation of the Terminal Side

An equation of the terminal side of an angle θ (in standard position) is given with a restriction on x. Sketch the situation with least positive angle θ, and find the values of the 6 trig functions.

a. $-6x - y = 0, x \geq 0.$

b. $x + y = 0, x \geq 0$

c. $x = 0, y \geq 0.$

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Lesson: Quadrantal Angles and Trig Functions

What were the quadrantal angles?

If one of the quadrantal angles is involved, how does this impact how we think about the trig functions? Draw a sketch to help illustrate this point. Explain how these are different from non-quadrantal angles, and how we choose points in these cases.

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Example Find the indicated value of: $\sin 90°$.

How should we think about something like $\sin^2 90°$?

Example Use trigonometric function values of quadrantal angles to evaluate: $\tan 0° - 6 \sin 90°$.

Example Use trigonometric function values of quadrantal angles to evaluate: $\sin^2 180° + \cos^2 180°$.

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Examples: Quadrantal Angles and Trig Functions

Find the function values:

a. $\cot 90°$

b. $\sec 180°$

c. $\sin(-270°)$

d. $3\sec 180° - 2\tan 360°$

e. $\sin^2 90° - 3\cos^2 90°$

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Lesson: The Unit Circle and Circular Functions

Explain how we create the unit circle:

What is the equation for the unit circle?

Recall the formula for arc length is $s = r\theta$. When the radius of the circle is 1, what does this formula become?

Why does this matter / what is cool about this?

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Since r=1 on the unit circle, what can we say about our previously studied trig functions? Just summarize what we discussed in the video as best as you can.

Write out the **circular functions**

1.

2.

3.

4.

5.

6.

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Explain the symmetry of the unit circle:

If a point (a, b) in on the unit circle, what other points are guaranteed to be on the unit circle?

How can we use symmetry with drawing the unit circle? How is it handy?

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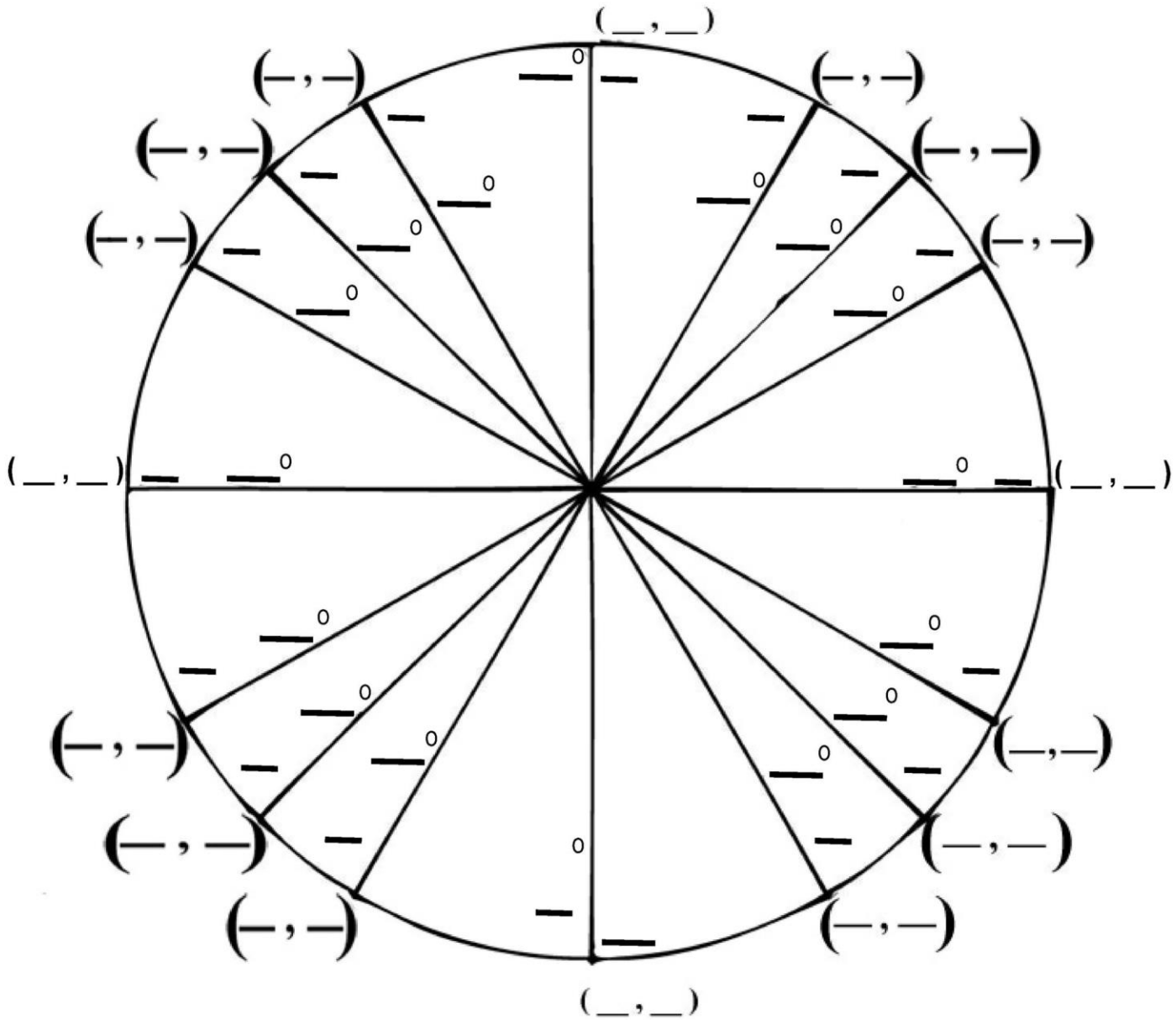
Label the unit circle. Note: you will have to have this memorized! These are the standard angle measures that you'd be expected to do without a calculator!



Now, going back to the circular functions we discussed, how do you view the x-coordinates?

How do you view the y-coordinates?

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Examples Use the unit circle to find the values of the following:

a. $\sin\frac{\pi}{3}$

b. $\cos\frac{5\pi}{4}$

c. $\sin\frac{11\pi}{6}$

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Lesson: Arc Length on a Circle and Some Applications

Using the unit circle, we can also determine the domains of our circular functions.

What is the domain of sine and cosine, and why?

What is the domain of tangent and secant, and why?

What is the domain of cotangent and cosecant, and why?

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Lesson: Values of Circular Functions

We have previously discussed the unit circle (you will want to have it handy). One the most important things about the unit circle is that it will quickly tell you the value of the sine and cosine functions.

On the unit circle, what does the y-coordinate represent?

On the unit circle, what does the x-coordinate represent?

When we think about the values of these functions, what do we assume?

What does this mean for your calculator?

<u>Examples</u> Use the unit circle to find the values of:

a. $\sin\frac{\pi}{2}$ b. $\cos\frac{\pi}{2}$ c. $\tan\frac{\pi}{2}$

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Examples Use the unit circle to find the values of:

a. $\sin\frac{5\pi}{4}$ b. $\cos\frac{5\pi}{4}$ c. $\tan\frac{5\pi}{4}$

Example Compare methods. Use reference angles and degrees to find the exact value of $\sin\frac{2\pi}{3}$, then confirm your answer with the unit circle.

For approximate values, we still must use a calculator. Remember, when you are not specifically told you are using degrees, then you assume you are using radians.

What must you ensure with your calculator?

Examples Use a calculator to approximate the function values.

a. $\cos(1.72)$

b. $\sin(-3.54)$

c. $\cot(1.438)$

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Lesson: Values of Inverse Circular Functions

What do the functions $\sin^{-1} x$, $\cos^{-1} x$, $\cot^{-1} x$, etc. represent?

Your calculator makes a few key assumptions if it uses these functions (the reasons why are explained later):

1. For all x in $[-1, 1]$, using $\sin^{-1} x$ with a calculator will return a value between:

2. For all x in $[-1, 1]$, using $\cos^{-1} x$ with a calculator will return a value between:

3. For all real numbers x, using $\tan^{-1} x$ with a calculator will return a value between:

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Sometimes you will be asked to find answers in different quadrants than what your calculator will return!

<u>Example</u> Solve for s, given that s is in the interval $\left[0, \frac{\pi}{2}\right]$: $\sin s = 0.3311$

<u>Example</u> Solve for s, given that s is in the interval $\left[0, \frac{\pi}{2}\right]$: $\tan s = 0.2155$

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Example Solve for s, given that s is in the interval $\left[\pi, \frac{3\pi}{2}\right]$: $\tan s = \sqrt{3}$

Example Solve for s, given that s is in the interval $[0, 2\pi)$: $\cos^2 s = \frac{1}{2}$

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Lesson: The Unit Circle and Circular Functions as Line Segments

The unit circle is really just an extension of how we have previously thought about trig functions as line segments / right triangles.

Draw the illustration in the video, then show how these sides relate to our trig functions.

From your illustration, how do we define the following?

$\cos\theta =$ $\tan\theta =$ $\csc\theta =$

$\sin\theta =$ $\cot\theta =$ $\sec\theta =$

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Example Using your illustration from the previous page, suppose the measurement of TVU is $45°$. Find the lengths of the following line segments:

OQ

OU

OV

VR

US

PQ

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Lesson: Sine and Cosine as Periodic Functions

What is a periodic function?

What are some examples?

What are 2 known periodic functions?

What can we say about the behavior of these functions as we move along the unit circle?

AS s CHANGES FROM	THE VALUE OF $\sin s$	THE VALUE OF $\cos s$
0 TO $\frac{\pi}{2}$		
$\frac{\pi}{2}$ TO π		
π TO $\frac{3\pi}{2}$		
$\frac{3\pi}{2}$ TO 2π		

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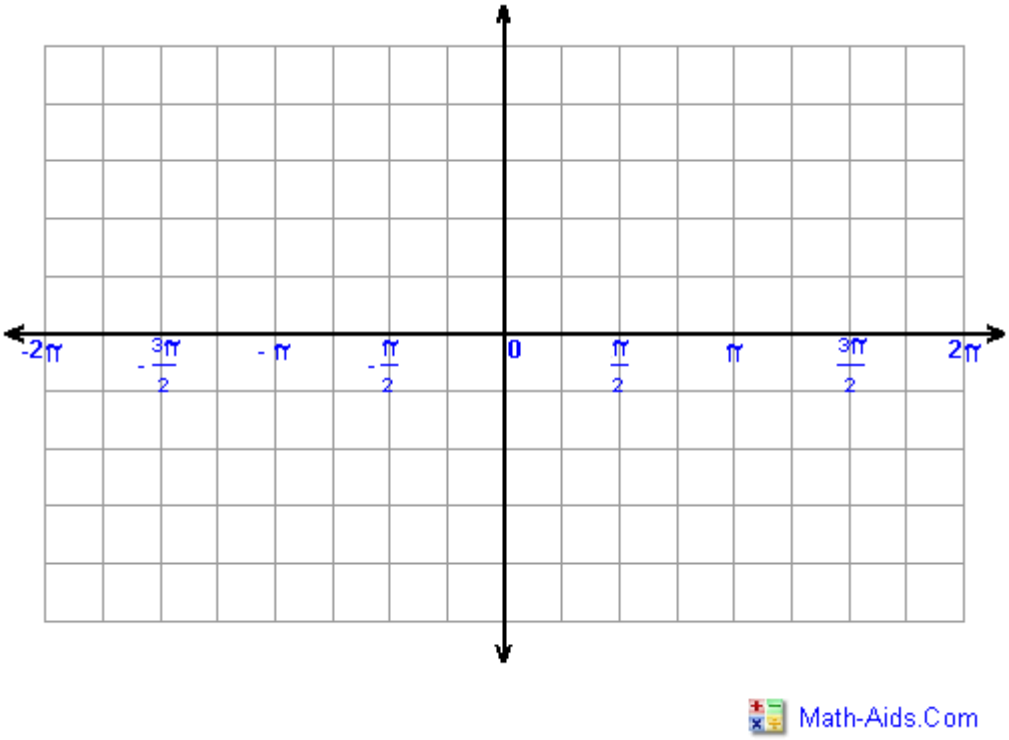
But let's change s to x going forward so we don't get confused.

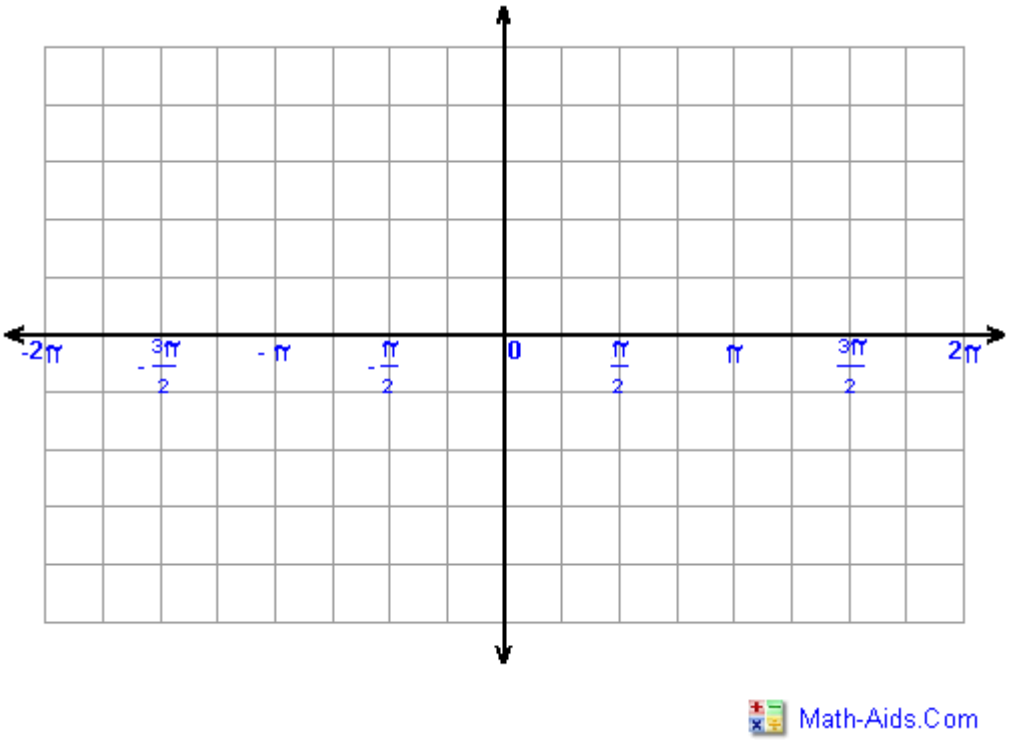
Draw the graph of sine, filling in the table and then using the graph paper.

x	$y = \sin x$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



Draw the graph of cosine, filling in the table and then using the graph paper.

x	$y = \sin x$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



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What term do we use to describe the graph of sine?

How do we describe the graph of cosine?

What is the period of both the sine and cosine functions?

Where are the graphs continuous?

What is the domain and range of the graphs?

How can we describe the x-intercepts of sine?

How can we describe the x-intercepts of cosine?

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What is the test for even or odd functions?

What does it mean to be an even function?

What does it mean to be an odd function?

How do you know that $y = \sin x$ is odd? How would you show it?

How do you know that $y = \cos x$ is even? How would you show it?

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Lesson: Using Amplitude and Period to Graph Sine and Cosine

What is amplitude?

Using the equation $y = a \sin x$, when will this graph stretch or shrink? Draw 2 comparison sketches to help you remember (just a quick sketch so you remember how these visually compare)

Using the equation $y = a \cos x$, when will this graph stretch or shrink? Draw 2 comparison sketches to help you remember (just a quick sketch so you remember how these visually compare)

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What happens when a is negative? Demonstrate this with some brief sketches so that you can remember.

What is period?

Using the equation $y = \sin bx$, watch the video and write an explanation with sketches to demonstrate period.

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Using the equation $y = \cos bx$, watch the video and write an explanation with sketches to demonstrate period.

What formula can you use to determine the period of a function?

<u>Examples</u> Determine the periods of the following functions:

a. $y = \sin 3x$

b. $y = \cos \frac{1}{2} x$

c. $y = \sin 2\pi x$

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Summarize amplitude and period when the function is of the form $y = a \sin bx$ and $y = a \cos bx$

<u>Examples</u> Graph the following over 2 period intervals, completing the table with strategic points to help you graph. Give the period and amplitude in each. You will have to sketch your own graph and create your own axes.

$y = \cos \frac{1}{3} x$

Period: **Amplitude:**

$\boldsymbol{x}$	$\boldsymbol{y = cos \frac{1}{3} x}$

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$y = 2\sin x$

Period: **Amplitude:**

$\boldsymbol{x}$	$\boldsymbol{y = 2\,sin\,x}$

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$y = 3\cos 2\pi x$

Period: **Amplitude:**

x	y

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Examples: Using Amplitude and Period to Graph Sine and Cosine

Graph the following over two-period intervals.

a. $y = -\frac{1}{2}\sin x$

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b. $y = 3\cos 2x$

d. $y = -\sin \pi x$

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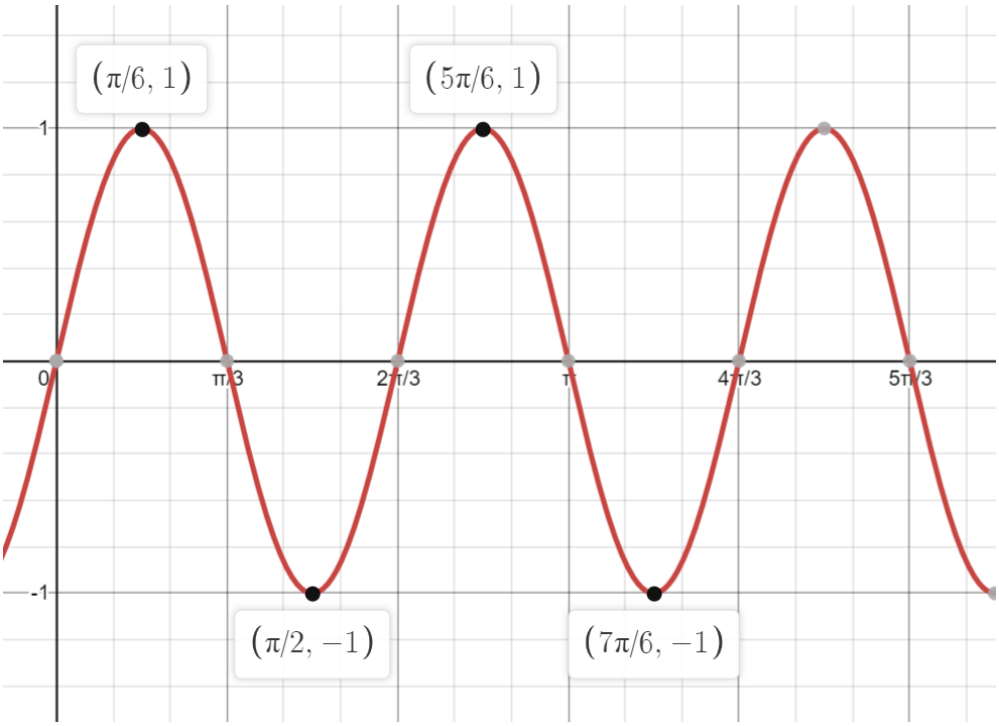
Lesson: Determining the Equations of Sine and Cosine from Graphs

In each graph, determine the equation of the form $y = a\cos bx$ or $y = a\sin bx$, with $b > 0$, for each graph.

Desmos.com was used to create these graphs.

a.



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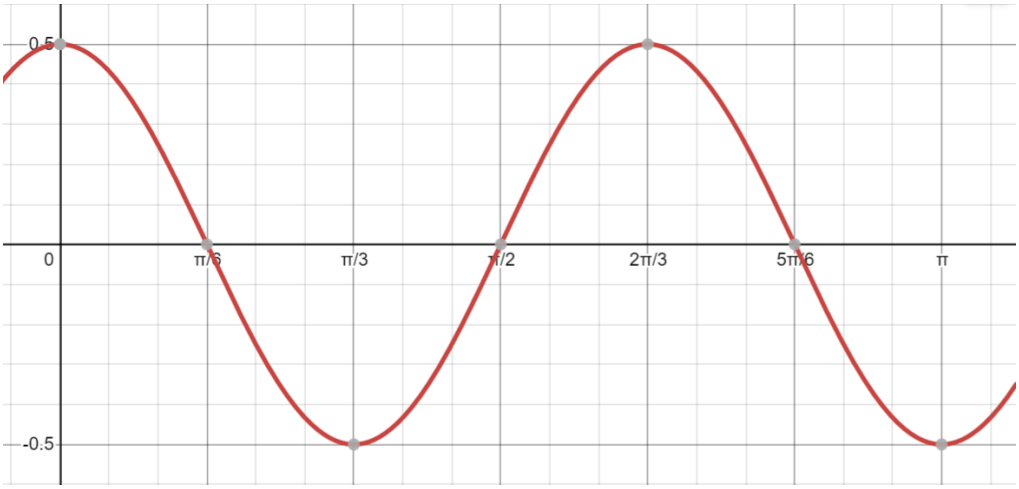

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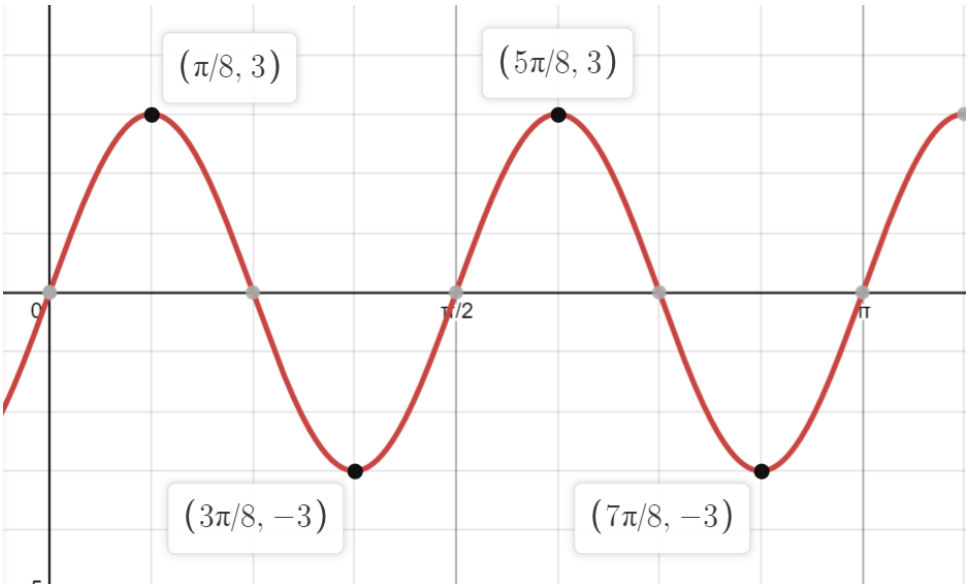
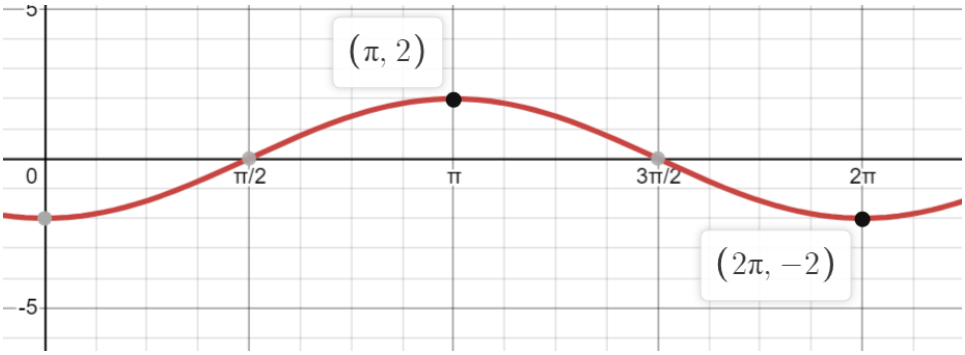


d.



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Lesson: Translations of the Sine and Cosine Graphs

How do you describe a horizontal translation? In your own words / explanation, summarize with sketches how the video show these translations work.

What is phase shift?

What is an argument?

How do you describe a vertical translation? In your own words / explanation, summarize with sketches how the video show these translations work.

What happens when a function is of the form: $f(x) = a\cos(bx + bd)$ with the phase shift?

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Examples Identify the amplitude, period, phase shift, argument, and vertical translation.

a. $y = \sin\left(x + \frac{\pi}{4}\right)$

b. $y = 2 - \sin x$

c. $y = 3 + \sin(x + 2)$

d. $y = -4 + \frac{1}{2}\cos x$

e. $y = 2 + \sin(2x - 2\pi)$

f. $y = 1 - 2\cos\left(3x + \frac{\pi}{2}\right)$

g. $y = 2 - 4\cos(3x + 1)$

h. $y = 3\pi - 2\sin(\pi x - 3)$

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Examples Show how to find the start and end points of an interval for us to graph.

a. $y = \sin\left(x + \frac{\pi}{2}\right)$

b. $y = 1 - 2\sin\left(3x - \frac{\pi}{2}\right)$

c. $y = 3\pi - 2\sin(\pi x - 3)$

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<u>Example</u> Using the graph $y = \sin\left(x + \frac{\pi}{4}\right)$, and *clearly write out the steps* required to graph any sine or cosine graph. Identify the amplitude, period, phase shift, and vertical translation.

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Example Graph $y = 1 + \cos x$. Identify the amplitude, period, phase shift, and vertical translation.

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<u>Example</u> Graph $y = -1 + \frac{1}{2}(\cos 2x - \pi)$. Identify the amplitude, period, phase shift, and vertical translation.

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Examples: Translations with Sine and Cosine

Graph the following: $y = \frac{5}{2}\sin\left(4\left(x + \frac{\pi}{4}\right)\right)$

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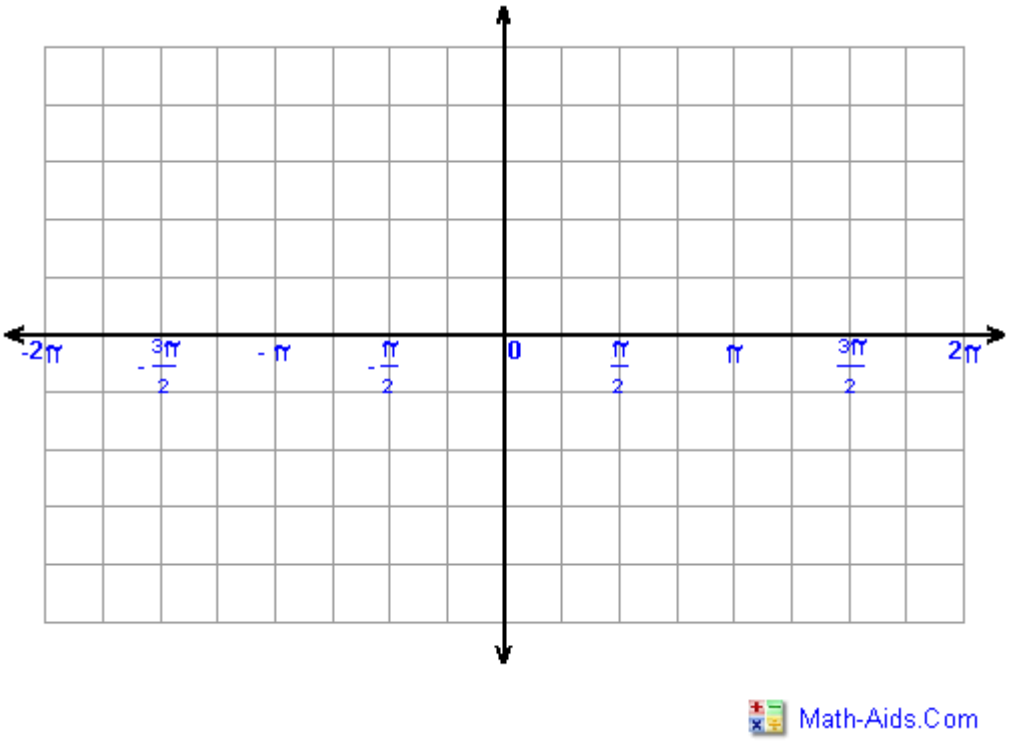
Lesson: The Graphs of Tangent and Cotangent

What is an asymptote? Draw a sketch to help explain.

Draw the graph of tangent, filling in the table and then using the graph paper.

x	$y = \tan x$
$-\frac{\pi}{2}$	
$-\frac{\pi}{4}$	
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{2}$	



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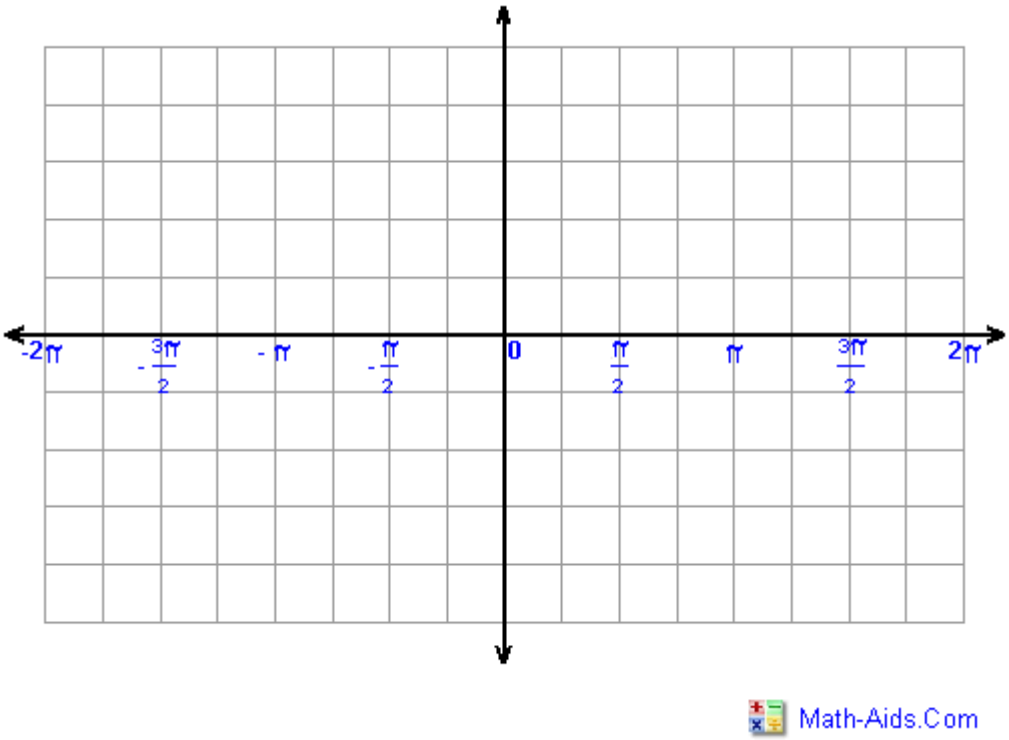
Draw the graph of cotangent, filling in the table and then using the graph paper.

x	$y = \cot x$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{2}$	
π	
$\frac{5\pi}{4}$	



How would we describe the period for these functions?

Where are the graphs continuous (and discontinuous)?

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What is the domain and range of the graphs?

How can we describe the x-intercepts of tangent?

How can we describe the x-intercepts of cotangent?

Are either function even or odd?

How do you interpret amplitude with these functions and why?

What is the key for graphing? (Be specific with the equations you need to leverage)

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<u>Example</u> Graph over a one-period interval: $y = \tan 2x$

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<u>Example</u> Graph over a two-period interval: $y = -\frac{1}{2}\cot 4x$

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Lesson: Translations with The Graphs of Tangent and Cotangent

Tangent and cotangent can experience the same types of translations that sine and cosine did (which we discussed in a previous lesson).

<u>Examples</u> Describe the period, phase shift, and vertical translations for each function.

a. $y = 3\tan 4x$

b. $y = -\cot(x + \pi)$

c. $y = 2\tan\left(3x - \frac{\pi}{2}\right)$

d. $y = 3 + \cot(x - 4)$

e. $y = -2 + \tan\left(2\pi x - \frac{\pi}{4}\right)$

f. $y = \frac{1}{2} - \cot(5x + 2)$

<u>Examples</u> Show how to find 2 adjacent asymptotes for graphing each of the following:

a. $y = 3\tan 4x$

b. $y = 2\tan\left(3x - \frac{\pi}{2}\right)$

e. $y = -2 + \tan\left(2\pi x - \frac{\pi}{4}\right)$

f. $y = \frac{1}{2} - \cot(5x + 2)$

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<u>Example</u> Graph $y = 3 + \cot\left(x + \frac{\pi}{4}\right)$

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<u>Example</u> Graph $y = -1 + 2\tan\,(4x + \pi)$

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Examples: Translations with The Graphs of Tangent and Cotangent

Graph over a two-period interval.

a. $y = \tan\left(3x + \frac{\pi}{4}\right)$

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b. $y = -2 + 3\cot(4x - \pi)$

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c. $y = 1 - \tan(2\pi x - \pi)$

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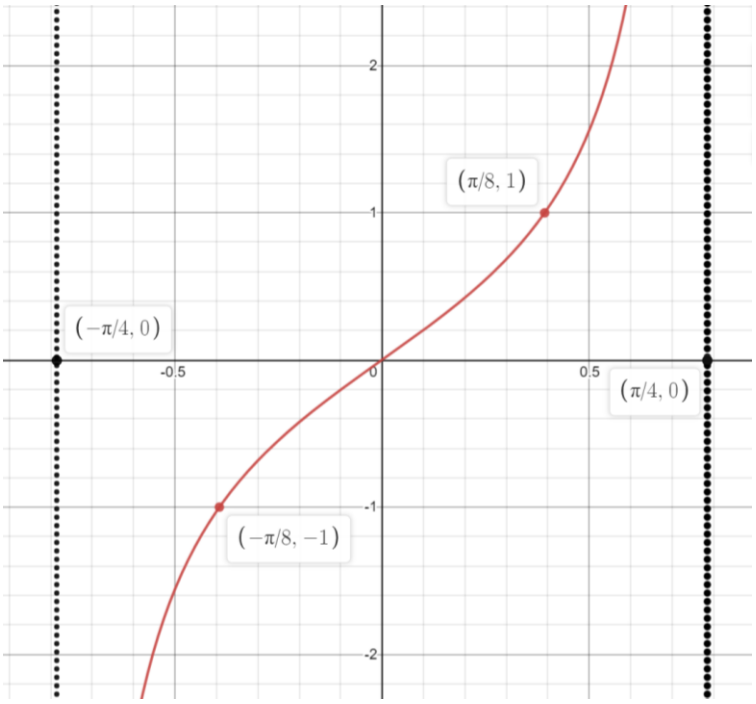
Lesson: Determining the Equations of Tangent and Cotangent from Graphs

Graphs from desmos.com of course!

a.



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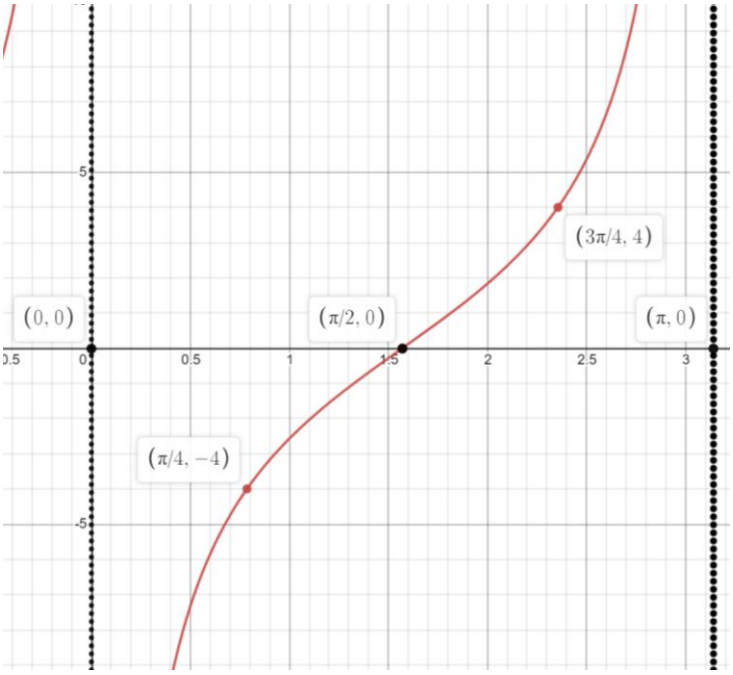

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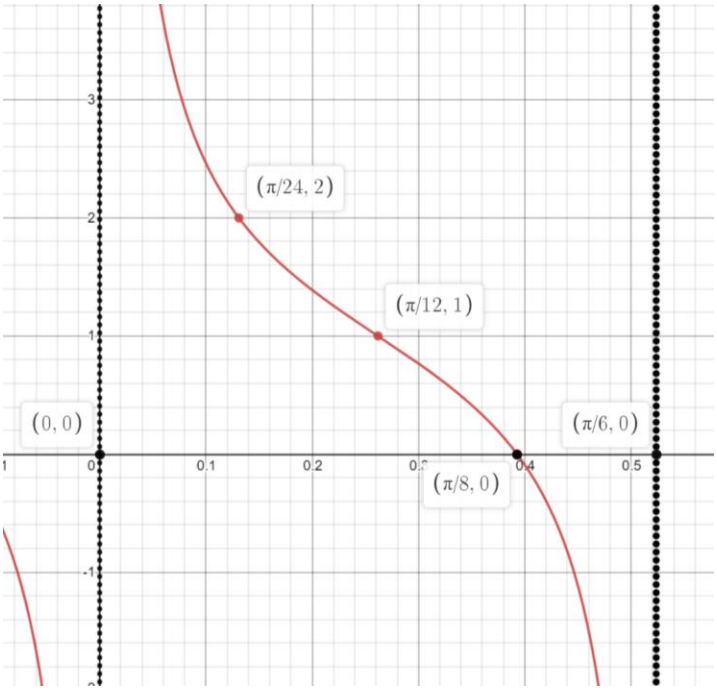
c.



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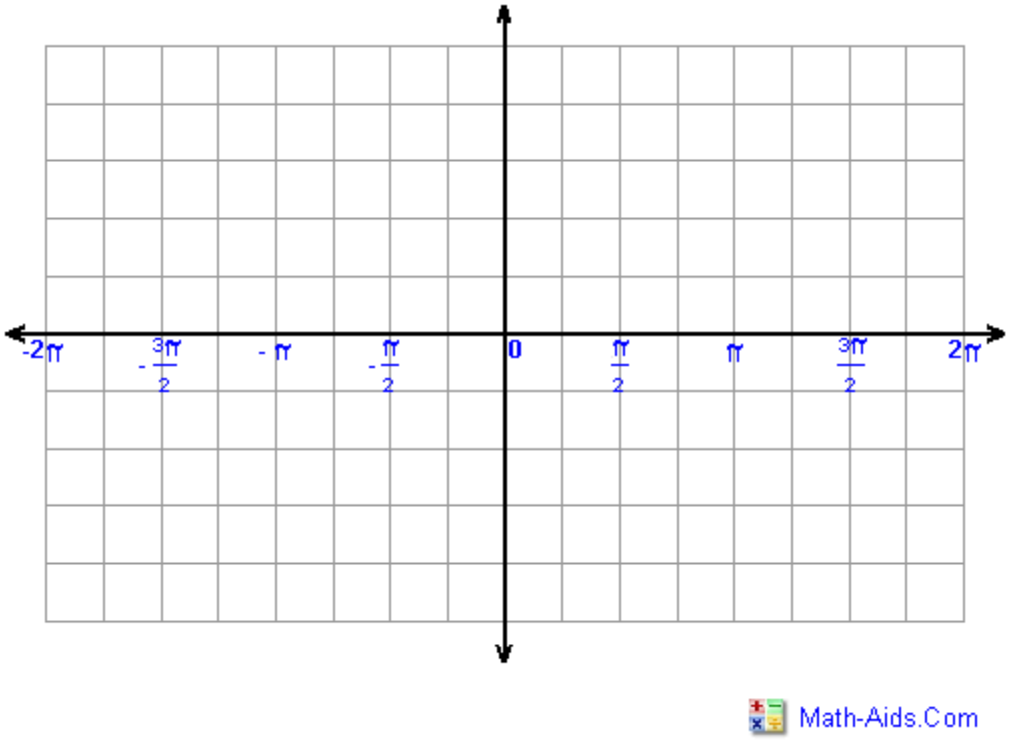
Lesson: Translations of The Graphs of Secant and Cosecant

Draw the graph of secant, filling in the table and then using the graph paper.

x	$y = \sec x$
$-\frac{\pi}{2}$	
$-\frac{\pi}{4}$	
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{4}$	
π	
$\frac{3\pi}{2}$	



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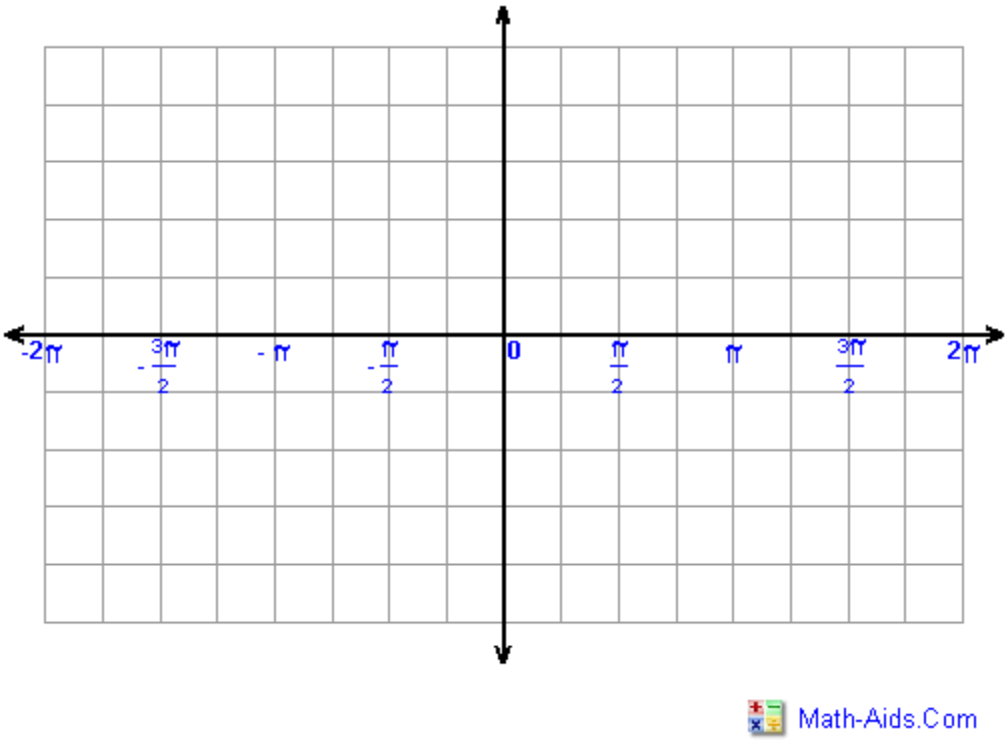
Draw the graph of cosecant, filling in the table and then using the graph paper.

x	$y = \sec x$
0	
$\frac{\pi}{6}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	
$\frac{2\pi}{3}$	
π	
$\frac{3\pi}{2}$	
2π	



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How would we describe the period for these functions?

Where are the graphs continuous (and discontinuous)?

What is the domain and range of the graphs?

How can we describe the x-intercepts of both graphs?

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Are either function even or odd?

How do you interpret amplitude with these functions?

What is the key for graphing? (Be specific with the equations you need to leverage)

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<u>Examples</u> Graph $y = 2\csc\frac{1}{2}x$

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<u>Examples</u> Graph $y = 1 + \frac{1}{2}\sec\left(x - \frac{\pi}{2}\right)$

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Examples: Translations of The Graphs of Secant and Cosecant

Graph the following over a one-period interval.

a. $y = -2\sec\left(x + \frac{3\pi}{4}\right)$

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b. $y = \frac{1}{2}\csc(2x - \pi)$

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c. $y = 2 + \csc\left(\frac{1}{2}x - \pi\right)$

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d. $y = 2 - 3\sec\left(2\pi x - \frac{\pi}{2}\right)$

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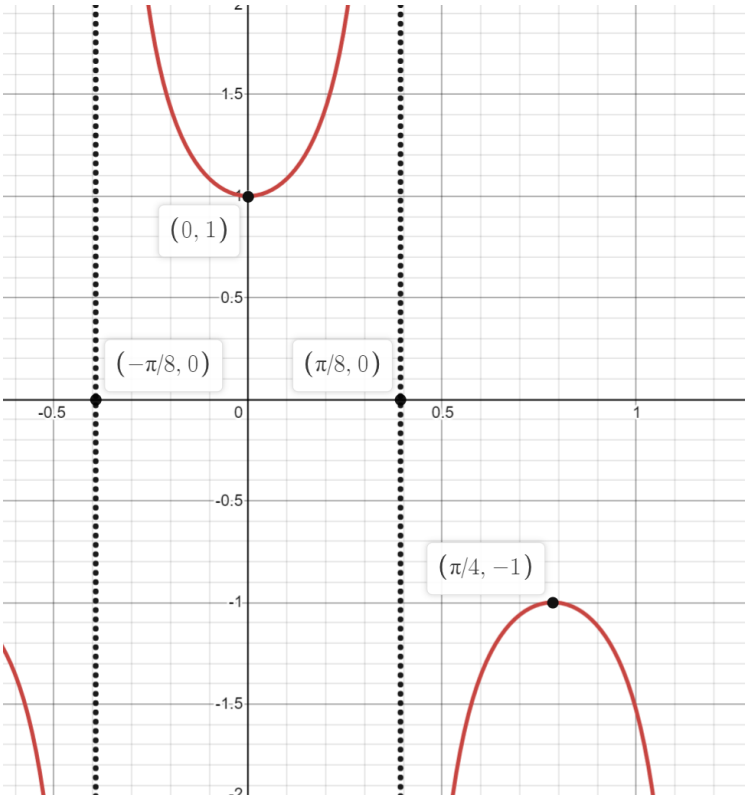
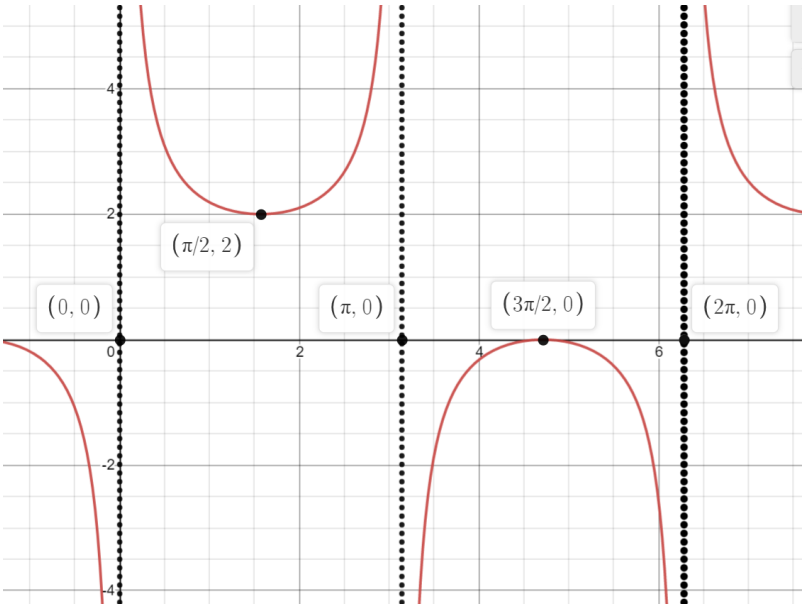
Lesson: Equations of Secant and Cosecant from Graphs

Determine the equation of secant or cosecant from the graphs.

a.



b.



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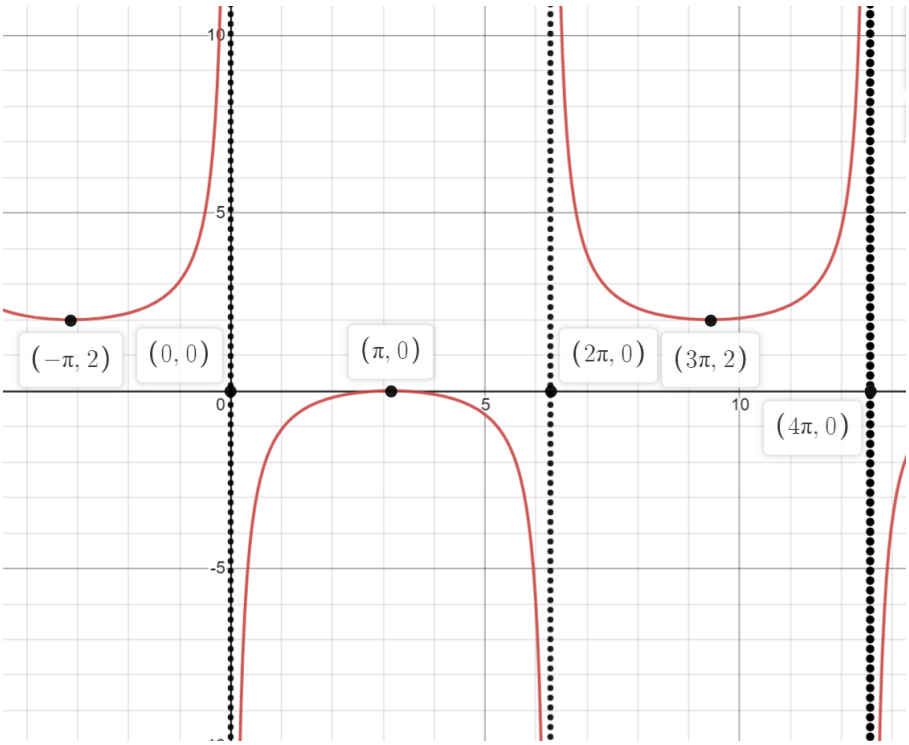
c.



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Lesson: Inverse Functions (a review)

What does it mean to be an "inverse function" ?

<u>Example</u> If a function includes the points $\{(2,3), (5,4), (-7,2), (-1,5)\}$, then what points will the inverse have?

What is the horizontal line test?

<u>Example</u> Draw the graphs from the video, then determine if they have inverses:

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How do you find the inverse of a function $f(x)$?

<u>Example</u> Find the inverse of the following:

a. $f(x) = 2x + 1$

b. $f(x) = \sqrt{x}$

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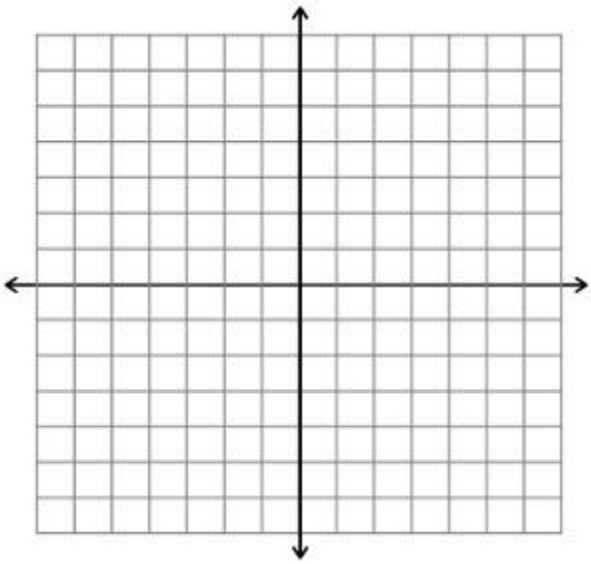
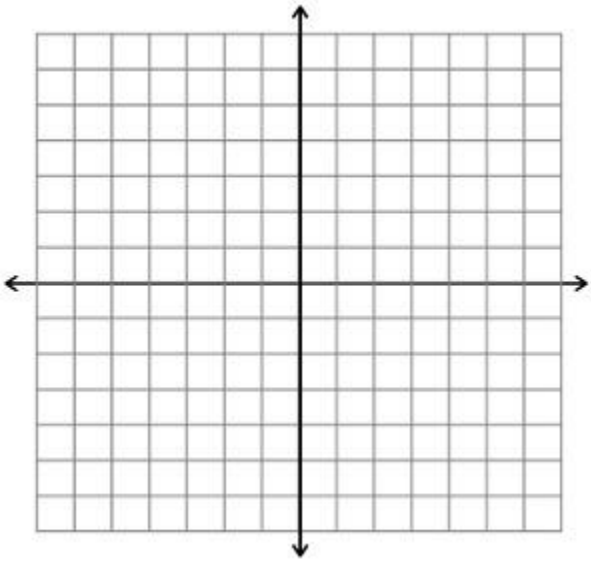
<u>Examples</u> Graph the following functions with their inverses:

a. $f(x) = 2x + 1$



b. $f(x) = \sqrt{x}$



From these graphs, we can see that inverses of functions are really just what?

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What can we say about the domain and range inverses of functions?

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Lesson: Inverse Trig Functions

Does the sine function have an inverse? Why or why not?

How can we manipulate the graph of the sine function to determine an inverse?

What are two ways that we denote the inverse sine function?

How do we “formally” define the inverse sine function?

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Show how to graph the inverse sine function. Draw both the table and graph discussed from the video:

<u>Example</u> Find the value of each of the following:

a. $y = \arcsin\frac{\sqrt{3}}{2}$

b. $f(x) = \sin^{-1}\left(-\frac{\pi}{4}\right)$

c. $y = \sin^{-1} -\sqrt{2}$

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Does the cosine function have an inverse? How should we think about this?

How do we denote the inverse of cosine?

How do we define the inverse cosine function?

Show how to graph the inverse cosine function. Draw both the table and graph discussed from the video:

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<u>Examples</u> Find the value of each of the following:

a. $y = \arccos 1$

b. $f(x) = \cos^{-1}\left(\frac{1}{2}\right)$

How do we denote the inverse of tangent?

How do we define the inverse tangent function?

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Show how to graph the inverse tangent function. Draw both the table and graph discussed from the video:

Define the other inverse circular functions:

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Summarize the domains, ranges, and quadrants of the inverse trig functions below (this is important, you'll need to study this for the next exam):

Function	Domain	Interval	Quadrants

<u>Examples</u> Find the degree measure of θ:

a. $\theta = \arctan\sqrt{3}$

b. $\theta = \csc^{-1}(-\sqrt{2})$

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Lesson: Finding Values of Inverse Trig Functions

Here we will just examine a wealth of examples of how to find different values of inverse trig functions.

<u>Examples</u> Use a calculator to determine the appropriate values.

a. Find y in radians if $y = \sec^{-1}(-4)$

b. Find θ in degrees if $\theta = \operatorname{arccot}(-0.25431)$

Before you evaluate those, you will have to watch some explanation of how / why we evaluate these the way that we do in the calculator. Write out the explanation of how/why we evaluate $\csc^{-1}(2)$ in a calculator below:

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Cheat Sheet – How to Evaluate Inverse Trig Functions That Don't Have a Button on a Calculator

Cheat Sheet (for Comparison) – How to Evaluate Regular Trig Functions That Don't Have a Button on a Calculator

<u>This might get confusing so maybe rip this sheet out and put it someplace where you can easily reference it??</u>

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Examples Use a calculator to find each value.

a. $\sin(\cos^{-1} 0.25)$

b. $\cot\,(\cos^{-1}(0.56331233)$

Examples Evaluate each expression without using a calculator:

a. $\cos\left(sin^{-1}\left(\frac{2}{3}\right)\right)$

b. $\sec\,(\sec^{-1} 2)$

c. $\sin\left(tan^{-1}\frac{4}{3} - cos^{-1}\frac{12}{13}\right)$

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<u>Examples</u> Write each expression as an algebraic expression in terms of u.

a. $\cot\left(\sec^{-1} u\right)\ , u > 0$

b. $\sec\left(\operatorname{arccot}\left(\frac{\sqrt{4-u^2}}{u}\right)\right.$

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Lesson: Oblique Triangles and the Law of Sines

First, we should review some facts about congruent triangles. When are 2 triangles congruent?

<u>Congruence Axioms</u>

Side-Angle-Side (SAS)

Angle-Side-Angle (ASA)

Side-Side-Side (SSS)

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When does Side-Angle-Angle (SAA) hold?

When a triangle is <u>not</u> a right triangle, what is it it called?

When can a triangle be solved?

What data is required for solving oblique triangles?

Case 1:

Case 2:

Case 3:

Case 4:

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Is there an Angle-Angle-Angle congruency?

When Case 1 or Case 2 is involved, what is required?

When Case 3 or Case 4 is involved, what is required?

This Law of Cosines will be discussed in another video.

Show how we derive the Law of Sines:

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State the Law of Sines:

What is an alternative form of the law of sines?

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Example Solve Triangle ABC if $A = 28.5°, C = 102.3°, c = 25.1$ in.

Recall the formula for the area of a triangle:

Using SAS, how else can you find the area of a triangle?

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Example Reference the sketch in the video. Find the area of the triangle.

Example Find the area of a triangle DEF if $D = 58°10', e = 31.2in, F = 73°30'$.

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Lesson: Applications of the Law of Sines

<u>Example</u> An engineer wishes to measure the distance across a river. Reference the sketch in the video.
If $C = 117.4°, A = 30.0°, b = 75.8$ ft

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Review the two methods of bearing:

<u>Example</u> The bearing of a lighthouse from a ship was found to be $N\ 52°$ W. The ship sailed 12.3 miles south, and the new bearing was $N\ 23°W$. Find the distance, to the nearest mile, between the ship and the lighthouse at each location.

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Example A balloonist is directly above a straight road, 2 miles long that joins the 2 villages. The closer town to him is an angle of depression of $45°$, and the farther town is at angle of depression of $41°$. How high above the ground is the balloon?

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Lesson: The Ambiguous Case of the Law of Sines

Describe the ambiguous case of the law of sines as describe in the video. There's going to be quite a bit of information presented – watch the entire explanation of why we need the law of sines and do your best to describe the various cases to sum up. Summarize the explanation so that it makes sense to YOU.

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What were some tips for applying the law of sines?

<u>Example</u> Solve triangle ABC if $a = 16.2cm, c = 14.1\ cm$, and $C = 72°30'$.

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<u>Example</u> Solve triangle DEF if $D = 62°8', a = 34.5in, b = 38.1in.$

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Explain the number of triangles satisfying the ambiguous case (SSA):

<u>Examples</u> Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

a. $B = 80°, C = 35°, b = 27$

b. $A = 60°, B = 70°, c = 5$

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Examples Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

c. $a = 15, b = 24, B = 33°$

d. $A = 25°, b = 8, a = 11$

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Examples Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

e. $A = 61.4°, b = 35.5, b = 39.1$

f. $a = 17.1, c = 13.1, C = 76°$

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<u>Example</u> Without using the law of sines, explain why no triangle ABC exists satisfying $B = 95°, b = 40cm, c = 46cm$.

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Lesson: The Law of Cosines

What is the property of triangle side length restriction?

Show how to derive the law of cosines:

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State the Law of Cosines:

Where is one place where we've seen this?

<u>Example</u> Two boats leave a harbor at the same time, traveling on courses that make an angle of $81°30'$ between them. When the slower boat has traveled 65.2 miles, the faster boat has traveled 84.3 miles. At that time, what is the distance between the boats?

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Example Solve triangle ABC if $B = 72.4°, a = 31.4ft, c = 44.3ft$.

Example Solve triangle DEF if $d = 24.2in, b = 42.6in, c = 59.1in$.

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Example The simplest type of roof truss is a triangle. Refer to the sketch in the video. Find angle C to the nearest degree for the truss shown.

Explain the 4 Cases of Solving Oblique Triangles

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Example: The Law of Cosines with SSS Triangles

Find the angles of the triangles given the following sides:

a. $a = 3ft, b = 5ft, c = 6ft$

b. $a = 324m, b = 421m, c = 298m$

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Lesson: Heron's Formula for the Area of a Triangle

Explain how to use Heron's formula (as explained in the video):

<u>Example</u> Find the area of triangle ABC where $a = 12m, b = 16m, c = 25m$

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Example Find the area of the Bermuda Triangle if the sides of the triangle have approximate lengths 800mi, 875mi, and 1250mi.

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Lesson: Fundamental Identities of Trig

Review the test for symmetry (and the results):

We know that the sine function is odd, show what this means:

We know that the cosine function is even, show what this means:

These are called **identities.**

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Using these identities, what other identity can we determine about the tangent function:

These are called the **even-odd identities.**

We have already seen a handful of identities, and we call these the fundamental identities. Summarize them below.

Reciprocal Identities

Quotient Identities

Pythagorean Identities

Even-Odd Identities

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Example If $\cos\theta = \frac{3}{4}$ and θ is in quadrant I, find $\sin\theta$.

Example If $\tan\theta = -\frac{1}{4}$ and θ is in quadrant IV, find the values of all the other 5 trig functions.

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We use the identities to work on problems called transformations.

Example Write $\tan x$ in terms of $\sec x$.

Example Write the following in terms of sine and cosine, and then simplify until no quotients appear and all functions are of θ only.

a. $\csc\theta \cos\theta \tan\theta$

b. $\cos\ \theta(\cos\theta - \sec\theta)$

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Examples: Using Fundamental Trig Identities to Find Values of Other Trig Functions

Find the five remaining trig functions given the following information.

a. $\csc\theta = -\frac{5}{2}$, θ in quadrant III.

b. $\sec\theta = \frac{7}{2}$, $\tan\theta < 0$

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Examples: Writing Trig Functions in Terms of Other Trig Functions (Fundamental Identities)

a. Write $\cot x$ in terms of $\sin x$

b. Write $\csc x$ in terms of $\cos x$

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Examples: Using Fundamental Identities to Write Expressions in Terms of Sine and Cosine

Write each expression in terms of sine and cosine, then simplify so that no quotients appear and all functions are in θ only.

a. $\cos\theta \csc\theta$

b. $\frac{(1+\cot\theta)}{\cot\theta}$

c. $\frac{\sec^2\theta-1}{csc^2\,\theta-1}$

d. $\frac{\csc\theta}{\cot(-\theta)}$

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Lesson: Verifying Trigonometric Identities

As you progress through higher mathematics, sometimes you will need to manipulate equations in very creative ways. One way to learn this level of creativity is by verifying trigonometric identities. We will go through a few techniques of how to do this.

<u>Strategy 1: Rewrite the More Complicated Side / Work from One Side</u>

a. Verify that $\frac{1-\sin^2 x}{\cos x} = \cos x$. Show *why.*

b. Verify that $\sin^2 x + \tan^2 x + \cos^2 x = \sec^2 x$

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Strategy 2: Express the Functions in Terms of All Sine and Cosine

a. $\frac{\tan x}{\sec x} = \sin x$

b. $\sec x\,(\sin x + \cos x) = 1 + \tan x$

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Strategy 3: Factor or Complete Indicated Algebraic Operations

Examples Factor.

a. $x^2 - 4$

b. $\sin^2 x - 4$

c. $(\tan x + \cot x)^2 - (\tan x - \cot x)^2$

d. $2\sin^2 x + 3\sin x + 1$

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Examples Verify the following identities.

a. $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

b. $\frac{1-\cos x}{1+\cos x} = (\cot x - \csc x)^2$

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Strategy 4: Manipulate Both Sides

a. $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = \sec^2 x - \tan^2 x$

b. $\tan^2 x \ \ sin^2 x = tan^2 x + cos^2 x - 1$

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Strategy 5: Use the Conjugate

What is the conjugate of the following:

a. $x - 4$

b. $\sqrt{2} - \sqrt{3}$

c. $\sin x + 1$

Verify the following identities: $\frac{1-\cos x}{1+\cos x} = \csc^2 x - 2\csc x \cot x + \cot^2 x$

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Examples: Verifying Trigonometric Identities

a. $\frac{\cos x}{\cot x \sin x} = 1$

b. $\frac{1}{\sec x - \tan x} = \sec x + \tan x$

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c. $\frac{1+\sin t}{1-\sin t} - \frac{1-\sin t}{1+\sin t} = 4\tan t \sec t$

d. $(\cos\theta - \sin\theta)(\sec\theta + \csc\theta) = \cot\theta - \tan\theta$

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Lesson: Solving Trig Equations, part 1 – linear methods

Recall how linear methods work first by solving: $2x + 4 = 8$

Now compare this approach with the equation: $3\tan\theta - \sqrt{3} = 0$, over the interval $[0°, 360°)$

How does this answer change if we want to solve for *all* solutions?

<u>Example</u> Solve: $2\sin x + 3 = 4$ over $[0, 2\pi)$

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Lesson: Solving Trig Equations, part 2 – quadratic methods

Using quadratic methods to solve trig equations relies on using the zero-factor property. Show how the zero-factor property works by solving $(x+5)(x-3)=0$.

Compare this approach by solving $\cos\theta\sin\theta = 0 = -\cos\theta$, over $[0°, 360°)$.

<u>Example</u> Solve $(\csc x + 2)\left(\csc x - \sqrt{2}\right) = 0$ for all solutions.

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Recall how to use quadratic methods by solving $x^2 + 5x + 6 = 0$.

Compare this approach by solving $\cos^2 x + 2\cos x + 1 = 0$ over $[0°, 360°)$.

<u>Example</u> Solve $2\cos^2 x + \cos x - 1 = 0$ for all possible solutions.

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Example Solve $2\cos^2 x - \sqrt{3}\cos x = 0$ over $[0, 2\pi)$

Example Solve $\tan x \csc x - \sqrt{3}\csc x = 0$ for all solutions.

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Example Solve $\csc^2 x - 2\cot x = 0$ over $[0°, 360°)$

Example Solve $\tan x - \sqrt{3} = \sqrt{3}\cot x - 1$ for all possible solutions.

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Lesson: Solving Trig Equations, part 3 – equations with half-angles

<u>Example</u> Solve $2\cos\frac{x}{2} - \sqrt{2} = 0$ over the interval $[0, 2\pi)$

How would the answers change if you needed to find all solutions?

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Example Solve $2\sqrt{3}\sin\frac{x}{2} = 3$ for all possible solutions.

Example Solve $\sin^2\frac{x}{2} - 2 = 0$ for all possible solutions.

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Lesson: Solving Trig Equations, part 4 – equations with multiple angles

<u>Example</u> Solve $\cos 2x = \sin x$ over $[0, 2\pi)$

<u>Example</u> Solve $2\cos^2 x - 2\sin^2 x + 1 = 0$ over $[0°, 360°)$

How would this change for all solutions?

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Example Solve $\cot 2x - \csc 2x = 1$ over $[0, 2\pi)$

Example Solve $\tan 2x - \sec 2x = 2$ over $[0, 2\pi)$

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Example Solve $\cos x - 1 = \cos 2x$ for all solutions.

Example Solve $\sin 2x = 2\cos^2 x$ for all solutions.

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Lesson: Equations Involving Inverse Trig Functions

<u>Example</u> Solve $y = 4\tan 3x$, where x is restricted to $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$.

<u>Example</u> Solve $y = \tan\,(2x - 1)$ for x in $\left(\frac{1}{2} - \frac{\pi}{4}, \frac{1}{2} + \frac{\pi}{4}\right)$

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Example Solve $3\tan^{-1} x = \pi$

Example Solve $6\arccos\left(x - \frac{\pi}{3}\right) = \pi$

Example Solve $\cot^{-1} x = \tan^{-1}\frac{4}{3}$

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Example Solve $\arccos x + 2\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

Example Solve $\sin^{-1} x + \tan^{-1} x = 0$

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Lesson: Proofs of the Sum and Difference Identities for Cosine

Write out the proof for $\cos(A - B) = \cos A \cos B + \sin A \sin B$. Do your best to understand the proof and then explain it in a way that you will understand.

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Write out the identity for $\cos\,(A - B)$ below (yes I know I stated it above, but YOU should write it so you have a better chance of remembering it):

Write out the proof for $\cos(A + B) = \cos A \cos B - \sin A \sin B$

Write out the identity below (yes, write it out so you don't forget!):

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Lesson: Using Sum and Difference Identities for Cosine

<u>Examples</u> Show how to use the cosine of a sum or difference to find the exact value of the expressions:

a. $\cos 105°$

b. $\cos \frac{\pi}{12}$

c. $\cos 40° \cos 50° - \sin 40° \sin 50°$

Show how the cofunction identities are an extension of the cosine of a difference:

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Write out all the **cofunction identities**

<u>Examples</u> Find one value of θ that satisfies the following:

a. $\sec\theta = \csc 72°$

b. $\tan\theta = \cot(-44°)$

c. $\cos\theta = \sin\frac{5\pi}{6}$

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Example Write $\cos(180° - \theta)$ as a trigonometric function of θ alone.

Example Suppose $\sin s = \frac{3}{5}$ and $\sin t = -\frac{12}{13}$, where s is in quadrant I and t is in quadrant III. Find $\cos\,(s + t)$ and $\cos(s - t)$.

Example Verify that $\sec(\pi + x) = -\sec x$

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Lesson: Sum and Difference Identities for Cosine

Write out the proof for $\sin(A + B)$

Write out the proof for $\sin(A - B)$

Write out the identities for the sum and difference of sine below:

Write out the proof for $\tan\,(A + B)$

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Write all identities for the sum and difference of tangent below:

<u>Examples</u> Find the exact value of each expression.

a. $\sin(-15°)$

b. $\tan\left(-\frac{5\pi}{12}\right)$

c. $\sin\left(\frac{5\pi}{9}\right)\cos\left(\frac{\pi}{18}\right) + \cos\left(\frac{5\pi}{9}\right)\sin\left(\frac{\pi}{18}\right)$

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<u>Examples</u> Write each function as an expression involving functions of θ alone.

a. $\cos(30° + \theta)$

b. $\sin\,(45° + \theta)$

c. $\tan\left(\frac{\pi}{4} + x\right)$

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Examples Suppose A and B are angles in standard position such that $\sin A = -\frac{3}{5}\ \frac{3\pi}{2} < A < 2\pi$ and

$\cos B = -\frac{7}{25}$, $\pi < B < \frac{3\pi}{2}$. Find the following.

a. $\sin(A + B)$

b. $\tan(A - B)$

c. the quadrant of A-B

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<u>Examples</u> Verify the identities.

a. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

b. $\frac{\sin(x-y)}{\sin(x+y)}=\frac{\tan x-\tan y}{\tan x+\tan y}$

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Examples: Sine of a Sum and Tangent of a Sum

Let $\cos s = \frac{5}{12}$, $\sin t = \frac{4}{5}$ where s and t are in quadrant I. Find:

$\sin(s + t)$

$\tan\,(s + t)$

The quadrant of $(s + t)$

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Lesson: Double-Angle Identities

Show how we derive the first identity for $\cos 2A$:

Show how we derive the second identity for $\cos 2A$:

Show how we derive the third identity for $\cos 2A$:

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Show how we derive the identity for $\sin 2A$:

Show how we derive the identity for $\tan 2A$:

Summarize the Double-Angle Identities Below:

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Example Give $\sin\theta = \frac{5}{8}$ and $\cos\theta < 0$, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

Example Find the values of the six trig functions give $\cos 2\theta = -\frac{11}{13}$ and $180° < \theta < 270°$.

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Example Verify the following identities:

a. $(\sin x + \cos x)^2 = \sin 2x + 1$

b. $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$

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Examples Simplify the expressions.

a. $\sin^2 15° - \cos^2 15°$

b. $\sin^2 \frac{3\pi}{5} - \cos^2 \frac{3\pi}{5}$

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Example Write $\cos 3x$ in terms of $\cos x$

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Lesson: Product-to-Sum Identities

Show what happens when we add the identities $\cos(A+B)$ to $\cos(A-B)$

Now show what happens when we subtract them:

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Examples Write the following as the sum or difference of 2 trig functions.

a. $2\sin 48° \cos 104°$

b. $8\sin 9x \sin 5x$

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Examples Write the expression as a product of trigonometric functions.

a. $\cos 6x + \cos 9x$

b. $\sin 106° - \sin 96°$

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Lesson: Half-Angle Identities

Show how we derive the identity for $\sin\frac{A}{2}$:

Show how we derive the identity for $\cos\frac{A}{2}$:

Show the first way we can derive an identity for $\tan\frac{A}{2}$:

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Show the second way we can derive an identity for $\tan\frac{A}{2}$:

Show the third way we can derive an identity for $\tan\frac{A}{2}$:

Summarize the half-angle identities below:

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Example Show how to use the half-angle identity to evaluate: $\sin 12.5°$

Example Find the exact value of $\cos 165°$

Example Find the exact value of $\tan 75°$ using the identity $\tan A = \frac{\sin A}{1+\cos A}$

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<u>Example</u> Given $\cos s = \frac{2}{7}$ with $0 < s < \frac{\pi}{2}$, find $\sin\frac{s}{2}$, $\cos\frac{s}{2}$, and $\tan\frac{s}{2}$

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<u>Example</u> Simplify each expression.

a. $\sqrt{\frac{1-\cos 40°}{2}}$

b. $\frac{1-\cos 64.72°}{\sin(64.72°)}$

c. $\pm\sqrt{\frac{1+\cos 10x}{2}}$

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Example Verify the identities.

a. $\cot^2 \frac{x}{2} = \frac{(1+\cos x)^2}{\sin^2 x}$

b. $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

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Lesson: Summarizing All of the Identities

Using all the trig identities that we've discussed, verify the following:

a. $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

b. $1 - \tan^2 \frac{x}{2} = \frac{2\cos x}{1+\cos x}$

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c. $\tan\left(\frac{a}{2}+\frac{\pi}{4}\right)=\sec a+\tan a$

d. $\sin 7x+\sin 5x+\sin 3x+\sin x=4\sin 4x\cos 2x\cos x$

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e. $\tan 4x = \frac{2\tan 2x}{2-\sec^2 2x}$

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Lesson: A Quick Review of the Basics of Solving Systems of Linear Equations

Systems of equations are 2 or more equations in 2 or more variables. They can look like:

$\begin{cases} 2x + 3y = 6 \\ x - 3y = 3 \end{cases}$ or $\begin{cases} 2x - y + z = 8 \\ 5x + 2y - z = 9 \\ x - 4y + z = 10 \end{cases}$ or $\begin{cases} 3x + 2y - z = 7 \\ x - 4y + 2z = 8 \end{cases}$

A solution of a system is an ordered pair that is true in each equation of the system. Some systems have no solutions, and some systems have infinite solutions.

In this lesson, we will briefly review the main techniques to solve systems. We will focus on systems that have a solution.

<u>Example</u> Use the substitution method to solve: $\begin{cases} 3x - y = -4 \\ x + 3y = 12 \end{cases}$

<u>Example</u> Use the elimination method to solve: $\begin{cases} 3x - y = -4 \\ x + 3y = 12 \end{cases}$

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<u>Example</u> Solve: $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 4 \\ \frac{3x}{2} + \frac{3y}{2} = 15 \end{cases}$

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<u>Example</u> Solve: $\begin{cases} 2x + y + z = 9 \\ -x - y + z = 1 \\ 3x - y + z = 9 \end{cases}$

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Examples: Systems of 3 Linear Equations with 3 Variables

Solve: $\begin{cases} x + y + z = 2 \\ 2x + y - z = 5 \\ x - y + z = -2 \end{cases}$

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Solve: $\begin{cases} x + 2y + 3z = -11 \\ 3x - y + z = 0 \\ -2x + 3y - z = 4 \end{cases}$

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Lesson: Systems with No Solutions or Infinite Solutions

<u>Example</u> Solve: $\begin{cases} x + 4y - 3z = 2 \\ 2x - 5y + 2z = -8 \\ -3x - 12y + 9z = 7 \end{cases}$

<u>Example</u> Solve: $\begin{cases} 7x + 2y = 6 \\ 14x + 4y = 12 \end{cases}$

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<u>Example</u> Solve: $\begin{cases} 5x - 4y + z = 9 \\ y + z = 15 \end{cases}$

<u>Example</u> $\begin{cases} 3x - 2y + z = 15 \\ x + 4y - z = 11 \end{cases}$

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<u>Example</u> Solve: $\begin{cases} 2x + y - 3z = 0 \\ 4x + 2y - 6z = 0 \\ x - y + z = 0 \end{cases}$

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Lesson: Using Matrices to Solve Systems of Linear Equations

What is a matrix?

Show how to setup the associated **augmented matrix** for the system: $\begin{cases} 3x + 4y = 4 \\ x - y = 13 \end{cases}$

What are the dimensions of this matrix?

In general, how do we state dimensions of a matrix (circle): rows x columns columns x rows

<u>Example</u> What is the dimension of

a.

$$\begin{bmatrix} 3 & 2 & 4 & 1 \\ 7 & 8 & 1 & 2 \\ 9 & 5 & -2 & -3 \end{bmatrix}$$

b.

$$\begin{bmatrix} 2 & 4 \\ 5 & 3 \\ 8 & 3 \\ -1 & -4 \end{bmatrix}$$

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With matrices, there are 3 **row transformations** that we can make. We are not allowed to do anything else to a matrix. Write down these 3 transformations.

Now, recall, our goal in this lesson is to use a matrix to solve systems of linear equations. What do we call this specific solving process?

What do we call the “ideal form” of a matrix in its solved form?

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The Gauss-Jordan method has a goal of transforming a matrix into reduced-row echelon form or diagonal form. Draw a few examples of how the FINAL matrix should look after we transform it:

When we use the Gauss-Jordan method, which direction do we want to work on the matrix?

What is the general strategy to remember with this method?

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Example Outline the steps for the Gauss-Jordan method for the system: $\begin{cases} -2x + 4y = -4 \\ x - y = 4 \end{cases}$

Example Use Gauss-Jordan method for the system: $\begin{cases} 3x + 4y = 4 \\ x - y = 13 \end{cases}$

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Lesson: Using The Gauss-Jordan Method for a System with 3 Equations

<u>Example</u> Use the Gauss-Jordan method for the system: $\begin{cases} x + y + z = 2 \\ 2x + y - z = 5 \\ x - y + z = -2 \end{cases}$

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Another page for you to show your work …

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Lesson: Using Matrices to Solve Systems of Linear Equations when There is No Solution or Infinite Solutions

<u>Example</u> Use the Gauss-Jordan method to solve: $\begin{cases} 3x + 5y - z = -2 \\ 4x - y + 2z = 1 \\ -6x - 10y + 2z = 0 \end{cases}$

<u>Example</u> Use the Gauss-Jordan method to solve: $\begin{cases} 3x + 5y = -2 \\ 9x + 15y = -6 \end{cases}$

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<u>Example</u> Use the Gauss-Jordan method to solve: $\begin{cases} x - 2y + 3z = 6 \\ 2x - y + 2z = 5 \end{cases}$

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<u>Example</u> Use the Gauss-Jordan method to solve: $\begin{cases} 5x - 4y + z = 0 \\ x + y = 0 \\ -10x + 8y - 2z = 0 \end{cases}$

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Examples: Using Matrices to Solve Systems of Linear Equations (any type of solutions)

Use the Gauss-Jordan method to solve the following.

a. $\begin{cases} x + 4y - z = 6 \\ 2x - y + z = 3 \\ 3x + 2y + 3z = 9 \end{cases}$

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b. $\begin{cases} 2x + y - 3z = 0 \\ 4x + 2y - 6z = 0 \\ x - y + z = 0 \end{cases}$

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c. $\begin{cases} 3x - 2y + z = 15 \\ x + 4y - z = 11 \end{cases}$

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Lesson: How to Calculate the Determinate of a Matrix

Every n x n matrix has a real number associated with it called the **determinant.** What is the notation to represent this?

<u>Example</u> Show, in detail, how to calculate the determinants of the following matrices.

a. $A = \begin{bmatrix} 2 & 4 \\ -5 & 7 \end{bmatrix}$

b. $B = \begin{bmatrix} -5 & 1 \\ -3 & 4 \end{bmatrix}$

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<u>Example</u> Show, with illustrations, how to calculate the determinant of $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & -3 \\ 3 & 4 & -2 \end{bmatrix}$

The 2x2 "sub" matrices that we calculate are called minors. The symbol M_{ij} represents the minor that results when row i and column j are eliminated. Using the matrix above, describe the following minors.

$M_{11} =$

$M_{32} =$

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The number that we multiply the minor against is called the **cofactor.** Draw an illustration to show whether a cofactor will be positive or negative:

<u>Example</u> Using the matrix below, describe the minor associated with each cofactor AND determine whether the cofactor is positive or negative. You do not need to calculate the determinant.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ -3 & -5 & -1 \end{bmatrix}$$

a. 2

b. 3

c. -3

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<u>Example</u> Show how to efficiently calculate the determinants of the following matrices. EXPLAIN YOUR REASONING FOR WHY YOU DID IT THE WAY THAT YOU DID.

a. $A = \begin{bmatrix} 2 & 7 & 8 \\ -1 & 0 & 1 \\ 3 & 5 & 1 \end{bmatrix}$

b. $B = \begin{bmatrix} 9 & -3 & 1 \\ 2 & 3 & 0 \\ 5 & 2 & 0 \end{bmatrix}$

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Lesson: Determinant Theorems

Theorem 1: If every element (or column) of a matrix is 0, then the determinant will be 0.

<u>Example</u>: find the determinant of $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 4 & 2 & 8 & 9 \\ 3 & 2 & 8 & 4 \\ 9 & 3 & 1 & 7 \end{bmatrix}$

Theorem 2: If the rows of a matrix A are the corresponding columns of a matrix B, then the determinants are equal.

<u>Example:</u> Calculate both determinants: $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 3 & 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$

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Theorem 3: If any 2 rows (or columns) of a matrix are interchanged, then the determinants are negatives of one another.

<u>Example</u> Find the determinants of: $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 2 & 4 \end{bmatrix}$

Theorem 4: Suppose matrix B is formed by multiplying every element in a row or columns in a matrix A by a real number k. Then the determinant is $|B| = k|A|$.

<u>Example:</u> Find the determinants of: $A = \begin{bmatrix} 2 & 1 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 4 \\ 3 & 6 \end{bmatrix}$

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Theorem 5: If two rows (or columns) of a matrix are identical, then the determinant will be 0.

<u>Example</u> Find the determinant of: $A = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$

Theorem 6: Changing a row (or column) or a matrix by adding it to a constant times another row (or column) does not change the determinant of the matrix.

<u>Example</u> Find the determinant of $A = \begin{bmatrix} 1 & 2 \\ 5 & 8 \end{bmatrix}$. Then create a matrix B by taking adding $3 *$ (row 3) to row 1. Find the new determinant.

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Theorem 7: If a matrix is in triangular form (all 0s either above or below the diagonal), then the determinant is found by multiplying the elements down the diagonal.

<u>Example</u>: Find the determinant of $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 8 & 7 & 3 & 1 \end{bmatrix}$

<u>Examples</u> Use the determinant theorems to find the following determinants.

a. $\begin{vmatrix} -4 & 1 & 4 \\ 2 & 0 & 1 \\ 0 & 2 & 4 \end{vmatrix}$

b. $\begin{vmatrix} 1 & 0 & 8 \\ 4 & 0 & 2 \\ 0 & 0 & 8 \end{vmatrix}$

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Lesson: Properties of Matrices: Equality, Addition, Subtraction, and Scalar Multiplication

When are 2 matrices equal?

<u>Example</u> What values of x, y, z, and w will make the matrices equal?

$$\begin{bmatrix} 2+x & 3-w \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 2-y & 2+z \end{bmatrix}$$

When can you add 2 matrices?

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What is the definition of matrix subtraction?

What is scalar multiplication with a matrix?

List the properties of scalar multiplication for matrices:

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<u>Example</u> Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}, B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 7 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 7 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Complete the following operations, if possible.

a. $A + C$

b. $B - D$

c. $B - E$

d. $2B + E$

e. $2A - 3C$

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Lesson: Matrix Multiplication

First, let's review dimension. What is the dimension of the following matrices?

$A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}$

$B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}$

To understand how to multiply matrices, let's use the following 2 examples:

a. $\begin{bmatrix} 2 & 7 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 7 & 2 \end{bmatrix}$

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<u>Example</u> Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}, B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 7 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 7 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Complete the following operations, if possible.

a. AC

b. CA

Using the above example, what can we say about matrix multiplication and commutativity?

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List the properties of matrix multiplication:

<u>Example</u> Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}, B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 7 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 7 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Complete the following operations, if possible.

a. BC

b. ED

c. CB

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Lesson: Inverse Matrices

What is the 2x2 identity matrix?

What is the 3x3 identity matrix?

How do we describe the general nxn identity matrix?

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<u>Example</u> Multiply $A = \begin{bmatrix} 3 & 2 & 1 \\ 8 & 1 & 7 \\ 3 & 1 & 0 \end{bmatrix}$ by the identity matrix as follows:

a. AI

b. IA

What can we generalize about the identity matrix?

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How do we describe a multiplicative inverse of a real number?

Some matrices have inverses. For a matrix A, what notation denotes its inverse?

What is the property of inverse matrices?

What are the steps for finding an inverse matrix?

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<u>Example</u> Find the inverses of the following matrices. Prove your answer by checking it: $\begin{bmatrix} 6 & -2 \\ 2 & 4 \end{bmatrix}$

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<u>Example</u> Find the inverses of the following matrices. Prove your answer by checking it: $\begin{bmatrix} 3 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

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<u>Example</u> Find the inverses of the following matrices. Prove your answer by checking it: $\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$

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Lesson: Using Inverse Matrices to Solve Systems

We've previously talked about using matrices to solve systems. We will look at another way to do that in this lesson. Show another way to set up the system using matrices (as well as the relevant notation):

$$\begin{cases} x + y + z = 6 \\ 2x + 3y - z = 7 \\ 3x - y - z = 6 \end{cases}$$

Now, using our setup above, explain the reasoning for how inverse can help solve a system:

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<u>Example</u> Use inverse matrices to solve: $\begin{cases} -x + y = 1 \\ 2x - y = 1 \end{cases}$

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<u>Example</u> Solve the system: $\begin{cases} x + y + z = 6 \\ 2x + 3y - z = 7 \\ 3x - y - z = 6 \end{cases}$ using inverse matrices.

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Lesson: Nonlinear Systems of Equations

Nonlinear systems of equations involve at least one equation that is not linear. Some examples look like this:

$\begin{cases} x^2 - y = 8 \\ x + y = -4 \end{cases}$ or $\begin{cases} x^2 + y^2 = 4 \\ |x| + y = 2 \end{cases}$

Similar to systems of linear equations, we can still use substitution or elimination to solve the system. However, the best method to use tends to depend on the type of system you are trying to solve. Additionally, you might have to use both methods.

What is one potential issue that can happen with solving systems of nonlinear equations and what should you do about this?

When one equation is linear and the other is not, it is usually best to use the substitution method.

<u>Example</u> Solve the system: $\begin{cases} x^2 + y = 8 \\ x - y = -2 \end{cases}$

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When both variables in both equations are squared, it's usually best to use elimination.

<u>Example</u> Solve the system: $\begin{cases} x^2 + y^2 = 25 \\ x^2 + 2y^2 = 9 \end{cases}$

Sometimes you will have to use both the substitution and elimination methods.

<u>Example</u> Solve the system: $\begin{cases} x^2 - xy + y^2 = 5 \\ 2x^2 + xy - y^2 = 10 \end{cases}$

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Now let's look at a few other examples to get a sense of how to use these methods!

<u>Example</u> Solve the system: $\begin{cases} x^2 + y^2 = 9 \\ -|x| + y = 0 \end{cases}$

<u>Example</u> Solve the system: $\begin{cases} 3x^2 + y^2 = 3 \\ 4x^2 + 5y^2 = 26 \end{cases}$

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Lesson: An Intro to Conic Sections

What are the 4 conic sections we will discuss? Draw an illustration with it.

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Lesson: Parabolas as Conic Sections

What is the previous equation we used for parabolas opening upwards or downwards? Draw a illustration to go with this.

How can we represent parabolas in similar notation for parabolas that open sideways?

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<u>Examples</u> Give the domain and range of the following parabolas, then graph the parabola.

a. $x + 1 = (y + 1)^2$

b. $-x = 3y^2 + 6y + 2$

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When dealing with conic sections, we shift how we think about things.

What is the geometric definition of a parabola? Draw an illustration to accompany this definition.

What equations do we use that align with the geometric definition?

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Examples Give the focus, directrix, vertex, and axis of symmetry for each parabola. Then use this information to graph the parabola.

a. $x^2 = 4y$

b. $(y-2)^2 = 24(x-3)$

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Lesson: Writing Equations of Parabolas

<u>Example</u> Write the equation of a parabola with the following characteristics.

a. focus $\left(0, -\frac{1}{2}\right)$ with the vertex at the origin

b. vertex (4, 3) and focus is (4, 5)

c. vertex (1, 2), directrix x=4

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Lesson: Equations of Circles and How to Graph Circles

Draw the circle in the video and label the relevant parts of it.

Using your sketch above, explain how the distance formula can be used to help us determine the equation of a circle (you will need to summarize the explanation in your own words).

What is the **Center-Radius Form** of the equation of a circle?

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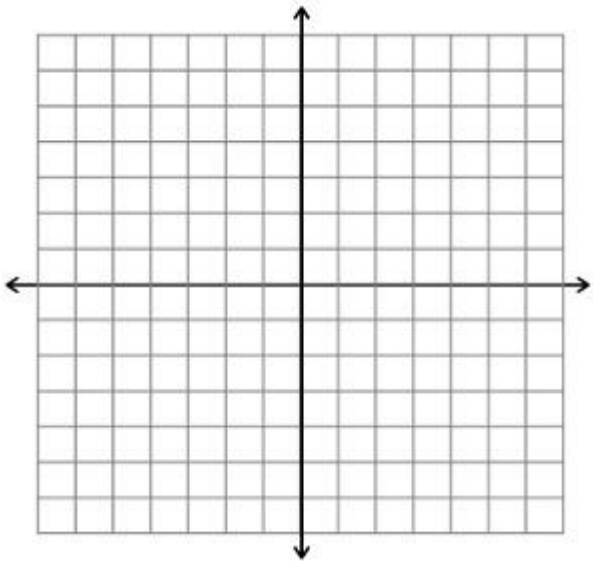
Example Find the center radius form of a circle using the given information.

a. center $(-2, 3)$ and radius 4

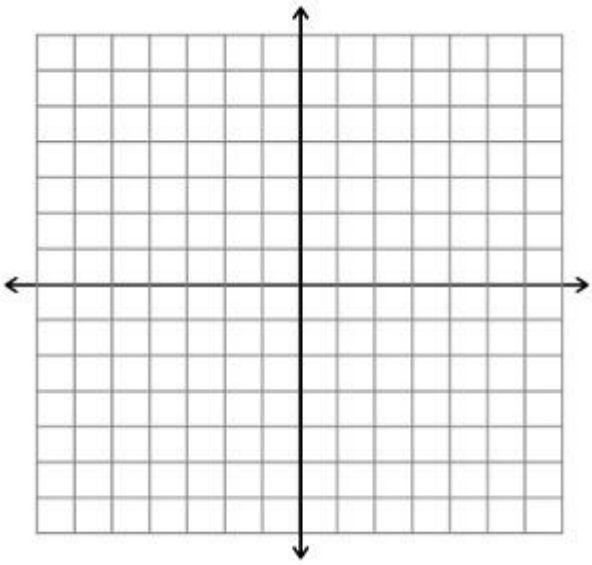
b. center $(0, -1)$ and radius 2

Example Graph the following circles.

a. $(x-1)^2 + (y+3)^2 = 4$



b. $x^2 + y^2 = 9$



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Expand $(x-1)^2+(y+3)^2=4$ and show how this creates the general form.

What is the **General Form** of the equation of a circle?

<u>Example</u> Review how to use completing the square with the equation: $x^2-6x+4=0$.

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<u>Example</u> Show how to use completing the square to write each in the center-radius form, and then sketch a graph of the circle.

a. $x^2 - 4x + y^2 + 8y - 44 = 0$

b. $x^2 + y^2 + 2x - 6y + 14 = 0$

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Examples: Working with the General Form Equation of a Circle

<u>Example</u> Show how to use completing the square to write in the center-radius form, and then sketch a graph of the circle if possible.

a. $4x^2 + 4y^2 + 4x + 16y - 19 = 0$

b. $x^2 + y^2 - 4x - 4y + 8 = 0$

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c. $x^2 + y^2 + 2x - 4y + 18 = 0$

d. $9x^2 + 9y^2 + 12x - 18y - 23 = 0$

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Examples: Graphing Circles Using Technology

You can use this page to take notes for how to solve the following 2 examples:

a. Find all coordinates of points whose distance from (-3, 2) and (-2, 1) is $\sqrt{5}$.

b. There's an earthquake! There are 2 receiving stations with the following coordinates:

$$A = (-1, 1), B = (-2, 1), C = (-3, -1)$$

The epicenter of the earthquake is 2 units from stations A and C, and $\sqrt{5}$ units from B. Where is the epicenter?

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Lesson: Ellipses as Conic Sections

Give the geometric definition of an ellipse. Draw an illustration to accompany this.

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What are the standard forms of an ellipse? Describe the parts and draw VERY CLEAR illustrations.

What is important to note with this equation?

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Example Graph each ellipse. Give the domain, range, center, vertices, endpoints of the minor axis, and focus.

a. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

b. $9x^2 + y^2 = 81$

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<u>Example</u> Graph each ellipse. Give the domain, range, center, vertices, endpoints of the minor axis, and focus.

c. $\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1$

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Lesson: Writing Equations of Ellipses

Example Write the equation of an ellipse with the following characteristics.

a. major axis 6, focus at (0,2) and (0, -2)

b. center at (-2, 7), major axis vertical with length 10, and c=2

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Lesson: Eccentricity with Ellipses

To discuss the concept of eccentricity, we need to discuss what the formal mathematical definition of conic sections is.

Define a conic:

The intuitive idea behind eccentricity can change slightly with each conic section, but it is generally a ratio of different measures within the conic section.

What does the eccentricity of an ellipse tell us?

How do we represent eccentricity?

How do you calculate the eccentricity of an ellipse?

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How does the eccentricity of a parabola compare to the eccentricity of an ellipse? Summarize the explanation from the video.

<u>Examples</u> Find the eccentricity of each ellipse.

a. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

b. $x^2 + 25y^2 = 25$

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<u>Example</u> Find the equation of an ellipse with $e = \frac{3}{4}$ with foci at $(-2,0), (2,0)$

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Examples: Using Complete the Square to Write an Ellipse Equation in Standard Form

Rewrite the following equations in standard form.

a. $x^2 + 12y^2 - 14x - 48y = -85$

b. $9x^2 + 36x + 16y^2 - 160y + 292 = 0$

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Lesson: Hyperbolas as Conic Sections

What is the geometric definition of a hyperbola? Draw an illustration with your definition.

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State the equations for the standard form of the hyperbola. Draw illustrations with it.

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<u>Examples</u> Graph each hyperbola. Give the domain, range, center, vertices, foci, and equations of the asymptotes.

a. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

b. $9x^2 - 25y^2 = 225$

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<u>Examples</u> Graph each hyperbola. Give the domain, range, center, vertices, foci, and equations of the asymptotes.

c. $\frac{(y-1)^2}{9} - \frac{(x+3)^2}{4} = 1$

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Lesson: Writing Equations of Hyperbolas

<u>Examples</u> Write the equation of the hyperbola with the following characteristics.

a. vertices at (0, 6) and (0, -6) with asymptotes $y = \pm\frac{1}{2}x$

b. foci at $\left(-3\sqrt{5}, 0\right), (3\sqrt{5}, 0)$ with asymptotes $y = \pm 2x$

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Examples Write the equation of the hyperbola with the following characteristics.

c. center at (1, -2) and focus at (-2, -2) and vertex at (-1, -2)

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Lesson: Eccentricity of Hyperbolas

How do you calculate the eccentricity of a hyperbola?

What can we say about the eccentricity of the hyperbola? What does it mean?

<u>Example</u> Find the eccentricity of the hyperbolas.

a. $\frac{x^2}{36} - \frac{y^2}{9} = 1$

b. $8y^2 - 16x^2 = 16$

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<u>Example</u> Find the equation of a hyperbola with $e = \frac{25}{9}$ and foci at (9, -1) and (-11, -1)

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Examples: Writing the Standard Form of a Hyperbola

Write each hyperbola in standard form.

a. $x^2 - 6x - y^2 + 4y = 1$

b. $y^2 - 14y - x^2 + 14x = 16$

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Lesson: Vectors

What is a scalar?

What are vector quantities?

What is a vector?

Draw some vectors showing the differences with magnitude and directions.

Explain some of the notation we use regarding these concepts:

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When are two vectors equal?

Is the sum of two vectors still a vector? (circle) Yes No

How do you find the sum of two vectors?

Vector addition is commutative. What does this mean?

For every vector, there is an opposite vector. Explain what this means:

The sum of **v** and **-v** results in what?

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What happens when you multiply a vector and a scalar?

Explain the geometric properties of parallelograms:

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Examples For each pair of vectors and the angle θ between them, sketch the result.

a. $|\boldsymbol{u}| = 12, |\boldsymbol{v}| = 22, \theta = 25°$

b. $|\boldsymbol{u}| = 20, |\boldsymbol{v}| = 40, \theta = 30°$

Examples Use the parallelogram in the video to find the magnitude of the resultant force for the 2 forces shown in the figure in the video.

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Lesson: Vectors and Applications of Magnitude

Example A boat is being towed by two other boats, with the first boat having a force of 840lbs and the second have 925lb. The angle between these forces is 26°. Find the direction and magnitude of the equilibrant.

Example Two forces of 150 lbs and 275 lbs act at a point. The resultant force between them is 350 lbs. Find the angle between the forces.

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Lesson: Vectors and Applications of Navigation

<u>Example</u> A ship leaves port on a bearing of $27°$ and travels 62.4miles. The ship then turns due east and travels 85.2 miles. How far is the ship from the port? What is its bearing from port?

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Lesson: Vectors and Applications of Incline

<u>Example</u> Find the force required to keep 2700-lb car parked on a hill that makes a $13°$ angle with the horizontal.

<u>Example</u> A force of 18.00 lb is required to hold a 74.0 lb crate on a ramp. What angle does the ramp make with the horizontal?

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Lesson: Vectors (Algebraic Interpretation)

Vectors are defined algebraically in the rectangular coordinate system with what notation?

Draw how we interpret this vector below.

In a previous video, we discussed how to define vectors geometrically. How is this definition different from our algebraic interpretation?

Show how to define a direction angle.

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How do we calculator the magnitude of a vector?

How do we calculation the direction angle of the same vector $\boldsymbol{u} = \langle a, b \rangle$?

<u>Example</u> Calculate the magnitude and direction for $\boldsymbol{u} = \langle 5, 2 \rangle$.

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Draw how we can view the horizontal and vertical components of vectors.

Example A vector has magnitude 36.0 and direction angle $35°$. Find the horizontal and vertical components.

Example Copy the sketch in the video. Write the vectors in the form $\langle a, b \rangle$.

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What are the vector operations?

<u>Example</u> Let $\boldsymbol{u} = \langle -3, 4 \rangle, \boldsymbol{v} = \langle 5, 7 \rangle$. Find and illustrate each of the following.

a. $\boldsymbol{u} + \boldsymbol{v}$

b. $-3\boldsymbol{v} + 2\boldsymbol{u}$

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What is the i,j form of vectors?

<u>Example</u> Write $\langle 3, 5 \rangle$ in i,j form

<u>Example</u> Write $2\boldsymbol{i} - 4\boldsymbol{j}$ in bracket vector form.

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Lesson: The Dot Product

What is the dot product?

What is another name for the dot product?

Which is true: the dot product results in a vector the dot product results in a real number

<u>Example</u> Find the dot products.

a. $\langle 3, 5\rangle \cdot \langle 2, -1\rangle$

b. $\langle 2, -3\rangle \cdot \langle -4, -5\rangle$

What are the properties of the dot product?

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<u>Examples</u> Let $\boldsymbol{u} = \langle 1, 3 \rangle, \boldsymbol{v} = \langle 2, 4 \rangle, \boldsymbol{w} = \langle -1, 2 \rangle$. Evaluate the following.

a. $\boldsymbol{u} \cdot \boldsymbol{v} - \boldsymbol{u} \cdot \boldsymbol{w}$

b. $(3\boldsymbol{u}) \cdot \boldsymbol{w}$

What are the possible outcomes of the dot product?

What is the geometric interpretation of the dot product? Draw a sketch.

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Example Find the angle between the vectors.

a. $\boldsymbol{u} = \langle 1, 3 \rangle, \boldsymbol{v} = \langle 2, 4 \rangle$

b. $\boldsymbol{u} = \langle 1, -2 \rangle, \boldsymbol{v} = \langle -1, -3 \rangle$

Explain how to interpret the dot product in the following scenarios …

The Dot Product is …	The Angle between the Vectors is …
Positive	
Negative	
Zero	

When the dot product is 0, what does this mean?

What's the better word for perpendicular? (fancier, makes you sound smart, you should definitely use it to impress people at parties):

Example Determine if the pair of vectors is orthogonal: $\boldsymbol{u} = \langle 1, 2 \rangle, \boldsymbol{v} = \langle -6, 3 \rangle$

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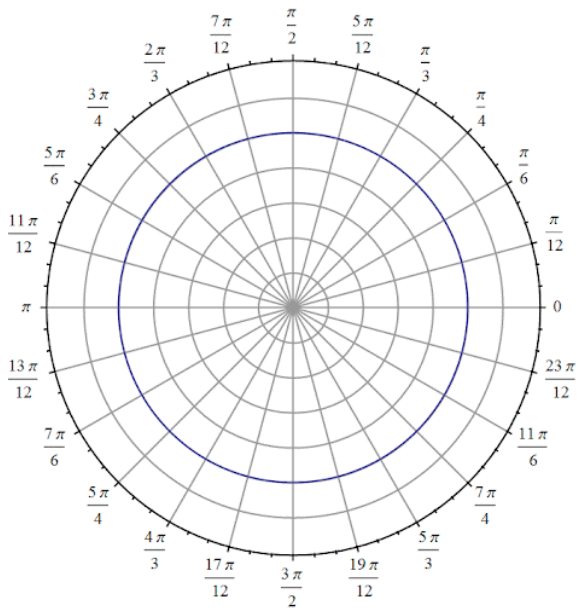
Lesson: Polar Equations and Graphs

There are other coordinate systems besides the rectangular coordinate system.

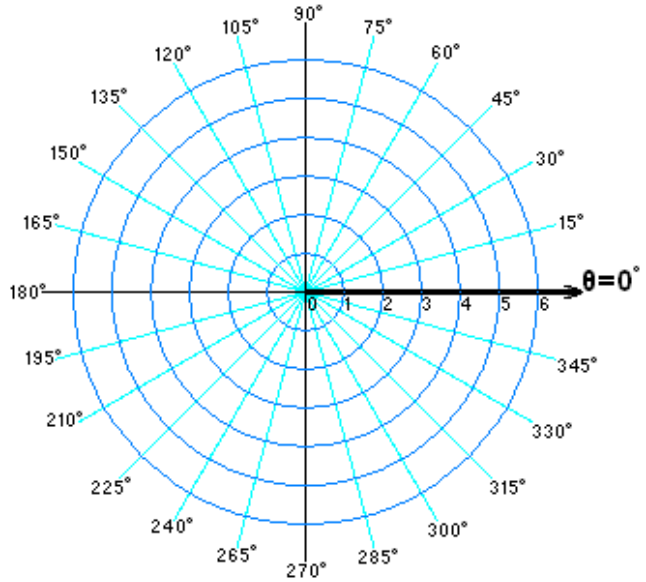
We will discuss the polar coordinate system!

What is a pole? What is the polar axis? Draw a sketch.

Here is what the polar coordinate system looks like:



It can also be in degrees:



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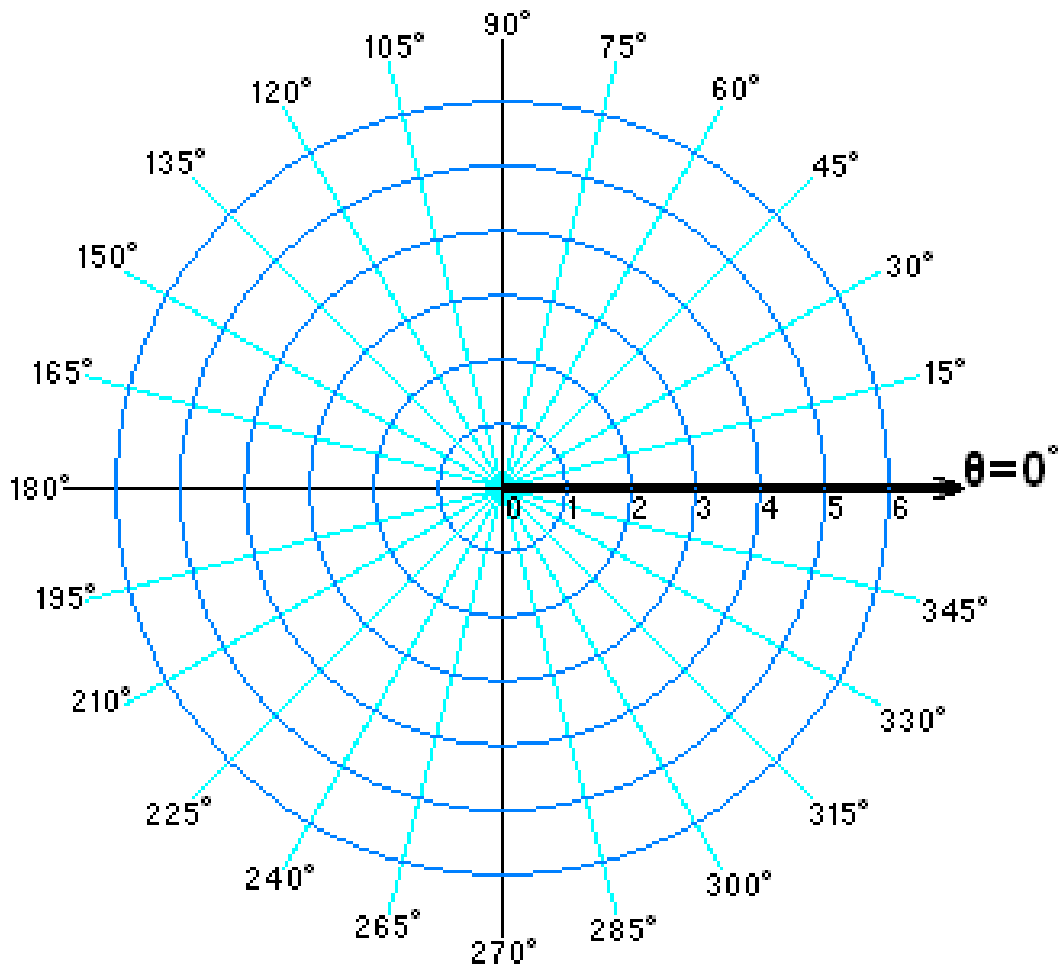
How do we convert between rectangular and polar coordinates?

<u>Examples</u> Show how to plot each point, and then determine the rectangular coordinates of each point.

a. $P(3, 45°)$



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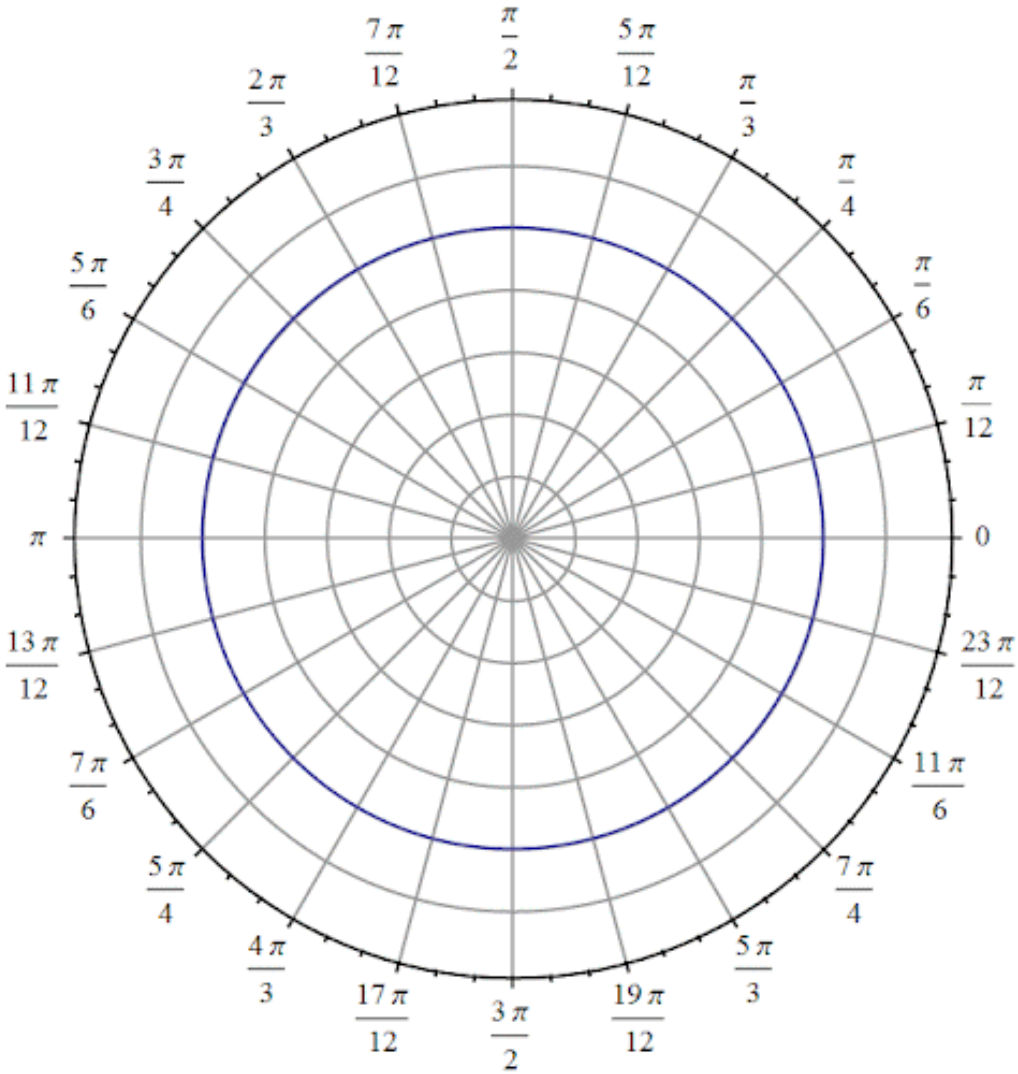
Examples Show how to plot each point, and then determine the rectangular coordinates of each point.

b. $P\left(-4, \frac{\pi}{3}\right)$



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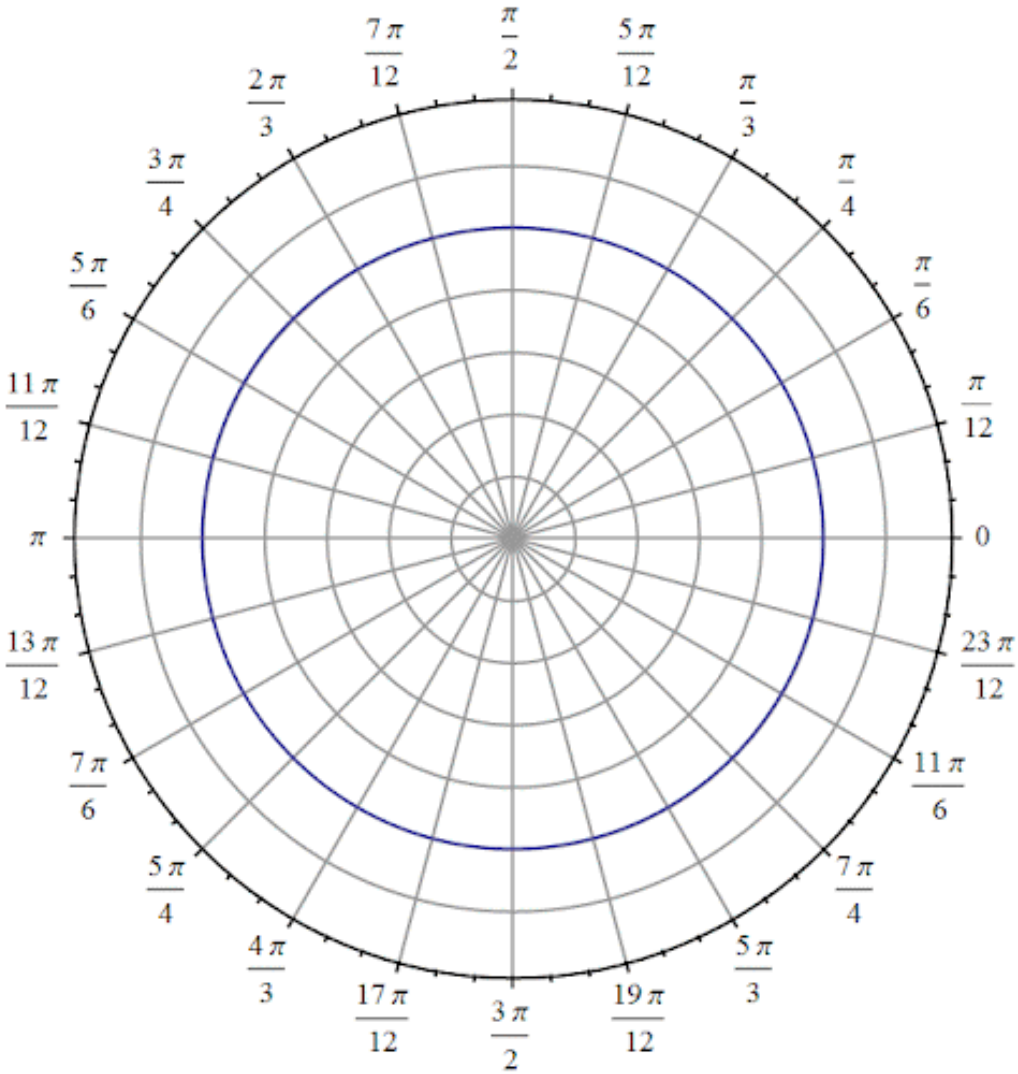
Examples Show how to plot each point, and then determine the rectangular coordinates of each point.

c. $R\left(3, -\frac{\pi}{2}\right)$



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While a given point in the plane can only have 1 pair of rectangular coordinates, how many pairs of polar coordinates can it have?

<u>Examples</u> Determine the following:

a. Three other pairs of coordinates for the point $P(3, -110°)$.

b. Two pairs of polar coordinates for the point with rectangular coordinates $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

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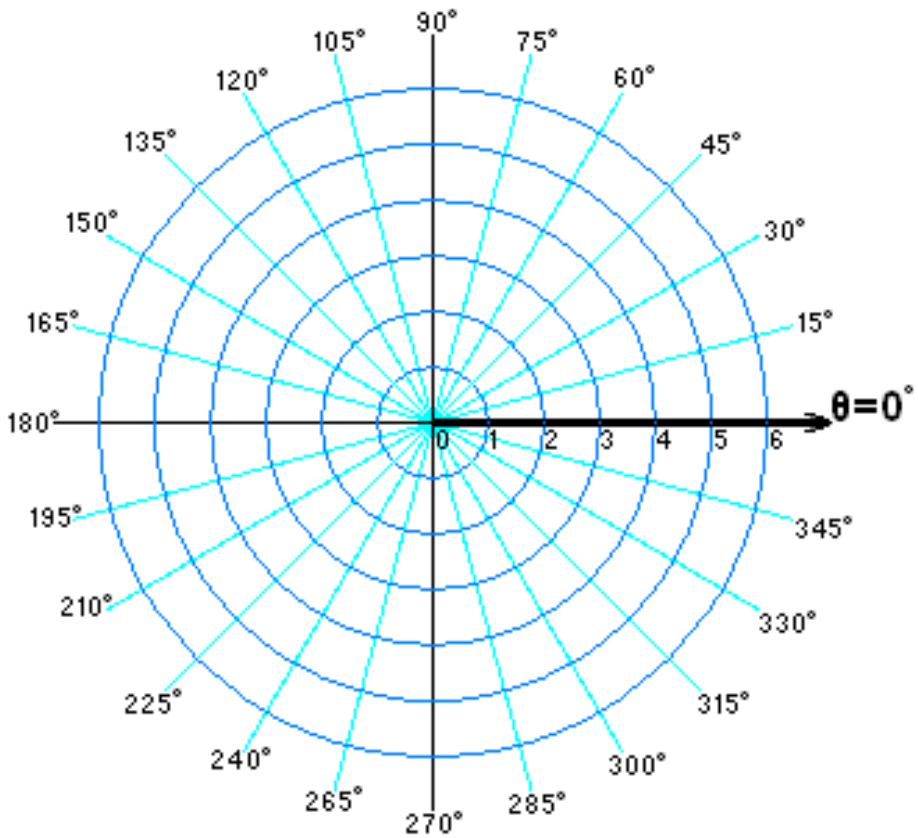
What makes an equation a “polar equation” ?

<u>Example</u> Convert the equation to a polar equation, and then graph it: $y = x - 1$



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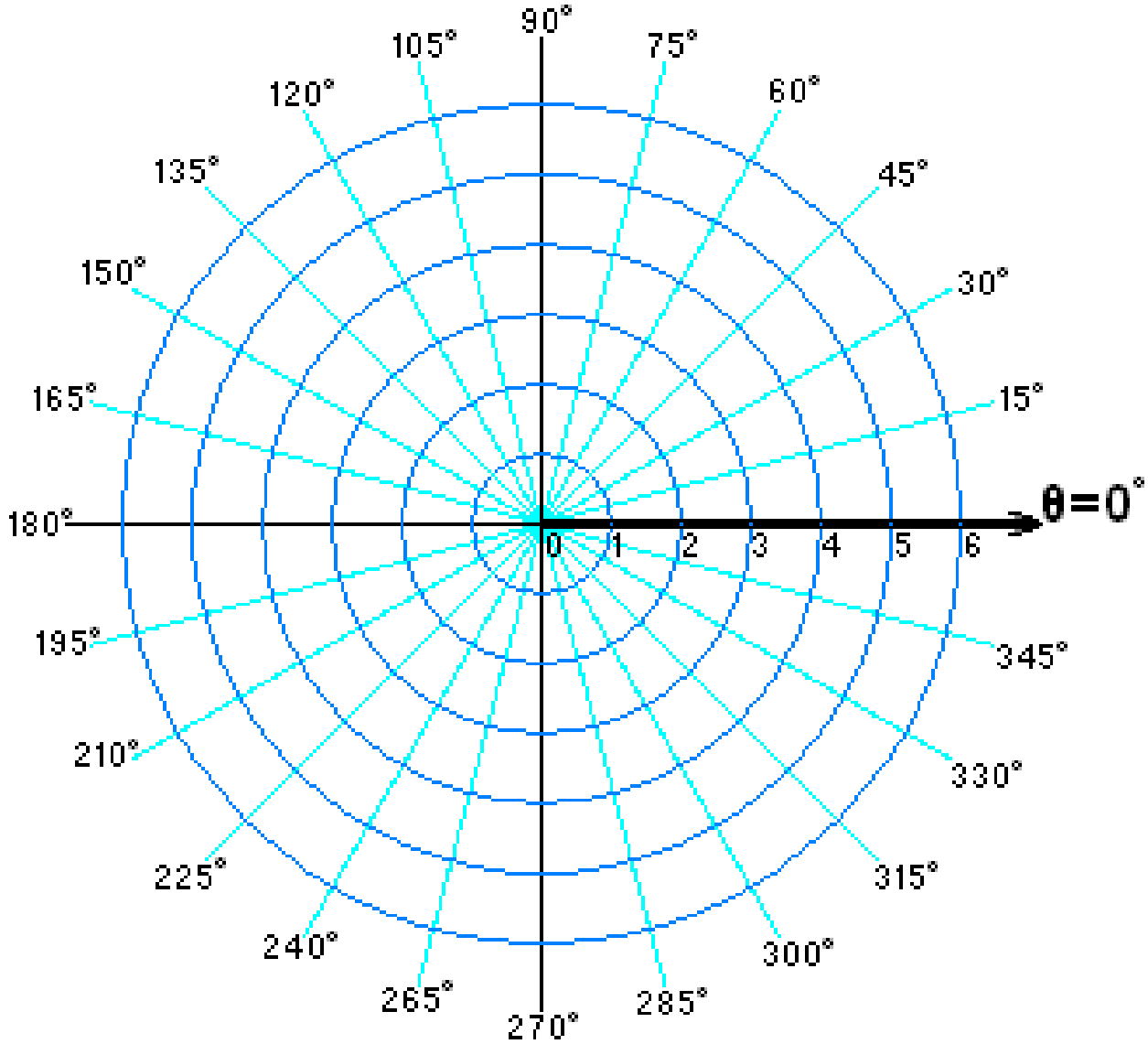
Example Convert the equation to a polar equation, and then graph it: $x^2 + y^2 = 25$



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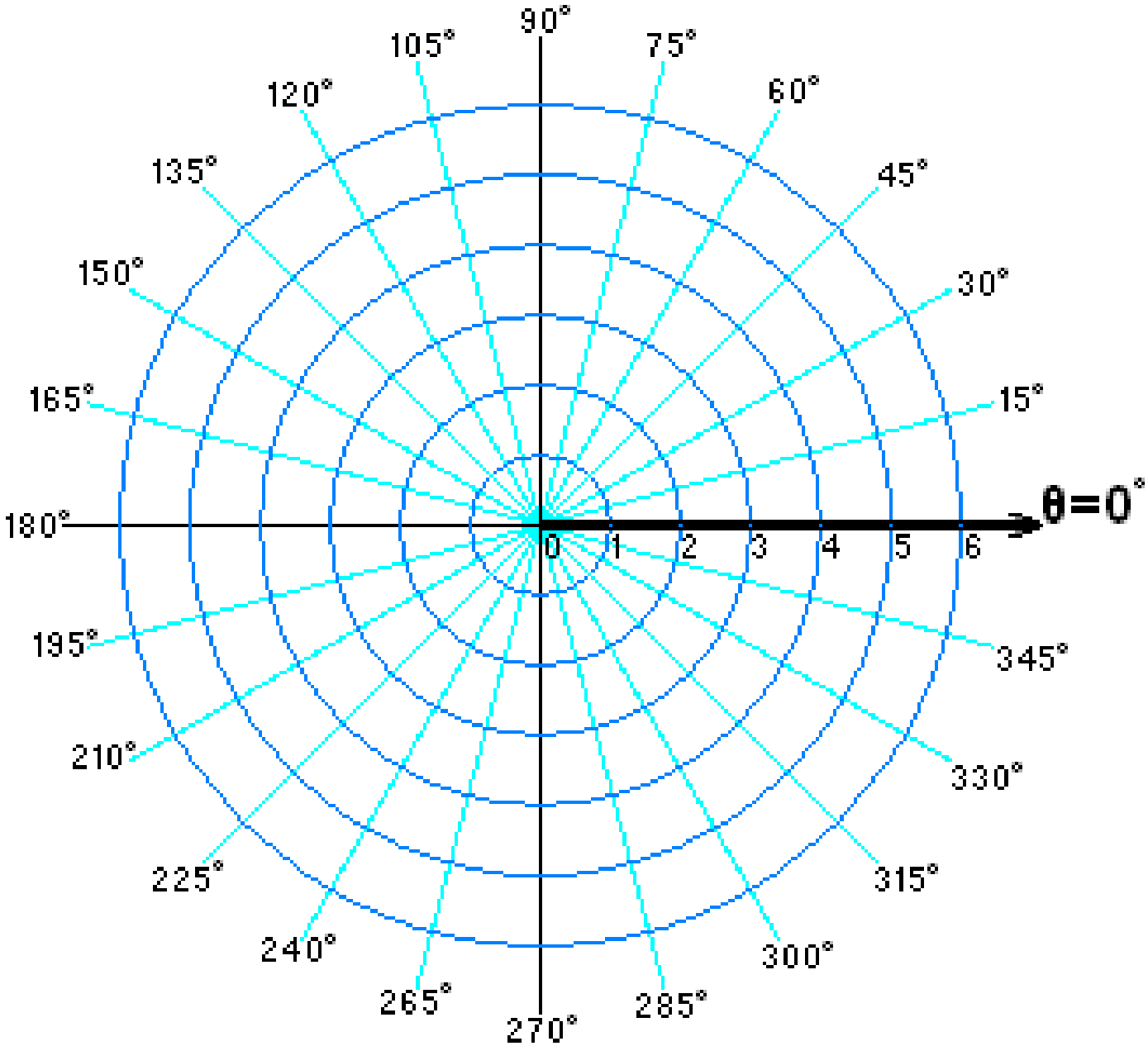
Example Graph $r = 1 - \sin\theta$



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Lesson: A Review of Imaginary Numbers

Please note: Knowing imaginary numbers if one of the "prerequisite" topics for this course. Ie – you should already know this (though I also realize you probably need a review!). If you have *never* seen imaginary numbers before, then I strongly suggest that you watch my series on imaginary numbers (because, yes, I have one). So if you felt like this went too fast and you want more details, please take responsibility for your education and go back to review this!

How do we represent an imaginary number?

What is i^2?

What is a complex number?

When are 2 complex numbers equal?

What does $\sqrt{-a}$ mean?

Examples Write using the definition of $\sqrt{-a}$

a. $\sqrt{-12}$

b. $\sqrt{-25}$

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Examples Complete the following operations.

a. $\sqrt{-5} * \sqrt{-3}$

b. $\frac{\sqrt{-12}}{\sqrt{-18}}$

c. $\frac{\sqrt{-50}}{\sqrt{9}}$

To add or subtract complex numbers, we can add/subtract the real parts and the imaginary parts similar to working with like terms.

Examples Find the sum or difference.

a. $(2 - 6i) + (8 + 2i)$

b. $(5 - 7i) - (-4 + 2i)$

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When multiplying complex numbers, recall that $i * i = i^2 = -1$!

<u>Examples</u> Multiply.

a. $(2 + 4i)(3 - 5i)$

b. $(4 - 5i)(3 - 2i)$

c. $(3 + 2i)(3 - 2i)$

Recall, the conjugate is found by using the opposite side for the second term!

<u>Examples</u> What is the conjugate of the following?

a. $4 + 2i$

b. $-3 - 4i$

c. i

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To find the quotient of 2 complex numbers, we multiply by the conjugate of the denominator.

Examples Find each quotient. Write your answer in *standard form,* ie $a + bi$ form

a. $\frac{3-4i}{2+i}$

b. $\frac{5}{i}$

How do the powers of i cycle through?

Examples Show how to simplify the following powers of i. Write your work in great detail so that you remember how to do this!

a. i^{17}	c. i^{-14}
b. i^{28}	d. i^{-5}

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<u>Examples</u> The following quadratic equations have imaginary solutions. Show how to solve them.

a. $x^2 = -25$ using the **square root property**

b. $x^2 + 75 = 0$ using the **square root property**

c. $2x^2 - 3x = -6$ using the **quadratic formula**

d. $x^4 - 16$ using **factoring** and also the **square root property**

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Lesson: Trigonometric (Polar) From of Complex Numbers

Complex numbers cannot be ordered like real numbers can. What is one way that we can organize complex numbers?

Show how we can graph complex numbers:

Complex numbers correspond to vectors. For a complex number $x + yi$, show how this corresponds to a vector (sketch), then label the sketch.

<u>Examples</u> Add $3 + 2i$ and $2 - 5i$, then graph both numbers and their resultant.

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Define the relationships between x, y, r, θ below (with a sketch)

What is the trigonometric form (or polar form) of a complex number?

How do we refer to the number r?

How do we refer to θ?

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Example Write $3(\cos 300° + i \sin 300°)$ in rectangular form.

How do you convert from rectangular from to trigonometric form?

Examples Write in trigonometric form.

a. $-\sqrt{2} + i$

b. -12

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<u>Examples</u> Write the complex number in its alternative form.

a. $8(\cos 200° + i \sin 200°)$

b. $-8 + 5i$

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Lesson: The Product and Quotient Theorems

First, let's review how to multiply $\left(2+i\sqrt{3}\right)(-2\sqrt{3}+2i)$

Let's rewrite both of those complex numbers in their trigonometric form:

Now multiply their trigonometric forms together:

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What do you notice about the result?

This problem demonstrates the product theorem.

State the Product Theorem:

<u>Example</u> Find the product of $3(\cos 45° + i \sin 45°)$ and $5(\cos 120° + i \sin 120°)$. Then write the answer in rectangular form.

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(PART A) Show how to find the quotient of $(2\sqrt{6} - 2i\sqrt{2})$ and $\left(\sqrt{2} - i\sqrt{6}\right)$

Now rewrite these complex numbers in their trigonometric form.

How can the "answer" to PART A be rewritten in trigonometric form?

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What do you notice about this result?

This demonstrates the quotient theorem.

State the Quotient Theorem.

<u>Example</u> Find the quotient $\frac{35\ cis\ 45°}{7\ cis\ (-90°)}$. Write the result in rectangular form.

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Example Use a calculator to find the following. Put your answer in rectangular form.

a. $(7.2\ cis\ 38.5°)(3.5\ cis\ 49.8°)$

b. $\dfrac{15.42\ (\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6})}{5.21\ (\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})}$

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Lesson: De Moivre's Theorem and Powers and Roots of Complex Numbers

Find the product: $[r\ (\cos\theta + i\sin\theta)]^2$

This is one example of De Moivre's theorem.

State De Moivre's Theorem:

<u>Example</u> Find $\left(\sqrt{3} + i\sqrt{5}\right)^8$ and write the answer in rectangular form.

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We can extend this theorem to find all nth roots of complex numbers.

Explain the concept of finding the nth root of a complex number.

State the nth Root Theorem

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Example Find the three cube roots of -8

Example Find all complex number solutions to $x^4 + 1 = 0$.

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Lesson: Sequences

Define a sequence.

<u>Examples</u> Write the first five terms of each sequence.

a. $a_n = \frac{1}{n}$

b. $a_n = \frac{n\mp 1}{n+3}$

c. $a_n = (-1)^{n+1} * (2n + 1)$

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What does it mean for a sequence to converge?

What does it mean for a sequence to diverge?

What is a recursively defined sequence?

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Examples Find the first four terms in the sequence.

a. $a_1 = 3; a_n = 2a_{n-1} - 4$

b. $a_1 = 2; a_n = a_{n-1}^2 + n - 2$

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Lesson: Series

What is the summation sign and what does it mean? What do we called the “i” in this formula?

Define a finite series.

Define an infinite series.

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<u>Examples</u> Evaluate the following sums.

a. $\Sigma_{i=1}^{4}\left(3^i + 1\right)$

b. $\Sigma_{j=3}^{6} a_j$

c. $\Sigma_{k=1}^{4}(4x_k - 1)$ if $x_1 = 2, x_2 = 3, x_3 = 5, x_4 = 8$

d. $\Sigma_{i=1}^{3} f(x_i)\Delta x$, if $f(x) = 3x, x_1 = 1, x_2 = 4, x_3 = 7, \Delta x = 3$

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What are the summation properties?

What are the summation rules for summing up terms?

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Examples Use the summation properties and rules to evaluate each series.

a. $\Sigma_{i=1}^{50} 3$

b. $\Sigma_{k=1}^{25} 3k$

c. $\Sigma_{j=1}^{33} (4j^2 + 1)$

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Lesson: Arithmetic Sequences

What is an arithmetic sequence?

Give some examples of an arithmetic sequence.

<u>Example</u> Determine the common difference, d, for the arithmetic sequence:

$$-7, -5, -3, -1, \ldots$$

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Example Find the first five terms of each arithmetic sequence.

a. The first term is 3, and the common difference is -2.

b. $a_1 = -5, d = 7$

How do we define the general nth term of an arithmetic sequence?

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Example Determine a_{11} and a_n for the arithmetic sequence $-2, 2, 6, 10, \ldots$

Example Determine a_n and a_{20} for the arithmetic sequence having $a_3 = 9$, and $a_4 = 16$.

Example An arithmetic sequence has $a_7 = -15$ and $a_{14} = -35$. Determine a_1.

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Lesson: Arithmetic Series

When we sum up the terms of an arithmetic sequence, how can we define that sum? (sum of an arithmetic sequence)

<u>Example</u> Consider the arithmetic sequence $-7, -3, 1, 5, \ldots$

a. Evaluate S_{10}

b. Evaluate the sum of the first 50 positive integers.

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Example The sum of the first 15 terms of an arithmetic sequence is 345. If $a_{15} = 65$, find a_1 and d.

Examples Evaluate each sum.

a. $\Sigma_{i=1}^{19}(-3 - 5i)$

b. $\Sigma_{k=4}^{15}(2 + 4k)$

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Lesson: Geometric Sequences and Series

Define a geometric sequence:

<u>Example</u> Determine a_6 and a_n for the geometric sequence 4, 12, 36, 108, ….

<u>Example</u> Determine r and a_1 for the geometric sequence with $a_2 = -18$ and $a_5 = 486$

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How do we define the sum of the first n terms of a geometric sequence?

<u>Example</u> Evaluate: $\Sigma_{i=1}^{8} 3 * 2^i$

If we start with 2 and use a common ratio of $\frac{1}{2}$, what eventually happens as the series continues infinitely? Summarize the explanation below.

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Examples Evaluate the following infinite sums.

a. The first term is 1 and the common ratio is $\frac{1}{3}$

b. $\Sigma_{i=1}^{\infty}\left(-\frac{1}{2}\right)\left(-\frac{3}{4}\right)^{i-1}$

c. $\Sigma_{i=1}^{\infty}\left(\frac{3}{7}\right)^{i}$

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Lesson: The Binomial Theorem

Below we can observe the pattern for binomial expansion:

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Focusing on the VARIABLES x and y only, how can we generalize the exponents for the variables?

Pascal's Triangle can explain how to find the coefficient. Draw out Pascal's triangle below.

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Having to draw out this pattern would be very difficult for larger expansions, so we will explore a few ways to simplify how to expand this pattern.

First, we need to understand factorials.

n-factorial: for any positive integer n,

$$n! = n(n-1)(n-2)\dots(3)(2)(1)$$

By definition: $0! = 1$

<u>Examples</u> Evaluate:

a. $5!$

b. $8!$

Now that we understand factorials, we can discuss the formula to find binomial coefficients.

For nonnegative integers n and r, $r \leq n$, the binomial coefficient is defined as follows:

<u>Examples</u> Evaluate each binomial coefficient:

a. $\binom{5}{2}$

b. $\binom{6}{0}$

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We can summarize how to expand $(x+y)^n$ as follows:

1. There are n+1 terms in the expansion

2. The first and last terms are x^n and y^n respectively

3. In each succeeding term, the exponent on x decreases by 1 and the exponent on y increase by 1

4. The sum of the exponents on x and y in any term is n.

5. The coefficient of the term with $x^r y^{n-r}$ or $x^{n-r} y^r$ is $\binom{n}{r}$

These observations are captured in the binomial theorem.

State **The Binomial Theorem:**

<u>Example</u> Find the first, last, and seventh term of the binomial expansion: $(3x+2y)^{10}$

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<u>Example</u> Use the binomial theorem to expand: $(a-b)^5$

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Lesson: Troubleshooting Limits With Algebraic Tricks (When You Get 0/0)

Trick #1 (name this trick here):

Note what happens if we try to evaluate $\lim_{x \to 2} \frac{x^2-4}{x-2}$

Draw what this situation looks like graphically.

How can we solve this problem?

<u>Example</u> Evaluate $\lim_{x \to 3} \frac{(x^3-27)}{x-3}$

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Trick #2 (name it):

<u>Examples</u> Evaluate:

a. $\lim_{x \to 0} \frac{\sqrt{x+9}-3}{x}$

b. $\lim_{x \to 1} \frac{2-\sqrt{x^2+3}}{x-1}$

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Trick #3 (name it):

<u>Examples</u> Evaluate

a. $\lim_{x\to 3} \frac{\left(\frac{1}{x}-\frac{1}{3}\right)}{x-3}$

b. $\lim_{x\to 1} \frac{\left(1-x^{-1}\right)}{1-x}$

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Examples: Algebraic Tricks for Limits

Evaluate the following:

a. $\lim_{x \to -4} \frac{\left(\frac{1}{\sqrt{12+x}} - \frac{1}{3}\right)}{x+4}$

b. $\lim_{x \to 0} \frac{\left(\frac{1}{3+x} - \frac{1}{3}\right)}{x}$

c. $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

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Lesson: One-Sided Limits

Use the following function to describe one-sided limits: $f(x) = \begin{cases} 3, & x > 2 \\ 1, & -1 \leq x \leq 2 \\ -2, & x < -1 \end{cases}$

Evaluate the following:

a. $\lim_{x \to 2^+} f(x)$

b. $\lim_{x \to 2^-} f(x)$

c. $\lim_{x \to 2} f(x)$

d. $\lim_{x \to -1^+} f(x)$

e. $\lim_{x \to -1^-} f(x)$

f. $\lim_{x \to -1} f(x)$

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Draw the graph presented in the video then evaluate the following:

a. $\lim_{x \to 3^+} f(x)$

b. $\lim_{x \to 3^-} f(x)$

c. $f(3)$

d. $\lim_{x \to -2^+} f(x)$

e. $\lim_{x \to 4^-} f(x)$

f. $\lim_{x \to 4^+} f(x)$

g. $\lim_{x \to 4} f(x)$

h. $f(4)$

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Write down the theorem that explains when $\lim_{x \to c} f(x)$ exists:

<u>Examples</u> Evaluate the following:

a. $\lim_{x \to 2^+} \sqrt{\frac{x-2}{x+1}}$

b. $\lim_{x \to 2^-} \left(\frac{1}{x+1}\right)\left(\frac{x-3}{x}\right)\left(\frac{4}{4-x}\right)$

c. $\lim_{x \to -3^-} \frac{(x+1)|x+3|}{x+3}$

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Examples: Basic One-Sided Limits

a. $\lim_{x \to 2^+} \sqrt{x^2 - 4}$

b. $\lim_{x \to 0^+} \frac{\sqrt{2x^2+3x+1}-1}{x}$

c. $\lim_{x \to 3^-} \begin{cases} 4, & x > 3 \\ -2, & x \leq 3 \end{cases}$

d. $\lim_{x \to 5^-} \frac{x-5}{|x-5|}$

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Examples: One-Sided Limits with Absolute Value

a. $\lim_{x \to 2^+} \frac{2-x}{|x-2|}$

b. $\lim_{x \to 5^-} \frac{(x-4)|x-5|}{x-5}$

c. $\lim_{x \to 2^+} \frac{\sqrt{3x}(x-2)}{|x-2|}$

d. $\lim_{x \to 0^-} \frac{\sin x}{|\sin x|}$

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Lesson: The $\boldsymbol{lim_{x\to 0}\frac{sin\ x}{x} = 1}$

This lesson is about the tricky but very important limit:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

Or equivalently:

$$\lim_{x\to 0}\frac{x}{\sin x} = 1$$

The proof is interesting and worth noting, you can write it down if you wish but you should minimally attempt to understand the reasoning behind it. It's difficult, but difficult things will expand your mind!

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Lesson: Limits that Do Not Exist

List the types of limits that do not exist, as well as an example and graph

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Lesson: The Sandwich Theorem and Basic Trig Limits

Explain the idea of the Sandwich Theorem:

State the Sandwich Theorem

<u>Example</u> Suppose there is a function $f(x)$ where $3 - x^2 \leq f(x) \leq 3\cos x$. Find $\lim_{x \to 0} f(x)$.

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Show how to use the Sandwich Theorem to evaluate the following limits:

a. $\lim_{x \to 0} \sin x$

b. $\lim_{x \to 0} \cos x$

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Examples: Basic Trig Limits

Evaluate the following:

a. $\lim_{x \to 0} \frac{3+x+\sin x}{4\cos x}$

b. $\lim_{x \to \frac{\pi}{3}} \sec x$

c. $\lim_{x \to \pi} \sin^2 x$

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Lesson: Finite Limits as $x \to \infty$ and Limits That Go To Infinity, the Conceptual Idea

Recreate the graph below, then find the following limits:

a. $\lim_{x \to \infty} f(x)$

b. $\lim_{x \to -\infty} f(x)$

c. $\lim_{x \to 3^+} f(x)$

d. $\lim_{x \to 2^-} f(x)$

e. $f(2)$

f. $\lim_{x \to 2^+} f(x)$

g. $\lim_{x \to 3} f(x)$

h. $\lim_{x \to -4} f(x)$

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Lesson: Finite Limits as $x \to \infty$ and Rational Functions

Do the limit laws apply to limits as $x \to \infty$? Yes No

How do you evaluate limits at infinity for rational functions?

In the following examples, we will see all the different ways this trick might be carried out:

<u>Examples</u> Evaluate the following:

a. $\lim_{x\to\infty} \frac{4x+3}{x^2+4x+3}$

b. $\lim_{x\to-\infty} \frac{5x^2+3x+1}{7x^2+5x-4}$

c. $\lim_{x\to\infty} \frac{6x^2+3x+1}{x+2}$

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This trick can be extended.

Examples

a. $\lim_{x\to\infty} \sqrt{\frac{x^2+5}{3x^2-6}}$

b. $\lim_{x\to-\infty} \frac{x^{\frac{2}{3}}-x^{\frac{1}{3}}+2}{x^2+3}$

c. $\lim_{x\to\infty} \frac{x+3}{\sqrt{x^2+3}}$

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Examples: Limits as $x \to \infty$

Find the horizontal asymptotes / evaluate the following:

a. $\lim_{x\to\infty} \frac{\sin x}{x^2}$

b. $\lim_{x\to\infty} \frac{7x^4-2x^3+8}{9x^4+2x^2+x-1}$

c. $\lim_{x\to-\infty} \frac{\sqrt{25x^2+x}}{x}$

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d. $\lim_{x\to\infty}\left(x-\sqrt{x^4+2x^2}\right)$

e. $\lim_{x\to-\infty}(5x^7+2x^3+8)$

f. $\lim_{x\to-\infty}\frac{x^3+2x+5}{x^2+3x+1}$

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Example: Limits as x approaches infinity

a. $\lim_{x \to -\infty} \frac{2x^6+3x^2-1}{6x^3-2x+1}$

b. $\lim_{x \to \infty} \frac{5x^{\frac{8}{3}}-3x^{\frac{5}{3}}+2}{x^{\frac{13}{5}}+2x-x^{\frac{1}{2}}}$

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Lesson: The Formal Definition of the Derivative

One of the things we learn in pre-calculus is about secant lines. You can write a review of this below:

When the distance between two points for a secant line goes to zero, what do we get?

What is the formal definition of the derivative?

What is the alternate formula for the derivative?

Which formula should you use?

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How do you find the derivative at a specific point?

What 4 things does the derivative at a point represent?

1.

2.

3.

4.

<u>Example</u> Calculate the derivative of $f(x) = x^2 + 2$

<u>Example</u> What is the value of the derivative of $f(x) = x^2 + 2$ at $(1, 3)$?

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Example Find the equation of the tangent line for $f(x) = x^2 + 2$ at $(1, 3)$

List all the different ways to represent a derivative below:

Write out the theorem that discusses differentiability and continuity

What types of functions will not have a derivative at a point?

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Examples: Calculating Derivatives Using the Formal Definition Only and Finding Tangent Lines

Use the formal definition of a derivative to calculate the follow derivatives of each function. Then determine the tangent line at the given point.

a. $f(x) = 4 - x^2; (-1, 3)$

b. $y = \frac{1-x}{2x}, (1, 0)$

c. $g(x) = x^{\frac{3}{2}}, (4, 8)$

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Examples: Using the Alternative Formula for Derivatives

Use the alternative formula for derivatives to calculate the derivative of the following function. Compare this calculation to the original formula.

a. $y = x^3 - x^2$

b. $y = \frac{1}{x+3}$

c. $f(x) = 2 + \sqrt{x}$

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Step-by-Step Examples: Comparing the Definition of the Derivative to the Alternative Formula

a. $f(x) = x + \frac{3}{x}$

b. $g(x) = x^3 - 5x^2 + 1$

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Example: Finding the Derivative of a Square Root with the Definition of the Derivative

Find the tangent line at (6,5) for $y = \frac{5}{\sqrt{7-x}}$ using only the definition of the derivative.

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Lesson: Other Notes about the Derivative

Define right-hand derivatives

Define left-hand derivatives

Explain how $y = |x|$ is differentiable on each side

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Interpreting Graphs of Derivatives

The derivative of $f(x) = x^2$ is $f'(x) = 2x$. Compare their graphs below:

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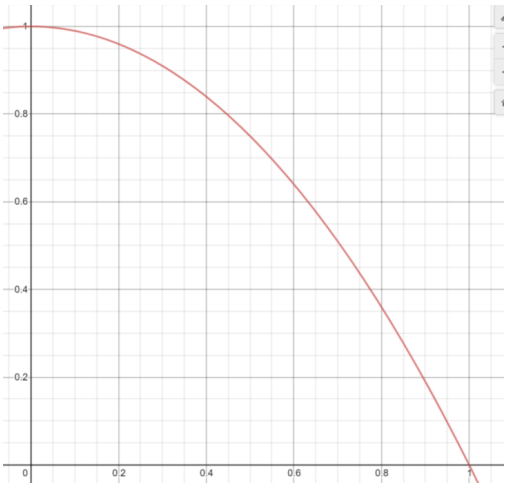
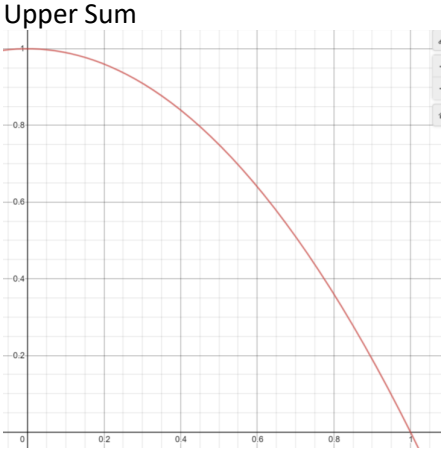
Area and Estimating with Finite Sums

First we will try to understand the idea behind this. Draw out one way to find the area of the curve $y = 1 - x^2$ below.

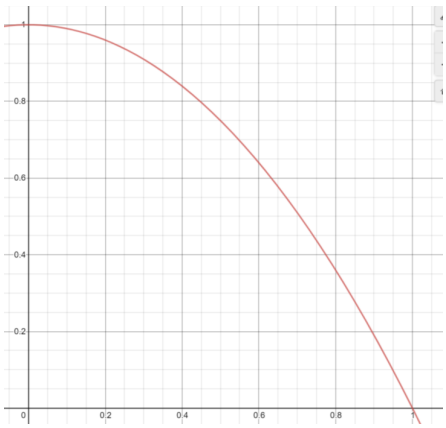


Now we need more formality around this idea. Show the idea of how to create sums using a more algorithmic approach. Detail what each sum means:

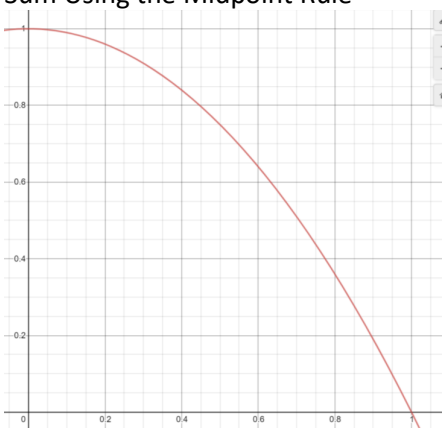
Upper Sum



Lower Sum



Sum Using the Midpoint Rule



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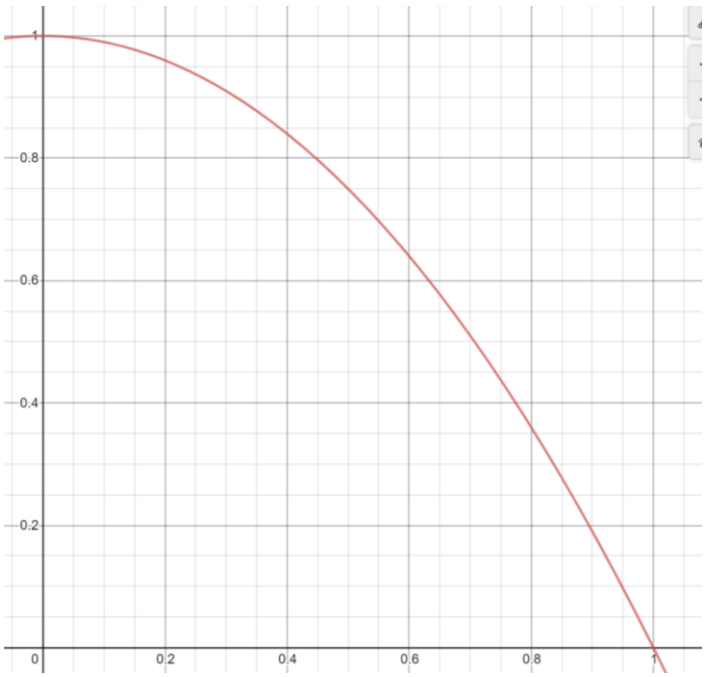
Now we need some vocabulary. Separate the graph in 4 parts and then label it with the correct terminology:



Define a Riemann Sum:

What is a left Riemann sum?

What is a right Riemann sum?

What is a midpoint Riemann sum?

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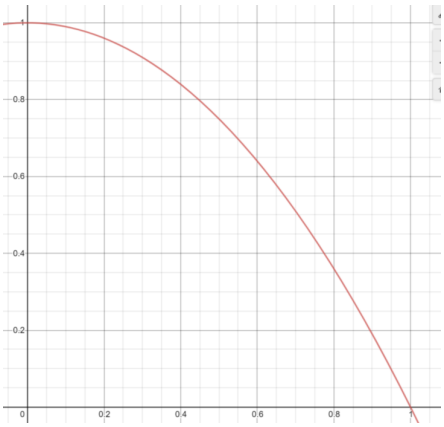
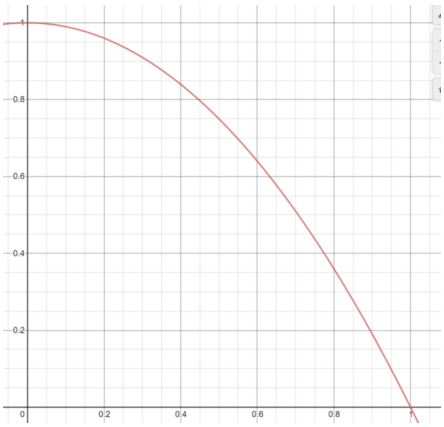
Show how you can make a more precise sum:

8 Subintervals

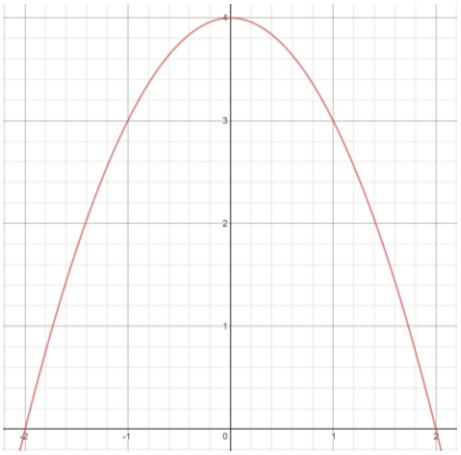


16 Subintervals

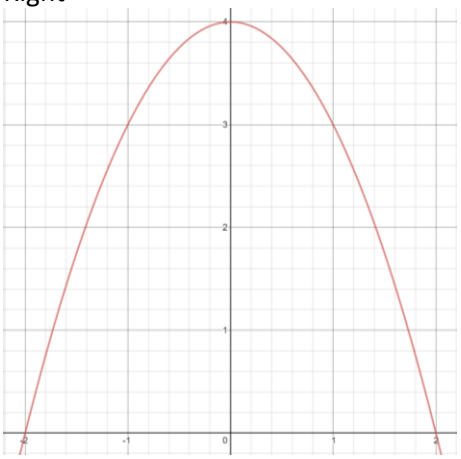


<u>Example</u> Compute the left, right, and midpoint Riemann sums for $f(x) = 4 - x^2$ between $[-2, 2]$ using 4 subintervals

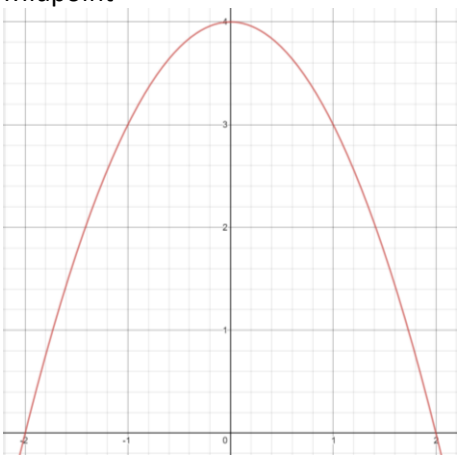
Left



Right



Midpoint



Graphs were created using Desmos.com. They have the greatest graphing software for free, check them out!

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Examples of Riemann Sums

Compute the left, right, and midpoint Riemann sum for $f(x) = 3 + x$ from $[1,3]$ using 4 subintervals.

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Examples of Riemann Sums – When It Goes Negative?

Compute the left, right, and midpoint Riemann sum for $f(x) = x^3$ from $[-2,2]$ using 4 subintervals.

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Examples: Different Ways to Compute Area Estimations

Using $f(x) = -x^2 + 2x + 3$, compute an area estimate using 4 subintervals from x=-1 to x=3.

Compute this estimate using an upper, lower, left-hand, and right-hand sum.

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Not included (no notes file):

112 (Lesson Introducing Where Calculus Starts)