Lesson: Algebraic Expressions

First we will define the terminology, which is already written out for you.

- **Constant:** any real number in an expression that is not attached to a variable
- **Variable:** a letter than can stand for any set of real numbers, and we can substitute values in for it if desired
- **Coefficient:** a number that is being multiplied by a variable
- **Term:** this is a single number, variable, or a number and a variable multiplied together, often separated by a plus + or minus – sign.
- **Factor:** a number or variable that is being multiplied with another number or variable
- **Algebraic Expression:** a mathematical string of constants and terms that can be added, subtracted, multiplied or divided together

This terminology will be used often, so it's important to understand what it means.

Example 1

Classify each part of the expressions

Expression	Constants	Variables	Coefficients	Terms	Factors
a. $5x$					
b. $2x + y$					
c. 8					
d. x					
e. $2x^2 - 4x + 7$					

Algebraic expressions are often used in everyday situations; we just don't realize it.

Example 2

You want to go to the movies, and on Tuesdays a theatre runs a deal so that all tickets cost $8. How much would it cost for each of the following situations?

a. A ticket for only you?

b. A ticket for you and 2 of your friends?

c. Tickets for 5 people, a giant tub of popcorn that costs $20.

d. Tickets for z people?

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In the last part of this example, we wrote an algebraic expression where z represented some quantity. Notice that if you input any of the previous number of tickets from examples a, b, and c into z, you would get the same answers.

To evaluate expression, you can simply plug in the desired values for a variable.

Example 3

Evaluate each expression for $x = -2$, $y = 4$, and $z = \frac{1}{3}$ as applicable:

a. $3x - 7y$

b. $6z - x^2 + 2y$

c. $x + z^2$

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Lesson: The Commutative, Associative, and Distributive Properties

Your math homework and English homework are due on the same day. Does it matter which one you do first?

This situation illustrates the idea of the Commutative Law**,** which we can now state below:

Commutative Law

Let a and b represent any two real numbers. Now we can define the following properties:

- The Commutative Law for Addition:

- The Commutative Law for Multiplication:

Example 1

Rewrite the following using the commutative law.

a. $3 + y$

b. $3c$

c. $\frac{2}{5}k$

d. $2 - z$

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You are buying chips, a drink, and a notebook. Does it matter what order the cashier rings up this items in?

These situations illustrate the idea of the **Associative Law**, which we can now state below:

Let a, b, and c represent any three real numbers. Now we can define the following properties:

- The Associative Law for Addition:

- The Associative Law for Multiplication:

Example 2

Rewrite the following at least 2 different ways using the associative law.

a. $4xz$

b. $8 + 2 + 7 + 3$

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These rules are helpful when we are computing longer expressions.

How to Simplify Calculations Using the Commutative and Associative Laws

a. Suppose we want to calculate: $2 + 3 + 8 + 7$
Using the commutative law, we more cleverly group these numbers together like this:

$$2 + 8 + 3 + 7$$

Usually we recognize 10s faster than some other quantities, so from here we can rearrange and group like this to find the final answer:

b. How can you rearrange $\left(\frac{1}{5} * \frac{1}{3}\right) \times (10 * 6)$ to make the calculations simpler to work with?

The Distributive Law

Let a, b, and c represent any three real numbers. Then the following property holds:

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The distributive property effectively gives us permission to either add or multiply certain expressions in the order that simplifies the calculation.

Example 3
Use the distributive property as needed to simplify the following calculations.

a. $10\left(\frac{2}{5}-\frac{1}{2}\right)$

b. $\frac{2}{3}(6+9)$

c. $20\left(\frac{1}{4}+\frac{2}{5}\right)$

d. $\frac{2}{5}(14-4)$

The distributive law works in the opposite direction as well. If we start with $ab+ac$, according to the distributive law what would this equal?

Another name for this process is called **factoring.**

Example 4
Use the distributive law to factor the following:

a. $km+kn$

b. $5x+10y$

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There are a few other properties we need to mention:

The Additive Identity
For any number a, $0 + a = a + 0 = a$

The Multiplicative Identity
For any number a, $1 \times a = a \times 1 = a$

The Additive Inverse (aka the opposite of a number)
For any number a, the addition of its opposite will result in zero, ie: $a + (-a) = 0$.

The Multiplicative Inverse (aka the reciprocal)
For any number a, it's inverse can be written in the form $\frac{1}{a}$ and has the property: $a \times \frac{1}{a} = 1$

It is interesting to note that there exist number systems (not discussed in this class) where these properties do **not** exist!

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Lesson: Like Terms and Combining Like Terms

In algebra, we often want to simplify expressions if possible to make our lives easier. This is where the discussion of like terms comes into play.

Like Terms

Like terms are terms that share the exact same variables with the exact same exponents. Only like terms may be added.

Do like terms work in real life? Ie, can you come up with an example of like terms that could still be added in the real world?

Building off of your example, what wouldn't be like terms and could NOT be added in that scenario?

Example 1

In the following list below, classify which terms are like terms.

Discuss the two examples with the class to ensure they understand how to identify like terms. Students may color code, underline, or draw shapes around anything in the lists to indicate they are like terms.

a. $2x, 7y, 6x^2, 8y, x, -2x^2$

b. $3x^2y^2, 7x^2y, 2xy^2, 8x^2y, x^2y^2, -3x^2, 5xy^2$

The idea of like terms only comes up when we are talking about addition or subtraction. We do not worry about this when we want to multiply two things together. You may only add or subtract terms that are like terms.

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Example 2

Combine like terms. The next page is also left blank if you need more space.

a. $3x + 2y - 5x + 9y$

b. $4x^2 + 2x + 6y^2 + 2x^2 + 8x$

c. $\frac{1}{2}x + \frac{2}{3}y - \frac{3}{2}x - 2\frac{1}{3}y$

d. $3x^2y^3 + 7x^2y^2 - 2x^2y^3 + x^2y^2$

e. $\frac{1}{4}x + \frac{2}{5}y - \frac{1}{2} + \frac{1}{5}y$

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Lesson: Translating English into Math

As we move through this course, one major goal is to learn how to recognize and interpret certain situations as math problems. There are particular words in English that denote certain operations in mathematics, and we will classify those here.

Addition	**Subtraction**	**Multiplication**	**Division**

Example 1

Translate the following sentences into an algebraic expression. There is nothing to solve for you, you are merely setting up expressions.

a. 3 more than Lucy's height

b. Two less than a number

c. Three more than twice his salary.

d. Five less than three times a number.

e. Two-thirds of the cost.

f. Three more than twice his and her salary combined.

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Lesson: Simplifying Fractions

Say we want to reduce the fraction: $\frac{55}{77}$

To do so, we need to understand what numbers must be multiplied to make this fraction. Write the steps below as outlined in the videos:

Example 1

Reduce the following fractions.

a. $\frac{15}{35}$

b. $\frac{12}{18}$

c. $\frac{18}{42}$

d. $\frac{42}{54}$

e. $\frac{105}{135}$

f. $\frac{113}{125}$

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How do you convert the mixed number $3\frac{4}{5}$ to an improper fraction?

Example 2

Convert the following to improper fractions

a. $5\frac{1}{2}$

b. $7\frac{2}{3}$

How do you convert the improper fraction $\frac{66}{5}$ to a mixed number?

Example 3

Convert the following to mixed numbers.

a. $\frac{15}{2}$

b. $\frac{25}{3}$

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Lesson: Multiplying Fractions

The Rule for Multiplying Fractions

Let a, b, c, and d be real numbers where $b \neq 0$ and $d \neq 0$. Then:

What is actually happening with this rule? Write down a simple non-math explanation of how to multiply fractions:

Is a common denominator required for this? Yes No

Example 1

Multiply the following. There will be no reducing required, just make sure you understand how to multiply these together:

a. $\frac{2}{5} \times \frac{3}{7}$

b. $\frac{7}{9} \times \frac{2}{5}$

c. $\frac{12}{7} \times \frac{10}{7}$

d. $\frac{1}{2} \times \frac{3}{5} \times \frac{1}{7}$

e. $5 \times \frac{2}{3}$

f. $\frac{2}{3} \times \frac{5}{7} \times 2$

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Reducing Fractions Before Multiplying

Occasionally you will multiply fractions that will need to be reduced. It can be easier to reduce them before multiplying them out. We now practice this.

Say you have this problem: $\frac{55}{54} \times \frac{45}{77}$

In this case, multiplying first would be messy so it is more advantageous to reduce before we multiply. Write down the steps below:

Example 2

Try this in the next problem. If you aren't sure what factor goes into each number, then break the numbers into their prime factorizations.

$$\frac{21}{28} \times \frac{16}{35}$$

Example 3

Cancel out any common factors before multiplying.

a. $\frac{35}{24} \times \frac{18}{49}$

c. $12 \times \frac{5}{6} \times \frac{1}{10}$

b. $\frac{60}{64} \times \frac{56}{72}$

d. $\frac{30}{75} \times 3\frac{2}{5}$

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Lesson: Dividing Fractions

The Rule for Dividing Fractions

Let a, b, c, and d be real numbers where $b \neq 0$, $c \neq 0$, $d \neq 0$. Then:

What is actually happening with this rule? Write down a simple non-math explanation of how to multiply fractions:

Is a common denominator required? Yes No

Example 1

Complete the following exercises:

a. $\frac{2}{3} \div \frac{5}{7}$

b. $\frac{2}{5} \div 3$

c. $7 \div \frac{2}{3}$

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Similar to multiplication, we can make cancellations *after we multiply by the reciprocal of the divisor*.

Example 2

Divide.

a. $\frac{15}{16} \div \frac{9}{14}$

c. $\frac{6}{7} \div 4\frac{1}{2}$

b. $20 \div \frac{4}{5}$

d. $\frac{24}{65} \div \frac{28}{70}$

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Lesson: Intro to Adding and Subtracting Fractions

Adding and Subtracting Fractions When Fractions Have a Common Denominator

When fractions have the same denominator, we do not change the denominator but add the numerators as shown ($c \neq 0$):

Is a common denominator required? Yes No

Note this is the first instance where the denominator actually matters in the discussion.

Example 1

Add or subtract.

a. $\frac{3}{5} + \frac{1}{5}$

b. $\frac{3}{10} + \frac{7}{10}$

c. $\frac{5}{3} + \frac{4}{3}$

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Adding and Subtracting A Whole Number with a Fraction

Suppose you want to complete the following example: $5 + \frac{1}{2}$

What denominator does the fraction have?

In this case, this is the common denominator we will want.

How to Turn a Whole Number into a Fraction

One property of all whole numbers is that we can rewrite them as fractions, so we know that:

$$5 = \frac{5}{1}$$

Once we convert 5 into this fraction form, we can then convert it to an equivalent fraction with the common denominator of 2. In this case, we say "whatever we do to the bottom, we also do the top." Show how to do this below:

Example 2

a. $5 - \frac{2}{7}$

c. $2 - \frac{11}{4}$

b. $3 + \frac{2}{3}$

d. $8 - \frac{1}{8}$

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Lesson: Adding and Subtracting Fractions when You Need to Find a Common Denominator

This a is a much longer process. The ultimate goal is to make two fractions have the same denominator so we can combine them like we did in the first case.

To add two fractions that do not have the same denominator, we must complete the following three steps:

1) Find the LCD between the two fractions.
2) Convert each fraction as a fraction with the LCD.
3) Add or subtract the fractions and reduce is necessary.

Example 1

Find the LCD between: $\frac{1}{6}, \frac{1}{10}$

What is the largest number that divides both 6 and 10?

Now show how you can factor 6 and 10 by using this number that divides each:

Show how to find the LCD:

Example 2

Find the LCD between: $\frac{1}{12}, \frac{1}{18}$

What is the largest number that divides both 12 and 18?

Now show how you can factor 12 and 18 by using this number that divides each:

Show how to find the LCD:

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Example 3

Find the LCD of the following fractional pairs. Put a box around your final answer to clearly show what the LCD is.

a. $\frac{3}{28}; \frac{5}{42}$

b. $\frac{5}{24}; \frac{1}{42}$

A Complete Example Showing How to Add Two Fractions with Unlike Denominators

Add the fractions: $\frac{1}{12} + \frac{4}{15}$

First, find the LCD between the denominators first:

What do you have to multiply 12 by to get the LCD?

What do you have to multiply 15 by to get the LCD?

Show how to finish the problem below:

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Example 4

Use the guided steps to add $\frac{3}{10} + \frac{2}{15}$

What is the LCD between the fractions?

What must you multiply 10 by to get to the LCD?

What must you multiply 15 by to get to the LCD?

Complete the problem and put a box around the final answer:

Example 5

Add $\frac{1}{24} - \frac{3}{18}$ using the steps in the previous example to guide you.

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Examples : Adding and Subtracting Fractions when You Need to Find a Common Denominator

Add or subtract the following:

a. $\frac{5}{16} - \frac{5}{6}$

b. $\frac{7}{12} + \frac{5}{18}$

c. $\frac{9}{10} - 3\frac{1}{2}$

d. $\frac{4}{15} + \frac{6}{25}$

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Lesson: Exponents and the Order of Operations

Exponents

Label the parts of this exponent:

$$2^3$$

What does 2^3 literally mean?

Example 1

Evaluate the following exponents

a. 3^3

b. 5^3

c. 2^4

We need to understand exponents so that we can fully understand the order of operations. The order of operations is one of the handiest concepts in arithmetic that is often confused.

Order of Operations

Write down what the order of operations is:

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Classic Example of the Order of Operations

Evaluate: $10 \div 5 \times 6$

Example 2

Evaluate the following. This page and the next page are both blank so that you have plenty of space to work out your calculations by hand.

a. $3 + 2^3 - 6 \div 3 \times 7$

b. $2 + 3^2 + 4 \times 2 - 5 \times 2$

c. $3(1 + 2^2 - 3)^2$

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Examples: The Order of Operations

a. $10 - 3(1 + 2 \times 3) - 2^2 \times 4$

b. $3(1 + 2^2 - 3)^2 + 5(4 - 2 \times 3)$

c. $\frac{1}{2}(5 + 3) - 2^2(5 - 6)$

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Lesson: Advanced Notes on Exponents

Multiplicative Property of Negative One

Explain what is the difference between -5^2 and $(-5)^2$ from the standpoint of the order of operations:

Example 1

Evaluate the following.

a. $(-3)^2$

b. $(-3)^3$

c. -3^2

d. -3^3

e. -7^2

f. $(-2)^5$

Example 2

What is the sign on the following problems? Do not worry about the final answer, you only need to circle if the final answer is positive or negative.

a. $(-4)^7$ Positive Negative

b. $(-58)^{12}$ Positive Negative

c. -72^{57} Positive Negative

d. $(-13)^{72}$ Positive Negative

e. -64^{14} Positive Negative

f. $(-23)^{91}$ Positive Negative

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There is one final rule we need to discuss.

Zero Exponents
For any real number a,

Example 3
Evaluate the following:

a. 5^0

b. x^0

c. 35^0

d. -6^0

e. $(-7)^0$

f. $3^0 + 7^0$

g. $-5^0 + 2^0$

h. $\left(-3 + \sqrt{52}\right)^0$

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Lesson: Solving Linear Equations, A Review of the Basics

To begin, let's review some of what we've learned about working with equations and using graphs to solve them.

Linear Equation in One Variable
An equation that can be written in the form ax+b=0 where a and b are real numbers

Solution
A value that when substituted into the variable of an equation makes the equation true

Example 1
Determine if $x = -1$ is a solution to the following equations.
a. $5 - 4x = 9$

b. $2x + 3 = 1 - 3x$

We will now work with some basic ideas of solving linear equations. There are two key concepts we must understand to do this.

The Addition Principle
Let a, b, and c be real numbers. If a=b then:

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Example 1

Use the addition principle to solve the following:

Choose at least one from each category. Label the examples you choose as a, b, c, etc to keep the class organized.

a. $x - 4 = 2$

b. $x + 3 = 3$

c. $x - \frac{1}{3} = \frac{1}{2}$

The Multiplication Principle

Let a, b, c be real numbers. If a=b then:

Example 2

Use the multiplication principle to solve the following:

a. $3x = 15$

b. $5x = 0$

c. $-\frac{2}{5}x = 10$

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Example 3

Solve the following examples:

a. $5 - 4x = 33$

b. $3x + 2 = 2 + 4x - 8$

c. $3(4 - 2x) = 3x + 6$

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Lesson: Solving Linear Equations, No Solution and Infinite Solutions

There are 2 special outcomes we have to watch out for:

An Example Where There Is No Solution

$3(2x-1)=2(2x+1)-2(3-x)$

An Example Where There Are Infinite Solutions

$4x+5=3(2x+1)-2(x-1)$

A Summary of the 3 Possible Solution Types for Linear Equations

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Examples: Solving Linear Equations, Basic Examples with No Fractions or Decimals

a. $\mathbf{3x - 5 = 7}$

b. $\mathbf{6 - 2x = 6}$

c. $\mathbf{4x + 7 = 2x - 9}$

d. $\mathbf{3 - 4x - 7 + 2x = 4 - 6x - 8 + 4x}$

e. $\mathbf{4x + 7 + 6x = 7(1 - 2x)}$

f. $\mathbf{3(5 - 2x) - 6(3 - 2x) = 5(1 + 2x)}$

g. $\mathbf{4(2x - 1) - 2(6 - 3x) = 2(3x - 4)}$

h. $\mathbf{3(2x - 5) - 4x + 1 = 4(x - 3) - 2(1 - 3x)}$

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Lesson: Solving Linear Equations with Fractions

To begin, we will look at 3 examples to demonstrate how to clear fractions without parentheses. Take notes as shown in the video.

a. $\frac{1}{2}x + \frac{1}{5} = \frac{6}{5}$

b. $\frac{1}{2}x + \frac{1}{5} - \frac{1}{10}x = 1 + \frac{3}{10}x - \frac{2}{5}$

c. $\frac{1}{2}x + 1 + \frac{1}{4}x = \frac{3}{4} + \frac{2}{3} + \frac{1}{3}x$

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Now we will look at how to clear fractions in equations with parentheses.

a. $\frac{3}{4}(3+2x)-\frac{1}{2}(5-3x)=\frac{5}{4}$

b. $\frac{1}{2}(x+2)+\frac{1}{3}(2x-5)=\frac{1}{3}(4+2x)$

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Examples: Solving Linear Equations with Fractions (No Parentheses)

a. $\frac{1}{5}x - \frac{1}{3} = \frac{2}{3}$

b. $\frac{1}{3}x - 1 + \frac{1}{5}x + \frac{2}{5} = \frac{2}{5}x + \frac{1}{5}$

c. $\frac{1}{4}x - \frac{1}{10} + \frac{1}{5}x - \frac{4}{5} = \frac{3}{20}x + \frac{1}{10} + \frac{3}{10}$

d. $\frac{2}{3}x + \frac{1}{3} - \frac{1}{2}x = \frac{5}{6}x - \frac{1}{6} - \frac{2}{3}x + \frac{1}{2}$

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Examples: Solving Linear Equations with Fractions (With Parentheses)

a. $\frac{1}{2}(3x-2)-\frac{2}{5}(2x-3)=\frac{1}{5}(x+1)$

b. $\frac{3}{4}(4x+5)+\frac{1}{2}(2-3x)=\frac{3}{4}(2x+1)$

c. $\frac{1}{2}(2x+3)-\frac{1}{3}(2x-1)=\frac{1}{4}(x+7)$

d. $\frac{3}{4}(2x-1)+\frac{5}{8}(2-4x)=\frac{1}{4}(2-4x)$

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Lesson: Solving Linear Equations with Decimals

To begin, we will look at 3 examples to demonstrate how to clear decimals without parentheses. Take notes as shown in the video.

a. $0.03x + 0.04 = 0.19$

b. $0.05x + 0.1 - 0.03x = 0.08$

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Now we will look at 3 examples to demonstrate how to clear decimals with parentheses. Take notes as shown in the video.

a. $0.4(x+2)-0.3(2x+1)=0.09$

b. $0.1x+0.03(4-2x)=0.5(x+3)$

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Examples: Solving Linear Equations with Decimals (no parentheses)

a. $0.1x - 0.06 - 0.2x - 0.04 = 0.3$

b. $0.3x - 0.6 + x - 0.2 = 0.6x - 0.8 + 0.7x$

c. $1 + 0.2x - 0.7 + 2x = 0.5 - 0.4x + 1.8 + 0.6x$

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Examples: Solving Linear Equations with Decimals (with parentheses)

a. $0.4(3x + 1) - 1.5(2x + 1) = 0.6(3 - 3x)$

b. $0.2(3x + 1) - 0.05(2x + 8) = 0.1(2x + 4)$

c. $0.09(2 - 4x) - 0.1(2x - 1) = 0.2(2 - x) - 0.06(2 + 6x)$

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Lesson: Solving a Linear Equation with Multiple Variables

Examples

a. Solve for x: $x + y = 7$

b. Solve for y: $4y = z$

c. Solve for a: $3a - 6b = 2c$

d. Solve for b: $4a + 8b = 16c$

e. Solve for k: $\frac{1}{3}k = m$

f. Solve for a: $3(a - 2b) = 6c$

g. Solve for b: $\frac{2}{3}(3a + 2b) = c$

h. Solve for a: $-\frac{2}{5}(2a - 4b) = 4$

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Lesson: The Basics of Applications and Word Problems

As a reminder, here is a review of some of the key words that imply certain operations in math:

Words That Imply Addition	**Words That Imply Subtraction**	**Word That Imply Multiplication**	**Words That Imply Division**
Add	Subtract	Multiplied by	Divided by
Increased by	Difference	Times	Quotient of
Altogether	Less than	Of	Out of
Sum	Fewer	Product of	Ratio
Longer than	Decrease	Twice	
Plus	Minus		
Above	Take away		
More than	Shorter than		

Example Juan is memorizing a list of names for a history class quiz. So far, he has memorized 8 of the names, which is one-fourth of the list. How many names are on this list?

Example Corey wants to rent a truck for the day. It costs $30 for the base fee, plus $0.23 for each mile driven. When he returned the truck, he owed $41.19. How many miles did he drive?

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Example A café serves both tea and coffee. On Sunday, the café sold 95 teas and coffees combined. The café sold 9 more coffees than teas. How many teas were sold on Sunday?

Example Latoya bought a new sweater and a pair of jeans for $64. The jeans cost $8 less than the sweater. How much was the sweater? How much were the jeans?

Example There are red, orange, and green balls in a ball pit. There are 12 more orange balls than red balls, and there are 9 more green balls than red balls. How many are there of each color?

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Lesson: Consecutive Integer Word Problems

How do you represent consecutive integers with variables?

Example The sum of 2 consecutive integers is 73. Find the integers.

Example The sum of 3 consecutive integers is -33. Find the integers.

How do you represent even consecutive integers with variables?

How do you represent odd consecutive integers with variables?

What is noteworthy about how you represent even or odd consecutive integers?

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Example The sum of 2 consecutive even integers is 110. Find the integers.

Example The sum of 3 consecutive odd integers is -87. Find the integers.

Example Find 3 consecutive odd integers such that their sum is seven more than four times the largest integer.

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Lesson: A Review of Percentages

What is a percent? What does it represent?

Examples Convert the following percentages to decimals:

a. 34%

b. 122%

c. 4.57%

Examples Convert the following decimals to percentages:

a. 0.052

b. 0.995

c. 0.00078

Examples Convert the following fractions to percentages:

a. $\frac{3}{4}$

b. $\frac{3}{5}$

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Examples

a. Find 34% of 87

b. What is 17.5% of 32?

c. 12 is what percent of 36?

d. What percent of 18 is 3?

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Lesson: Applications with Percents – General Applications and $ Applications

Example In the incoming freshman class of English majors, 52% of them were female. If there were 2,300 females majoring in English that year, what was the total number of English majors?

Example When Hector bought his house, it was worth $232,000. After a few years, the house's value increased by 22%. What is the increase in value of the house and what is the actual current value?

Example Devin wanted to buy a shirt that normally costs $18. When he go to the register to buy it, he found out there was a 55% discount. What is the price of the shirt now?

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Example Rachel buys dresses for her clothing store at $8 wholesale. When she gets to the store, she marks them up 23%. How much does she sell each dress for?

Example Mia is buying a new laptop for $800. There is an 6.25% sales tax in her county. What is the final price of the laptop?

Example Rose found a shirt she wanted for $24. When the cashier rang up the shirt, she owed $25.98. What percentage of tax did she pay?

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Lesson: Applications with Percents – Mixing with Percentages

Example Lala deposited $500 into a simple interest account that paid 7% interest per year, and $1500 into another account that paid 9% interest per year. After one year, how much interest did she make? What is the total percent of interest earned on the 2 deposits?

Example Steff works at a coffee shop and she is mixing coffee roasts. The Breakfast Blend contains 20% of the beans from Columbia. She mixes 3 pounds of the Breakfast Blend with 7 pounds of another roast that is 100% pure Colombian coffee. How many pounds of this mixture contain Colombian beans? What percentage of this new blend contains Colombian coffee?

Example Larry has mixture A with 55% acid and mixture B with 12% acid. He has 200 ml of mixture A. How much of mixture B did he use if his new solution is 48% acid?

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Example A company owns 2 different factories to make furniture. Factory A produced 2 times as many dining chairs as Factory B. Five percent of the chairs from Factory A and 3% from Factory B were defective. How many chairs did Factory B produce if 770 of the chairs were defective?

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Lesson: Ratios and Proportions

What is a ratio?

Example The unit price of an item is the ratio of the price of an item to the amount of the item. Find the unit price if 5oz of peanuts costs $1.

Example Express the ratio of 12 boys to 14 girls.

What is a proportion?

Examples Solve the following proportions:

a. $\frac{x}{5} = \frac{6}{10}$

b. $\frac{x+2}{12} = \frac{x-4}{8}$

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Examples: Proportions

a. $\frac{3}{x} = \frac{9}{21}$

b. $\frac{3x+10}{15} = \frac{2}{5}$

c. $\frac{2x-1}{6} = \frac{4x}{10}$

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Lesson: Proportion Word Problems

It 3 cookies cost $1.36, how much will 7 cookies cost?

At a motorcycle race, the ratio of parents to children was 5 to 3. If 500 children attended the motorcycle race, how many parents were at the race?

One euro is equivalent to $1.18 US dollars. How many euro would you get for $25?

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Lesson: Distance, Rate, and Time Problems

Two cars leave Glen Ellyn, IL, with one driving north and the other driving south. The northbound car is traveling 8mph faster than the southbound car, and after 2 hours they are 420 miles apart. Find the speed of each car.

Marshall and Lilly live together. Marshall leaves the house at 2pm, and then Lilly leaves the house at 2:30pm. If Marshall is driving 30 mph and Lilly is driving 35 mph in the same direction, how long will it be until their cars meet?

A Lamborghini and a Ferrari pass one another at noon on a highway in California. The cars are going opposite directions. If the Lamborghini is traveling 75 mph, and the Ferrari is traveling 80 mph, how long will it take for the cars to be 310 miles away from one another?

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Lesson: Money Word Problems

Sarah has five more nickels than quarters. When combined, the coins are worth $4.75. How many nickels and quarters does she have?

Maria has $75 worth of $1 and $5 bills. In total, she has 27 bills. How many $5 bills does she have?

Tickets at the movie theater course $15 for an adult and $10 for a child. In the last hour, sales were $550. How many adult tickets versus child tickets were sold?

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Lesson: Number Lines, Interval Notation, and Set Notation

Show how to set up the number line, interval notation, and set notation for the following:

a. $x > 5$

b. $x \leq 7$

c. $x \geq 0$

d. $-3 < x < 5$

e. $-7 \leq x < -1$

f. $3 < x \leq 8$

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Lesson: How to Solve Linear Inequalities in One Variable

Linear inequalities in one variable have the form $ax + b < 0$. Alternatively, they could use the symbol $\leq, >, \geq$.

Solving linear inequalities has some similarities and some differences when compared to solving for linear equations.

The Addition and Subtraction Properties

Let a, b, c be real numbers. If $a < b$, then:

This also holds for $\leq . >, \geq$

Examples Solve the following. Put your final answer both on a number line and in interval notation.

a. $x + 4 > 9$

b. $x - 8 \leq 12$

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The Multiplication and Division Properties

Let a, b, c be real numbers. If $a < b$, then:

- if c is positive $(c > 0)$:

- if c is negative $(c < 0)$

This also holds for $\leq . >, \geq$

Why is this rule different? Use the following example.

To begin, is $-2 < 4$ a true statement? True False

Now multiply both sides of the inequality by 3 and write down the new result below:

This new statement is True False

Now multiply both sides of the inequality by -3 and write down the new result below:

This new statement is True False

Now divide both sides of the inequality by -2 and write down the new result below:

This new statement is True False

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Examples Solve the following. Put your final answer both on a number line and in interval notation.

a. $3x \leq 15$

b. $-2x > 8$

c. $5x < -10$

d. $-4x \geq -12$

e. $-9x \leq 27$

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From here, you can start to combine the rules to solve more types of inequalities.

Examples Solve the following. Put your final answer both on a number line and in interval notation.

a. $2x - 4 < 6$

b. $3x + 7 > 5x - 9$

c. $\frac{1}{2}x + \frac{1}{5} > \frac{3}{4}x + \frac{7}{10}$

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Examples: Solving Linear Inequalities – Basic Examples

Solve the following. Put your final answer both on a number line and in interval notation.

a. $2x + 7 \geq 13$

b. $4x - 6 < 6x - 10$

c. $3(5 - 2x) > 5(3 + 2x)$

d. $2(4 - 2x) - 3(4 - 4x) \leq 4(3x + 3)$

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Examples: Solving Linear Inequalities – Examples with Fractions Decimals

Solve the following. Put your final answer both on a number line and in interval notation.

a. $\frac{1}{3}x - \frac{1}{5} \leq \frac{1}{3} + \frac{2}{5}x$

b. $0.03x + 0.2 > 0.02$

c. $\frac{3}{5}(2x - 1) - \frac{3}{4}(2 - 3x) < \frac{1}{10}(4 - 3x)$

d. $0.2(2x - 4) + 0.03(6 - 10x) \geq 0.06(x + 9)$

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Lesson: Three-Part Inequalities

Show how to solve the following three-part inequalities. Put your final answer both on a number line and in interval notation.

a. $4 < x + 2 \leq 10$

b. $-3 \leq 2x - 1 < 11$

c. $-5 < 3x + 4 < 19$

d. $4 \leq -1 - 5x < 14$

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Lesson: The Basics of Compound Inequalities

The word "and" has special implications in mathematic. One place it is often used is with sets, and we represent "and" with the intersection symbol ∩.

The word "or" also has special implications in mathematic. It is also used is with sets, and we represent "or" with the intersection symbol ∪.

Examples Given $A = \{1, 2, 3, 4, 5\}, B = \{2, 4, 6, 8, 10\}, C = \{1, 3, 5, 7\}$, find the following:

a. $A \cap B$

b. $A \cap C$

c. $A \cup C$

d. $B \cup C$

We use similar ideas with compound inequalities.

Show how to represent a solution to $x \geq -2$ and $x < 5$

Example Represent the following with a number line and interval notation.

a. $x > 0$ and $x \leq 7$

b. $x > 8$ and $x \leq -2$

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Show how to represent a solution to $x \geq 4$ or $x < -3$

Example Represent the following with a number line and interval notation.

a. $x \leq 0$ or $x > 5$

b. $x > 0$ or $x \leq 5$

We can create more complicated problems where we first have to isolate x, but then we can represent in the same manner as shown above.

Example Represent the following with a number line and interval notation.

$3x - 1 > 5$ and $5x + 4 \leq 24$

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Examples: Compound Inequalities

Put your final answer both on a number line and in interval notation.

a. $x - 4 > 8$ or $x + 2 < -3$

b. $2x + 1 \geq 7$ and $5x - 10 < 20$

c. $4x + 2 > 18$ or $3x - 1 < 17$

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d. $2 - 3x > 8$ and $5x - 2 > 12$

e. $5x + 9 < 16$ or $4 - 3x > 14$

f. $3 - 2x > 9$ and $5 - x < -12$

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Absolute Value Equations
Video 1: The Basics

Absolute value equations look like this:

$$|x + 2| = 7 \quad \text{or} \quad |3x - 5| + 4 = 2$$

To understand how to solve these types of equations, first think about: what are the solutions to $|x| = 5$? Explain why.

From here, we can see that we must consider 2 possible solution sets to find all solutions to absolute value equations. Write down the general strategy for solving absolute value equations:

Solve: $|2x + 3| = 9$

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Examples Solve the following:

a. $|3x - 9| = 6$

b. $|7 - 4x| = 3$

To solve $|5x - 4| + 2 = 13$, what must we do first?

Solve:

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Examples Solve:

a. $|3x - 6| + 4 = 10$

b. $\left|\frac{1}{2}x + \frac{1}{3}\right| + 5 = 10$

The next video will discuss a special situation and a more complicated equation type.

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Absolute Value Equations
Video 2: More Complicated Situations

In this video, we will discuss how to solve equations like:

$$|x+2| = -7 \quad \text{or} \quad |3x-5| = |2x+7|$$

What can we say about the solutions to $|x| = -5$?

Examples Solve the following:

a. $|2x-7| = -8$

b. $|3x+5| + 7 = 2$

Absolute value equations can also look like this: $|3x+1| = |2x-5|$. Explain how to solve this.

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Examples Solve the following:

a. $|5x - 10| = |10x + 15|$

b. $|3x - 5| = |5x + 7|$

c. $|4x - 6| = |3x - 8|$

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Example/Question: Why Do You Only Make One Side Negative with Solving $|ax + c| = |bx + d|$ type problems?

What is the approach for solving $|x + 1| = 5$?

What is key to notice here?

How would you solve $|x + 1| = |5x|$?

How would you solve $|x + 1| = |5x - 3|$?

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A review of number lines, interval notation, and set notation

This is a general review of these concepts. We will look at how to represent solutions for problems like:

$x > 5$ or $-3 \leq x < 4$ or $x > 2$ or $x < -3$

Use the example $x < 4$ to describe the 3 types of inequality notation. Write down any special symbols associated with it.

Number line:

The special signs/symbols are:

Graph $x < 4$ on a number line:

Interval Notation:

The special signs/symbols are:

Write $x < 4$ in interval notation:

Set Notation

The format of this is always:

Write $x < 4$ in set notation:

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Examples Write the following inequalities on a number line, in interval notation, and in set notation.

a. $x \geq 5$

b. $x < -2.5$

c. $-4 \leq x < 7$

d. $x > 4$ or $x < -5$

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Examples Write the following inequalities on a number line, in interval notation, and in set notation.

a. $x \leq -3$

b. $x > 12$

c. $0 < x \leq 8$

d. $x > 3$ or $x < 1$

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Lesson: Absolute Value Inequalities

Absolute value inequalities have 2 key aspects to solving them. What are those 2 key aspects?

Explain the setup to solve an absolute value inequality of the form $|P| \leq k$ (or $< k$).

Explain the setup to solve an absolute value inequality of the form $|P| \geq k$ (or $> k$).

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Let's try to understand why these are setup this way.

Explain the solutions to $|x| < 3$. Summarize what was explained in the video.

Explain the solutions to $|x| \geq 3$. Summarize what was explained in the video.

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Examples Solve the following inequalities. Graph your solution set on a number line and express your solution in interval notation.

a. $|2x - 4| < 6$

b. $|3x + 6| \geq 9$

c. $|x - 5| - 4 \leq 6$

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d. $5 \geq |3x - 4|$

e. $7 < 3 + |x - 4|$

f. $6 \leq 4 + |8 - 2x|$

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Lesson: Special Absolute Value Inequalities

We will discuss some special situations. Explain the reasoning for each of these situations:

a. $|2x + 3| + 7 < 4$

b. $|3x - 2| > -3$

c. $|5x + 1| \geq 0$

d. $|2x - 4| < 0$

e. $|6x + 2| \leq 0$

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Lesson: Intro to Linear Equations in Two Variables, Graphs, and Plotting Points

A **linear equation in two variables** is an equation of the form ax+by=c, where a, b, and c are real numbers, and both a and b are not zero at the same time.

Solutions for equations with two variables come as an ordered pair. Unless otherwise stated, you generally view the ordered pair in the alphabetical order of the equation.

Examples Decide if the following ordered pairs are solutions to the equations.

a. $\left(2, -\frac{1}{2}\right)$ for $3x - 2y = 4$

b. $(-5, -1)$ for $3a + 2b = 10$

c. $(3, 2)$ for $y = 5 - x$

Examples Complete the ordered pairs for the following equations

a. $(2, b)$ for $4a - 6b = 2$

b. $(x, -3)$ for $y = 2x + 1$

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Another way to represent ordered pairs is with a table.

Examples Complete the following tables for the linear equations:

a. $y = 2x + 4$

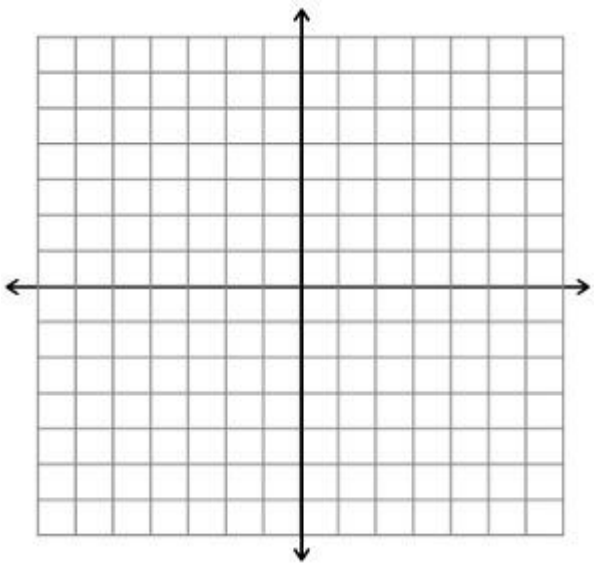
x	y
-3	
	10
0	

b. $3a - 2b = 12$

x	y
	0
0	
3	

Order pairs are often represented in coordinate systems. Below, draw the Cartesian (aka rectangular) coordinate system and label its parts.

Examples Plot the following points on the graph:



a. $(-2, 3)$

b. $(4, -1)$

c. $(1, 4)$

d. $(0, -4)$

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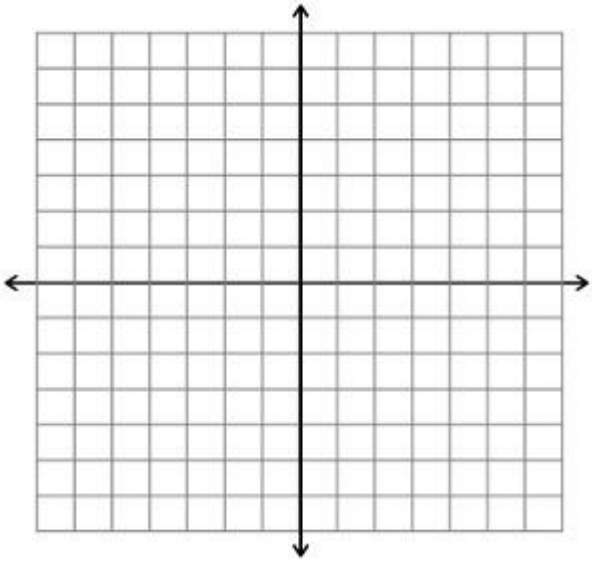
Lesson: Graphing Equations by Using a Table

One way to graph lines is by finding three points, and the plotting those points.

Examples Complete the following tables of ordered pairs, and the plot the points to form a line.

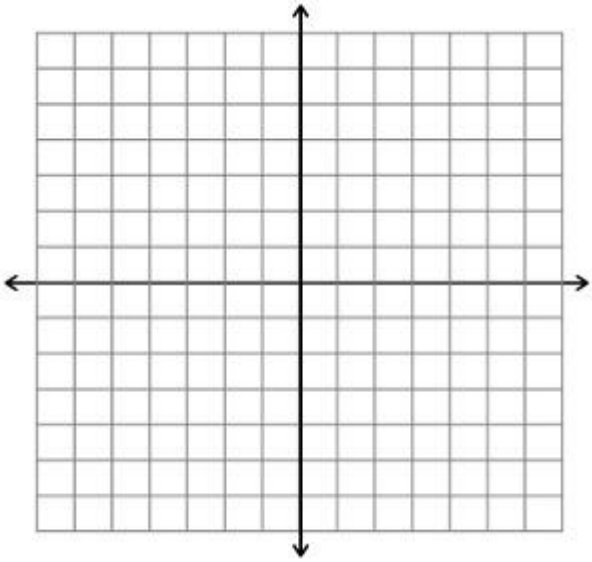
a. $y = 2x - 1$

x	y
2	
-2	
	0



b. $-2x + 6y = 12$

x	y
-3	
	0
3	

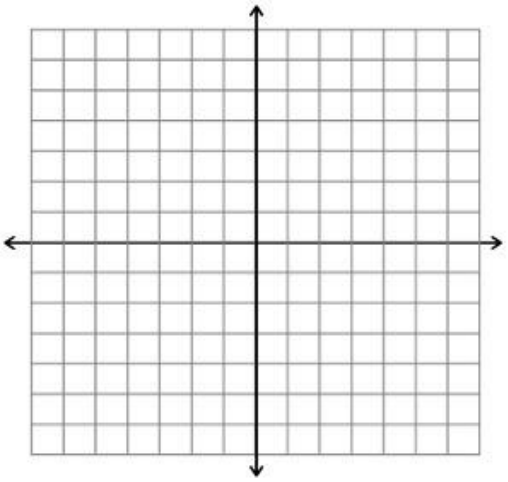


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Examples Find three points for the following equations, then plot those points to make a line.

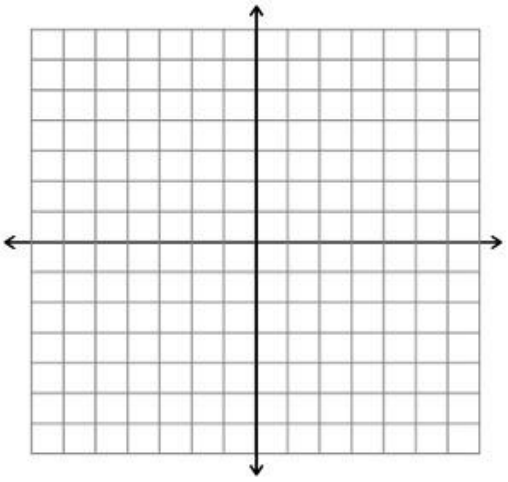
a. $y = -3x + 2$

x	y



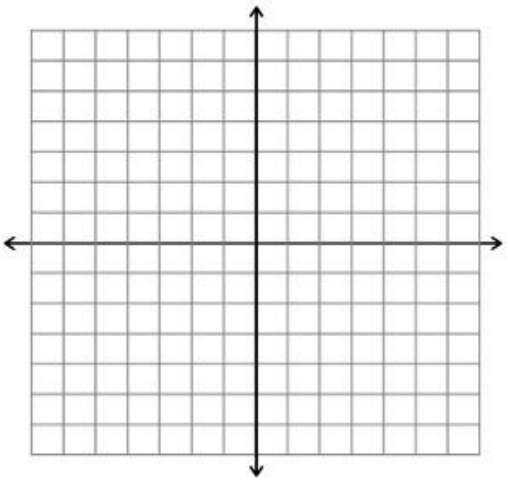
b. $y = -\frac{1}{2}x + 3$

x	y



c. $3x - 5y = 15$

x	y



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Lesson: Graphing Equations by Using Intercepts

What are x-intercepts? What are y-intercepts? Draw an illustration.

How do you find x-intercepts?

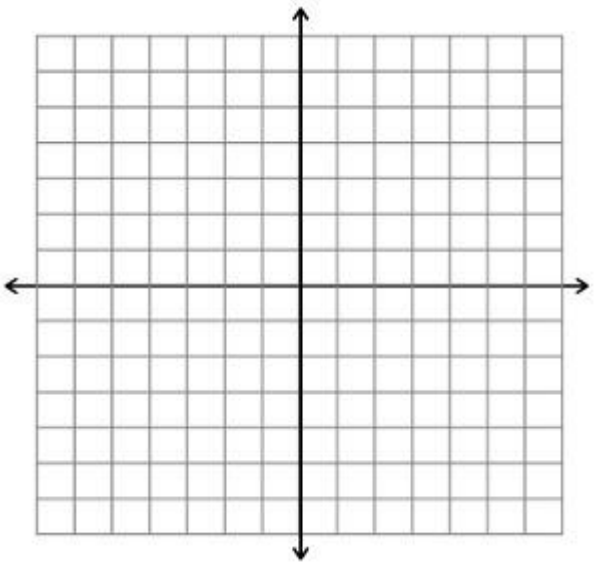
How do you find y-intercepts?

Examples Find the x- and y-intercepts of the following equations. Then find a third point point. Plot these three points to make a line.

$3x - 2y = 6$

x-intercept

y-intercept



Third point:

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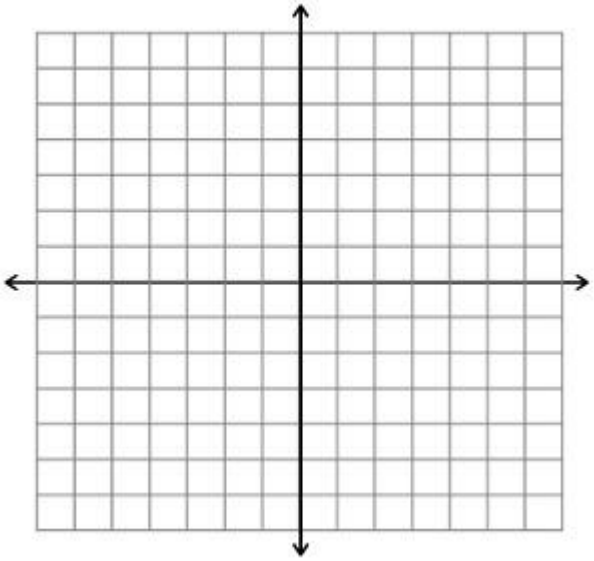
Examples Find the x- and y-intercepts of the following equations. Then find a third point point. Plot these three points to make a line.

$x - 4y = 4$

x-intercept

y-intercept

Third point:



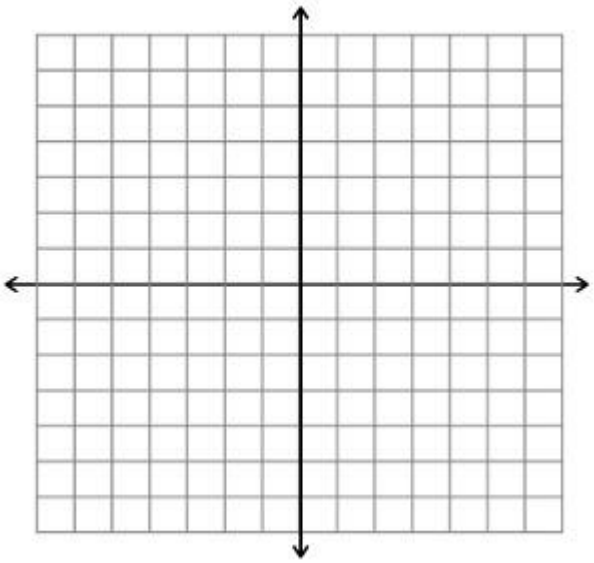
Examples Find the x- and y-intercepts of the following equations. Then find a third point point. Plot these three points to make a line.

$y = \frac{1}{2}x - 2$

x-intercept

y-intercept

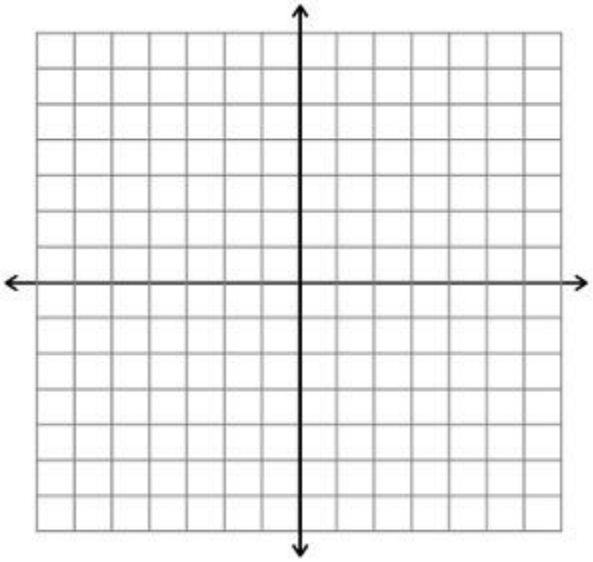
Third point:



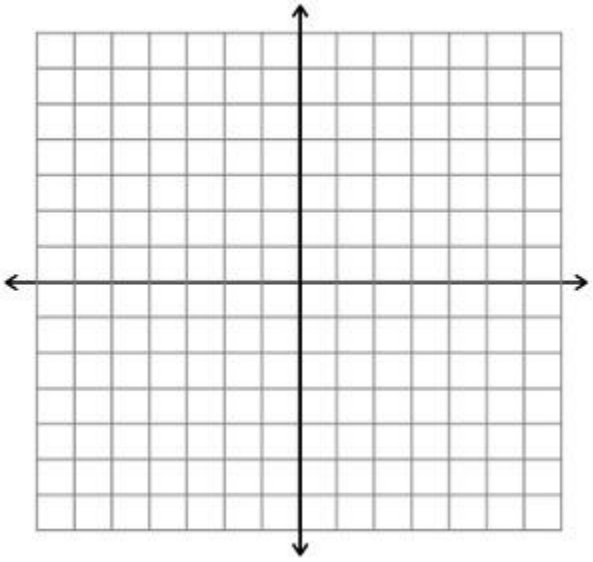
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Lesson: How to Graph Vertical and Horizontal Lines

Show how to graph $y = 3$



Show how to graph $x = -2$



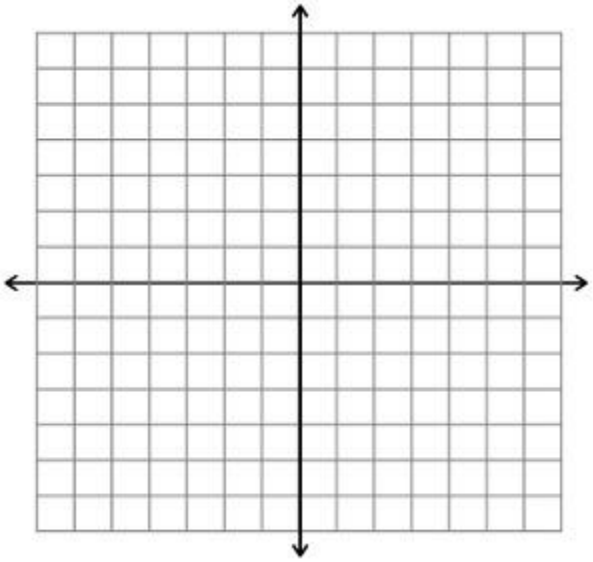
How do you know if you are working with a vertical line? How do you graph it?

How do you know if you are working with a horizontal line? How do you graph it?

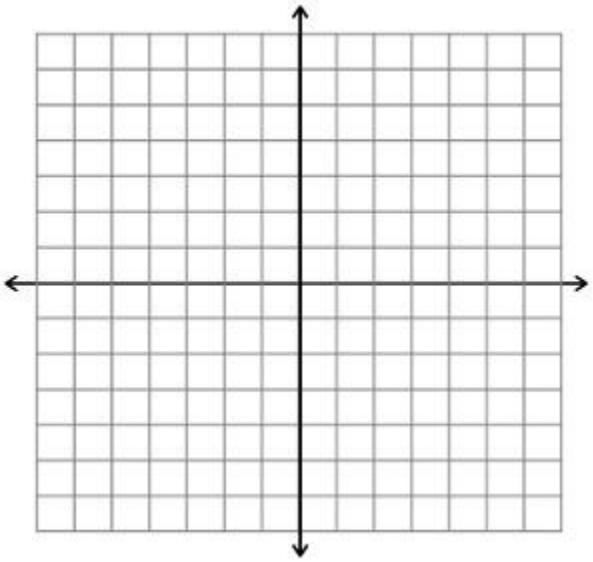
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Examples Graph the following:

a. $x = 4$



b. $y = -1$



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Lesson: The Slope of a Line

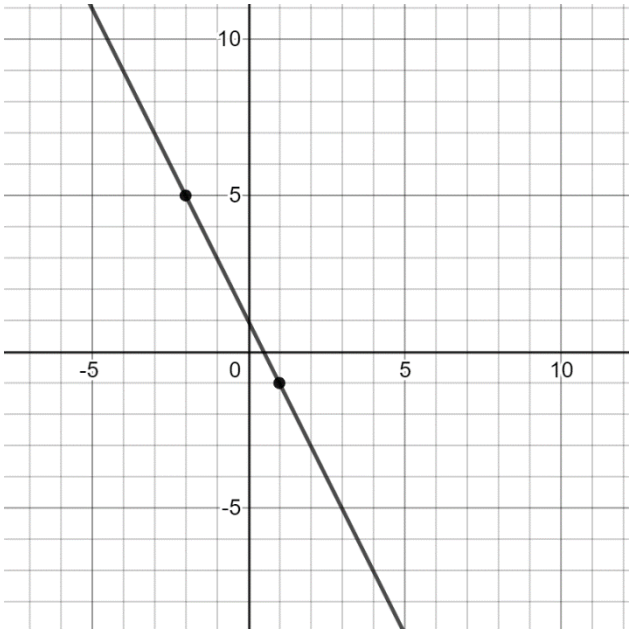
What does slope represent?

Draw the various possible slopes of lines:

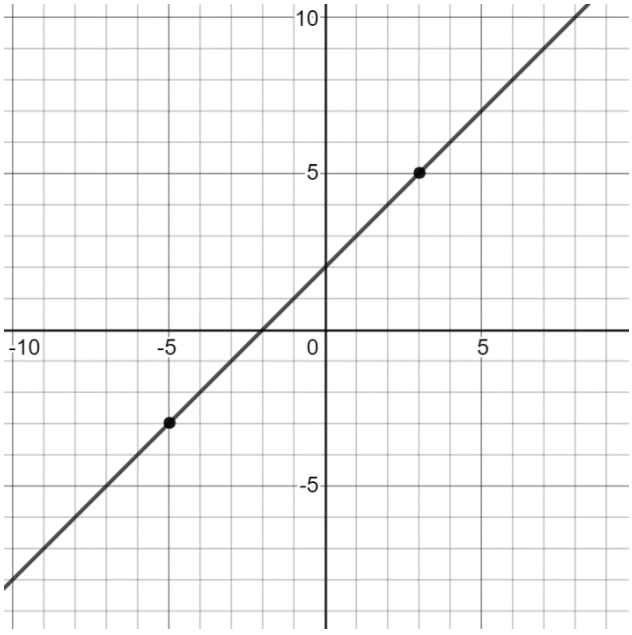
How do find the slope of a line?

Examples Find the slope of the following lines:

a.



b.



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What is the slope formula?

Examples Use the slope formula to find the slope between the pairs of points:

a. $(3, 5)$ and $(4, 8)$

b. $(6, -2)$ and $(-2, 4)$

c. $(7, 2)$ and $(7, 4)$

d. $(2, 4)$ and $(-1, 4)$

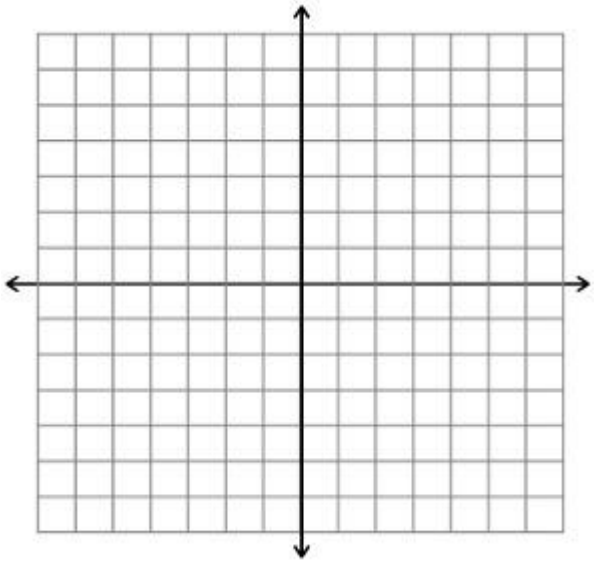
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Lesson: Using Slope to Graph Lines; Intro to Slope-Intercept Form

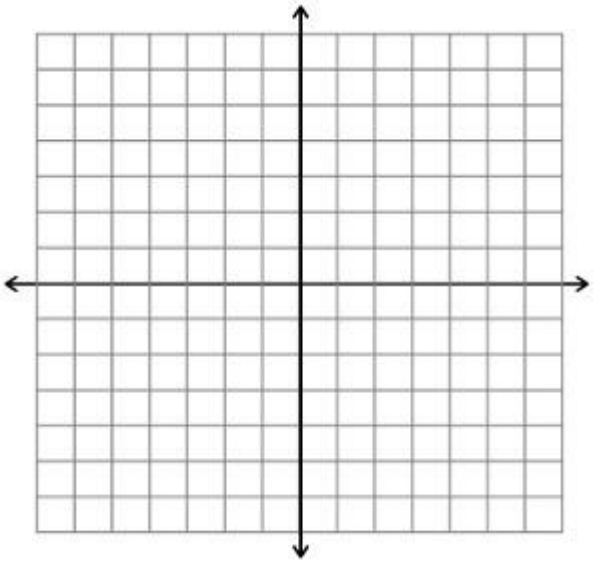
When the slope of a line and a point are known, a line can be easily graphed.

Examples Given the slope and the point, graph the following lines:

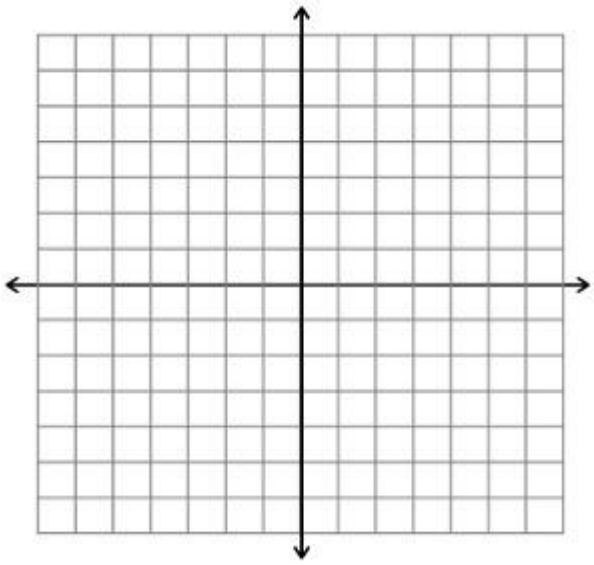
a. $m = 2$ and $(-1, 2)$



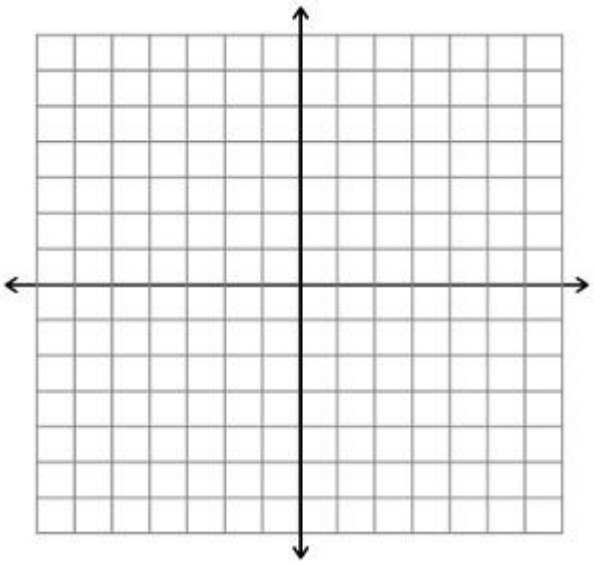
c. $m = 1$ and $(2, -1)$



b. $m = -\frac{1}{3}$ and $(-3, 4)$



d. $m = \frac{2}{3}$ and $(0, -3)$



What is the slope-intercept form of a line?

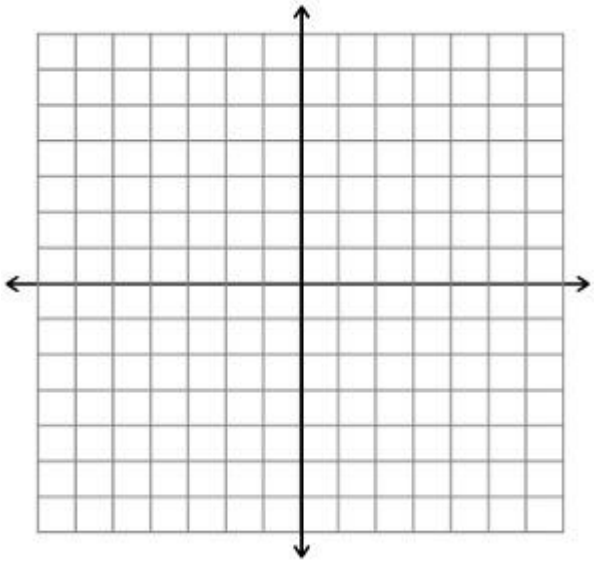
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Examples Identify the slope m and the y-intercept $(0, b)$ in each of the following equations:

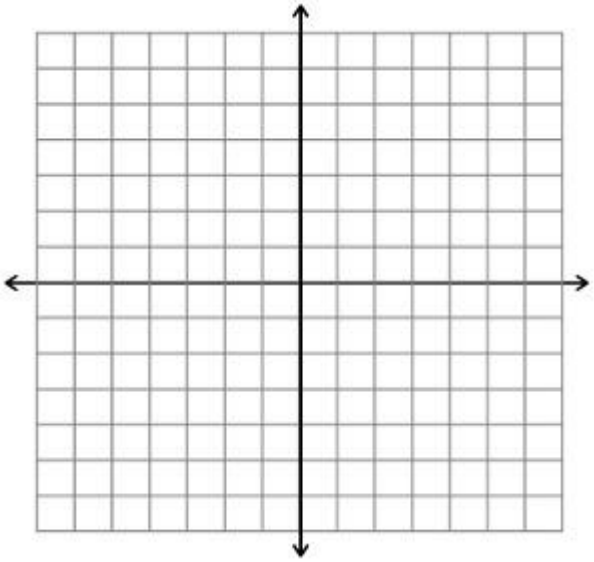
Equation	Slope, m	What stands for b	State the y-intercept (as a point)
a. $y = -2x + 3$			
b. $y = \frac{1}{4}x - 7$			
c. $y = 3x + 9$			
d. $y = x$			

Examples Graph the following lines:

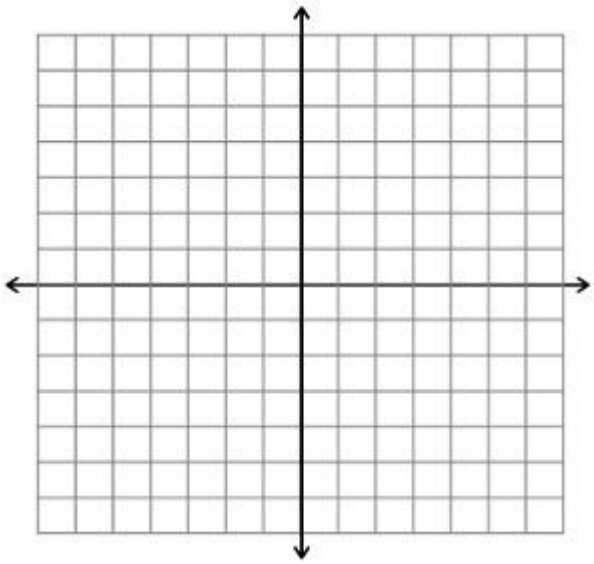
a. $y = 2x + 3$



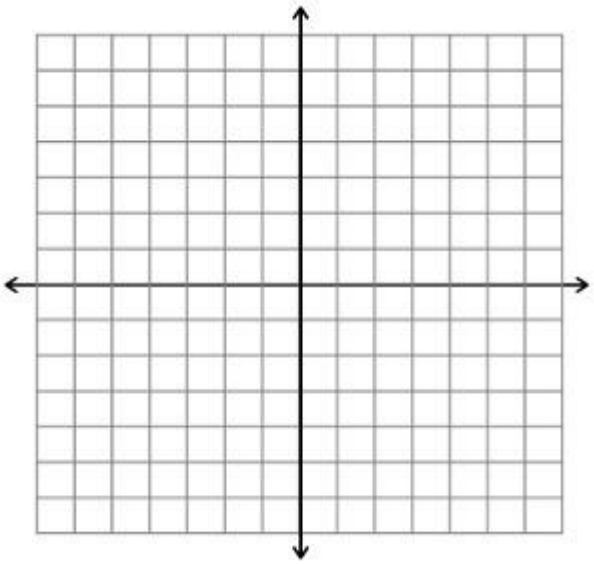
c. $y = \frac{1}{3}x - 4$



b. $y = -x + 4$



d. $y = 3x$



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Lesson: Rewriting Linear Equations into Slope-Intercept Form of a Line

What is the slope-intercept form of a line?

Explain how to put the equation $2x - 4y = 8$ into slope-intercept form.

Examples Put the following equations in slope-intercept form:

a. $6y - 3x = 12$

b. $-5x + 2y = -4$

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Lesson: Determining Whether Two Lines are Parallel or Perpendicular

What makes 2 lines parallel?

Examples Are the equations $y = \frac{2}{3}x + 5$ and $2x - 3y = 6$ parallel?

What makes 2 lines perpendicular?

Examples Determine the negative reciprocal of each of the following:

a. $\frac{2}{3}$

b. $-\frac{1}{5}$

c. -7

d. 1

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Example Are the equations $y = \frac{3}{5}x + 2$ and $-3x + 5y = -15$ perpendicular?

Examples Are the following pairs of lines parallel, perpendicular, or neither?

a. $3x - 2y = 6$ and $4y = -6x - 8$

b. $y - 3x = 0$ and $12x - 4y = 24$

c. $y = 3$ and $x = -2$

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Examples Lines A and B contain the given points. Determine whether the lines are parallel, perpendicular, or neither.

a. Line A: $(2, 11), (5, 10)$ Line B: $(1, 3), (-6, -18)$

b. Line A: $(-5, 5), (-7, -2)$ Line B: $(0, 11), (-4, 9)$

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Lesson: Putting Equations in Standard Form

What is the standard form of a line?

Examples Write the following equations in standard form:

a. $y = 2x - 4$

b. $y = -5x + 3$

c. $y = -\frac{1}{2}x + 1$

d. $y = \frac{2}{3}x - 2$

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Lesson: Writing Equations of Lines with The Point-Slope Form

What is the point-slope form of a line?

Examples Given the point and the slope, write the point-slope form of a line.

a. $m = -2, (1, 3)$

b. $m = -\frac{2}{3}, (5, -2)$

What is the slope-intercept form of a line?

What is the standard form of a line?

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Examples Given the point and the slope, write the equation of the line in the desired format.

a. $m = -1, (2, -5)$ in slope-intercept form

b. $m = 3, (5, -4)$ in standard form

c. $m = -\frac{1}{2}, (6, 3)$

d. $m = \frac{2}{3}, (-5, -1)$

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Lesson: Writing the Equation of a Line from Two Points

Write the equation of the line that contains the following 2 points in the desired format.

a. $(4, 2)$ and $(5, 5)$ in slope-intercept form

b. $(-2, 4)$ and $(-4, -2)$ in standard form

c. $(5, -3)$ and $(3, 5)$ in slope-intercept form

d. $(-1, -3)$ and $(1, 5)$ in standard form

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Lesson: Writing the Equation of a Line Parallel/Perpendicular to Another Line

What makes 2 lines parallel?

What makes 2 lines perpendicular?

Examples Write the equation of a line with the given conditions.

a. Parallel to $y = 3x + 1$ that goes through $(-2, -5)$ in slope-intercept form

b. Perpendicular to $y = -\frac{2}{3}x - 5$ that goes through $(6, -2)$ in standard form

c. Parallel to $2x - 3y = 6$ that goes through $(1, -2)$ in slope-intercept form

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Examples Write the equation of a line with the given conditions.

a. Perpendicular to $2x - 4y = 8$ that goes through $(-4, -2)$ in standard form

b. Parallel to $3x - y = 12$ that goes through $(-1, -5)$ in slope-intercept form

c. Perpendicular to $-2x + 3y = 12$ that goes through $(2, -5)$ in standard form

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Lesson: Linear Inequalities in Two Variables

What is a linear inequality in 2 variables?

Show how to graph $y \leq 2x + 1$

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Examples Graph the following:

a. $3x + 2y > -6$

b. $5x - 2y \leq 10$

c. $-3x + 4y < -12$

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Lesson: Compound Linear Inequalities in Two Variables

What is a compound linear inequality?

What does “and” mean in compound inequalities?

What does “or” mean in compound inequalities?

Show how to graph $y > 2x + 1$ and $y \leq -x - 3$

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Show how to graph $y \leq -2x + 1$ or $y > x + 2$

Examples Graph the following:

a. $3x - 2y > 6$ and $2x + y \leq 3$

b. $5x - 2y \leq 10$ or $2x + 6y > 12$

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Examples Graph the following:

a. $y \geq 4$ and $y < -2x + 1$

b. $x < 3$ or $4x - 2y \leq 8$

c. $x > -1$ and $y \geq 2x + 2$

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Lesson: An Intro to Functions

Define a relation:

Define a domain:

Define a range:

Example In the set of ordered pairs $\{(2,3), (4, -1), (7, 2)\}$, define the domain and range of the relation.

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Define a function:

What is the visual explanation that explains what a function is or is not?

Example In the set of ordered pairs $\{(2,3), (4, -1), (7, 2)\}$, is this a function?

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Example Are the following functions?

a. $\{(4,-1),(3,2),(7,-1)\}$

b. $\{(2,4),(3,5),(2,-1)\}$

What is the notation for a function? How do we describe/use this notation?

Example In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, what is $f(4)$?

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Example Suppose $f(x) = 2x + 1$ and $g(x) = x^2 - 3$. Find the value of the following:

a. $f(2)$

b. $g(1)$

c. $f(a)$

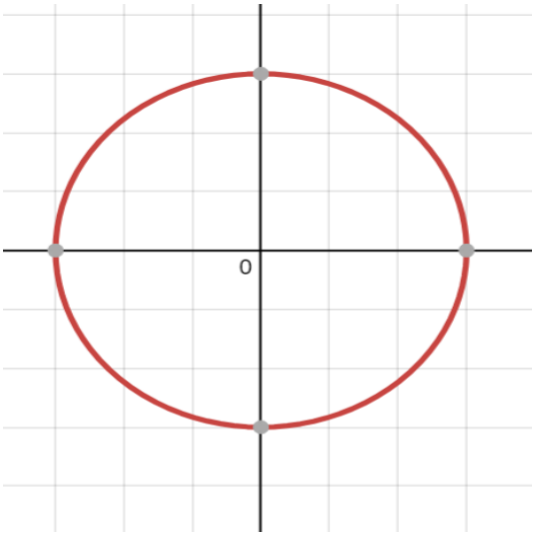
d. $g(h + k)$

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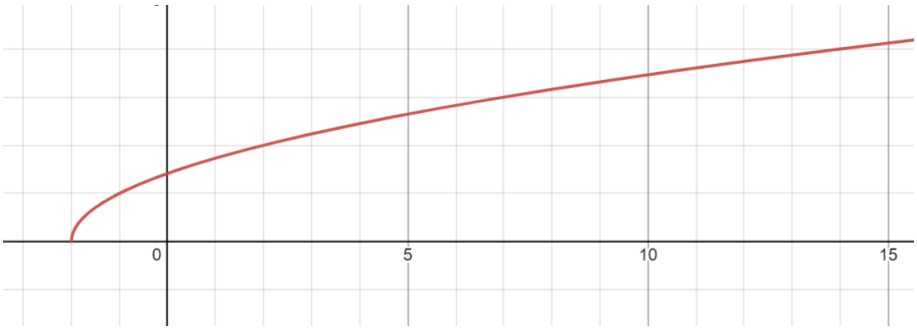
What is the vertical line test?

Example Do the following graphs represent a function?

a.



b.



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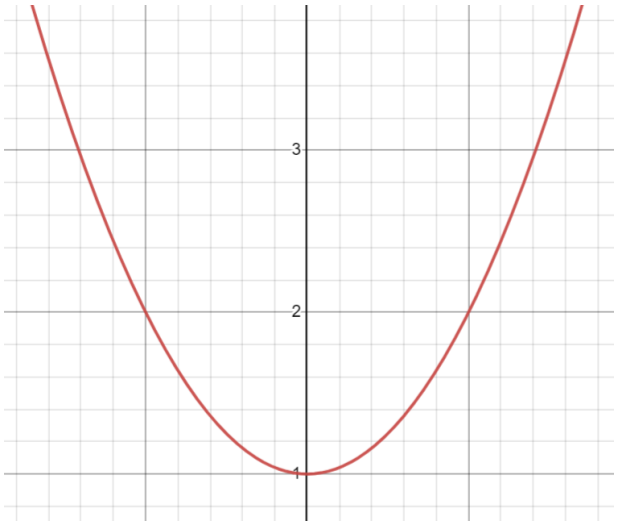
Lesson: Determining the Domain and Range of Functions In General

How do you determine the domain and range of a set of points like $\{(2,4), (5,8), (4,6)\}$?

What is the domain and range of that set of points?

In general, it is easiest to determine the domain and range when we have the graph in front of us.

For the following graph, write out how you should determine the domain and range.



Write a sentence on how you should find the domain:

State the domain:

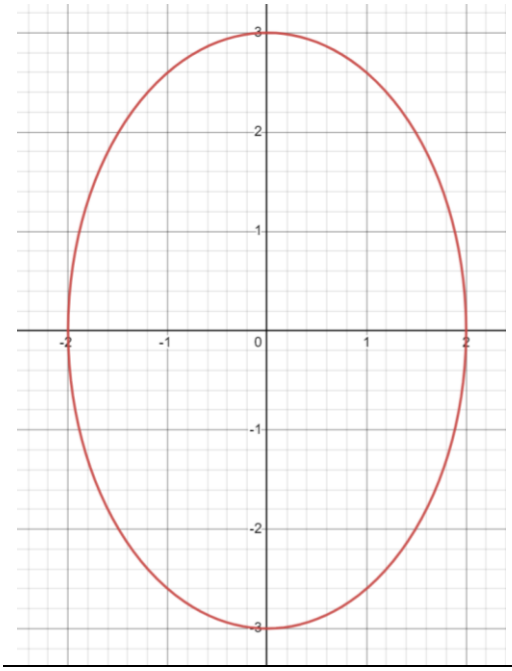
Write a sentence on how you should find the range:

State the range:

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Examples Determine if the following graphs represent functions, and then state the domain and range.

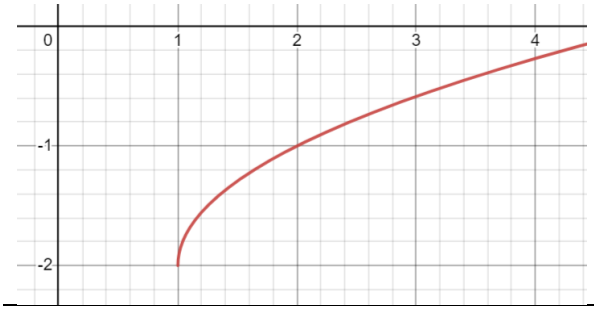
a.



Domain:

Range:

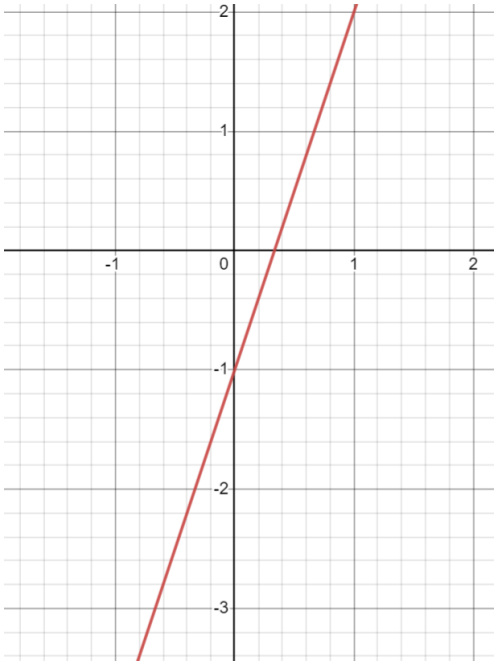
b.



Domain:

Range:

c.



Domain:

Range

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Lesson: The Domain of Some Specific Types of Functions

What is a polynomial? Write some examples.

What is the domain of any polynomial function?

What is a rational function?

What is the domain of any rational function?

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Examples Determine the domain of the following.

a. $f(x) = 3x - 1$

b. $g(x) = \frac{4}{x+2}$

c. $h(x) = 3x^3 - 2x^2 + 4x - 1$

d. $f(x) = \frac{2x+1}{x^2-9}$

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What is a square root function?

What is the domain of a square root function?

Examples Find the domain of the following:

a. $f(x) = \sqrt{x-2}$

b. $g(x) = \sqrt{4x-1}$

c. $h(x) = \sqrt{4-x}$

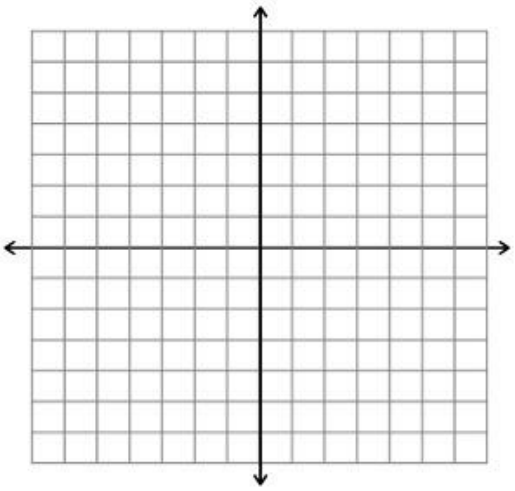
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Lesson: How to Graph a Function

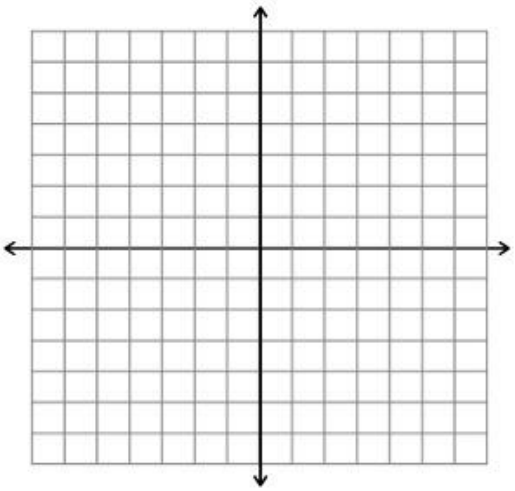
How do you graph a function?

Examples Graph the following functions.

a. $f(x) = 3x + 1$



b. $g(x) = 4$



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Lesson: A Few Basic Graphs

There are a few graphs that are helpful to immediately know as we move forward. Let's break these down.

The Parabola: $f(x) = x^2$

Create table of relevant points and then sketch your own graph.

Absolute Value: $f(x) = |x|$

Create table of relevant points and then sketch your own graph.

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X cubed: $f(x) = x^3$

Create table of relevant points and then sketch your own graph.

Square Root: $f(x) = \sqrt{x}$

Create table of relevant points and then sketch your own graph.

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Cube Root: $f(x) = \sqrt[3]{x}$

Create table of relevant points and then sketch your own graph.

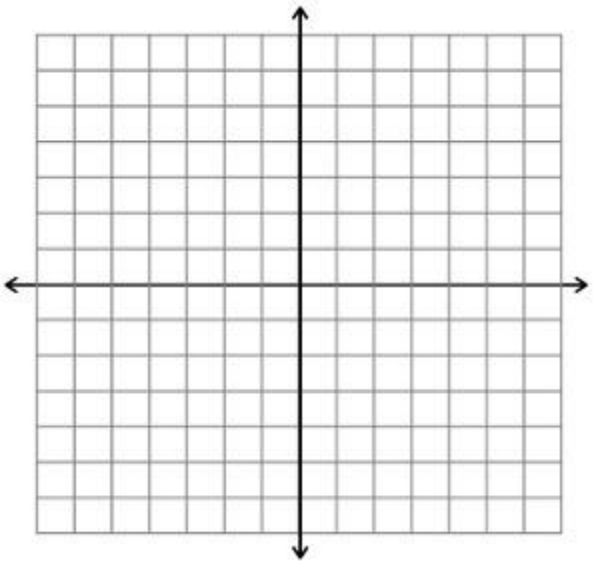
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Lesson: An Intro to Translations and Reflections of Graphs

We are going to develop the intuition for this concept before we actually graph anything.

Vertical Translations

Draw the base graph of $f(x) = x^2$ in one color and then use some other colors to show how the translations work. Label the graphs.



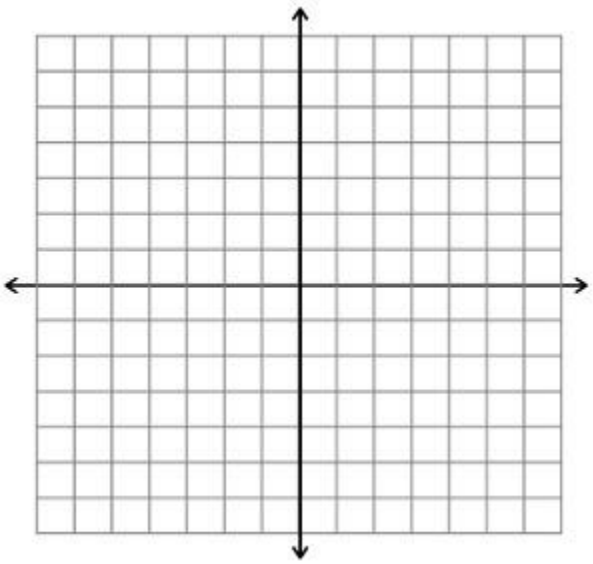
Explain the general way that vertical shifts work:

Example How does $f(x) = \sqrt{x} + 6$ shift?

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Horizontal Translations

Draw the base graph of $f(x) = |x|$ in one color and then use some other colors to show how the translations work. Label the graphs.



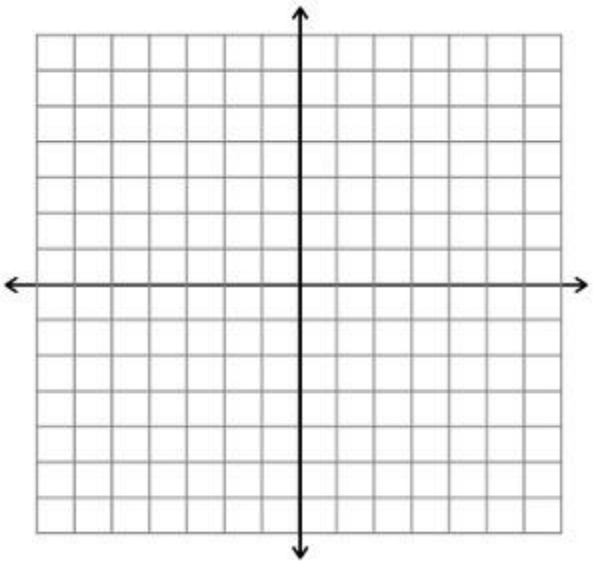
Explain the general way that horizontal shifts work:

Example How does $f(x) = \sqrt[3]{x+3}$ shift?

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Reflections Over the X-Axis

Draw the base graph of $f(x) = \sqrt{x}$ and then graph the reflection in another color. Label the graph of the reflection. Label the graphs.



Explain the general way that reflections over the x-axis work:

Example How does $f(x) = -x^2$ look?

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Example Describe the translations and reflections of the following.

a. $f(x) = -x^2 + 3$

b. $g(x) = \sqrt{x+2} - 1$

c. $h(x) = (x-5)^3 + 2$

d. $f(x) = -|x+2| + 5$

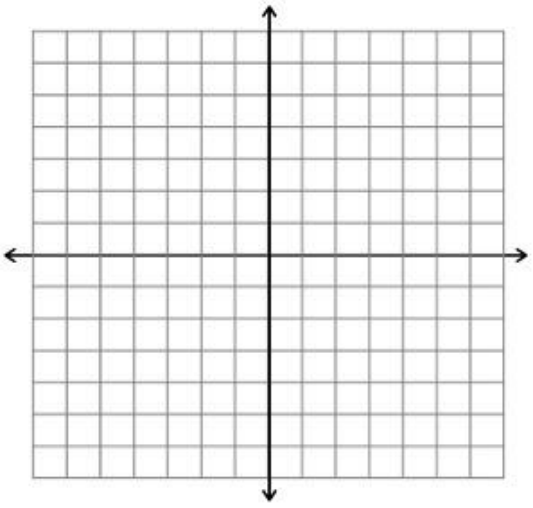
e. $g(x) = -\sqrt[3]{x-1} - 4$

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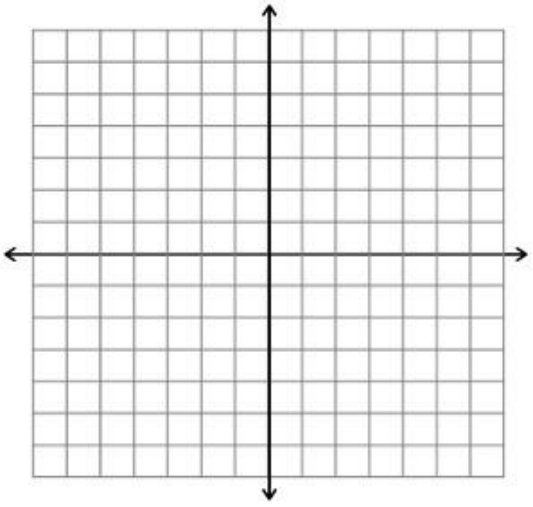
Lesson: Graphing with Translations and Reflections

Examples For each of the following, describe the translations and reflections. Then graph with 2-3 points.

a. $f(x) = x^2 + 1$
Describe any translations or reflections:

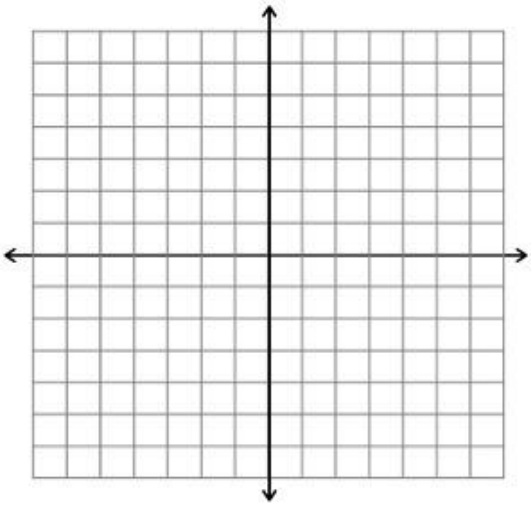


b. $g(x) = \sqrt{x - 2}$
Describe any translations or reflections:

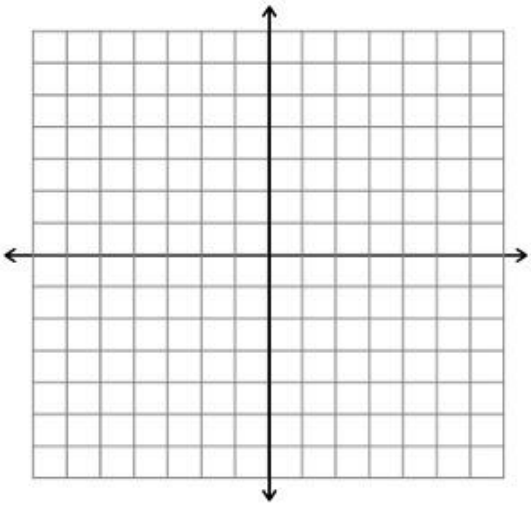


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c. $f(x) = (x + 2)^3 - 1$
Describe any translations or reflections:

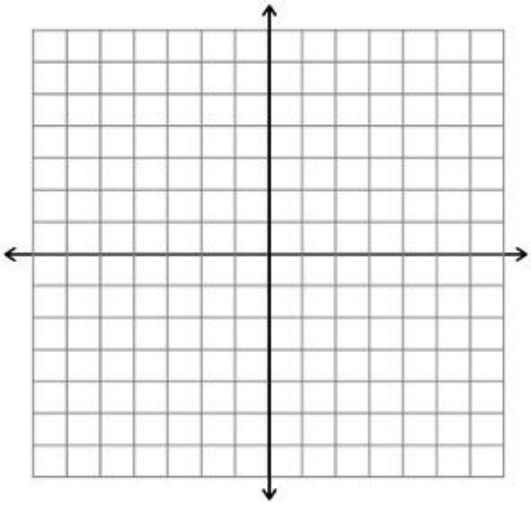


d. $h(x) = |x - 3| + 2$
Describe any translations or reflections:

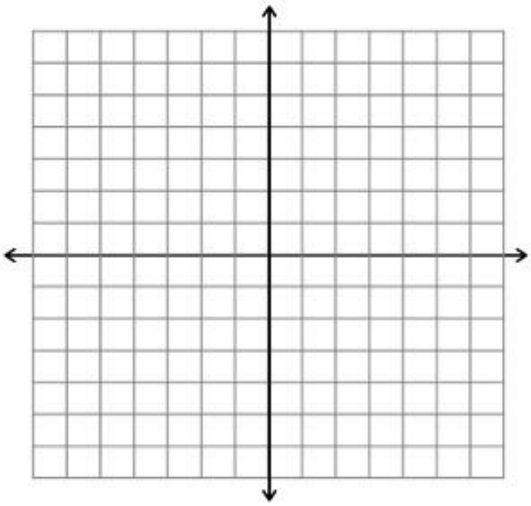


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e. $f(x) = -|x - 3| - 2$
Describe any translations or reflections:



f. $f(x) = -\sqrt{x + 3} + 1$
Describe any translations or reflections:



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Lesson: Writing Equations of Functions with Translations and Reflections

Example Write the equation of a function $g(x)$ where the following transformations are performed on the graph of $f(x)$

a. $f(x) = \sqrt[3]{x}$ is shifted 3 units down and 2 units right

b. $f(x) = x^2$ is shifted 2.5 units up and π units left

c. $f(x) = e^x$ is reflected about the x-axis, shifted 4 units down and 1 unit left

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Lesson: Solving Systems of Equations by Graphing

We will now turn our attention to systems of equations.

System of Equations

A system of equations refers to 2 or more equations that share 2 or more variables.

Solution of a System of Equations

A solution of a system of equations is an ordered pair that is true in all equations.

Determine if the following pairs are solutions to the systems.

a. $(1,3)$
$x - 2y = -5$
$2x - 3y = -4$

b. $(-4, 3)$
$x + 3y = 5$
$2x + 3y = 1$

There are several ways to solve systems. The first method we will focus on is how to solve by graphing.

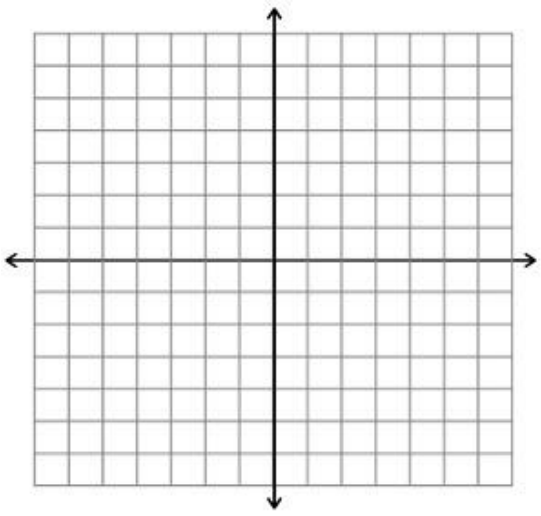
Solving by Graphing

In this method, we graph both lines and determine the point of intersection if possible. We will examine all the possibilities.

Example

Graph each line and determine the point of intersection to find the solution.

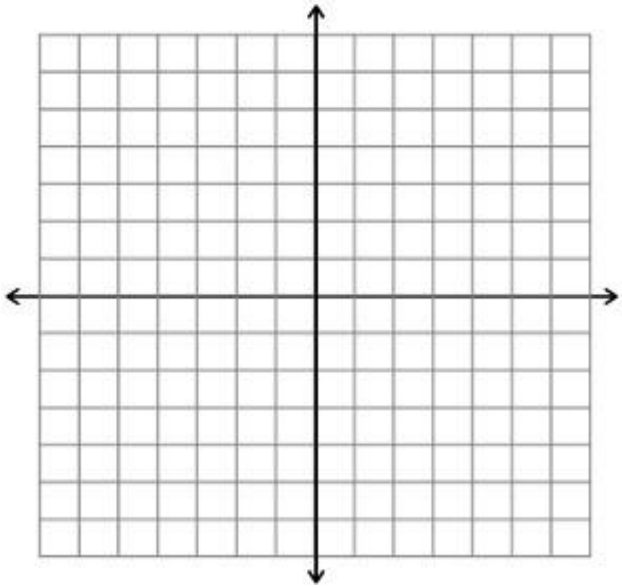
$y = 2x + 3$
$y = -\frac{1}{2}x - 2$



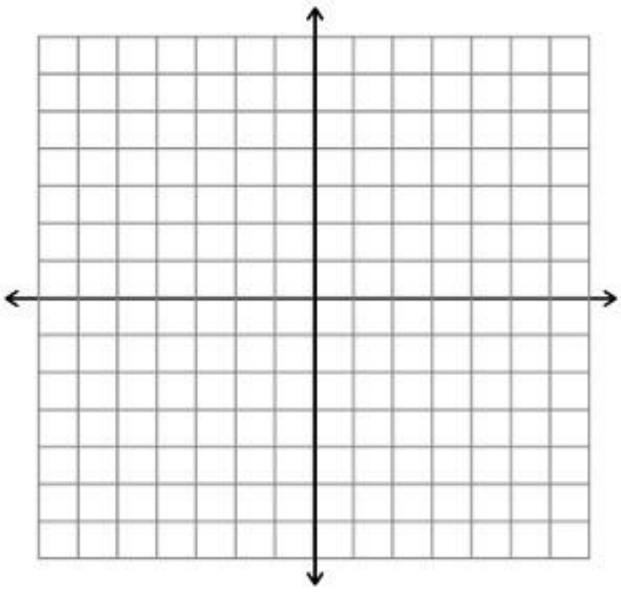
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Example

a. $y = -3x + 1$
$y = 2x - 4$



b. $y - \frac{1}{2}x = -4$
$y + 2x = 1$

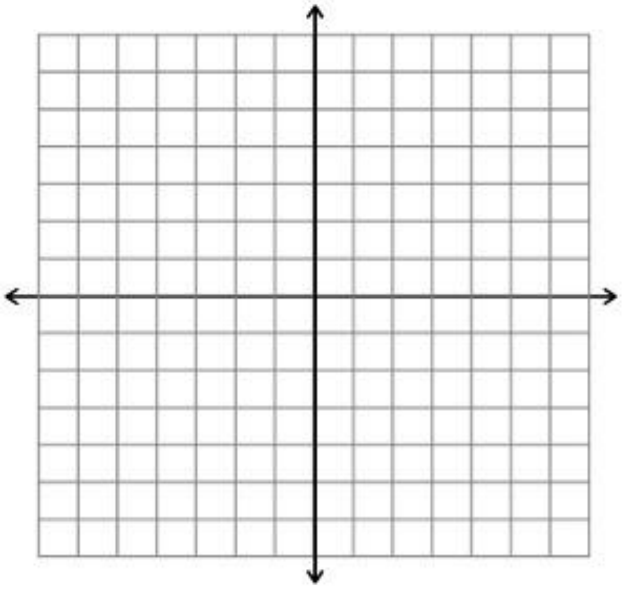


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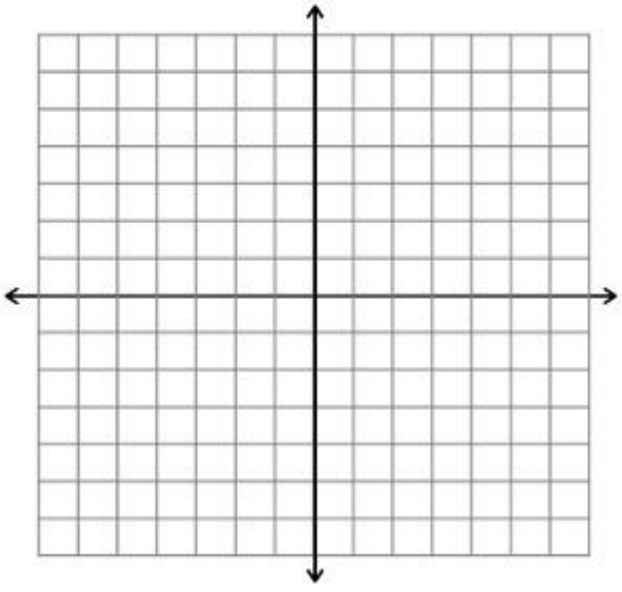
There are two special situations that can occur with graphing.

Example

a. $y = -3x + 2$
$9x + 3y = 6$



b. $y = -\frac{1}{2}x + 4$
$2x + 4y = 12$



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Lesson: Solving Systems of Equations by Substitution

In this lesson, we will focus on the second method for solving systems: substitution.

Show how to use substitution with this example:

$2x + y = 6$

$3x + 5y = 9$

Example Solve the system using substitution

$x - 2y = 10$

$5x - 7y = 20$

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Below are 2 more complicated problems using substitution. Outline the steps shown in the video:

$-2x + 4y = 4$

$-2x + 6y = 6$

$2x - 5y = -4$

$8x - 9y = 6$

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Now let's see what happens when we get a problem with no solution or infinite solutions.

a. $9y - 18x = 5$

$2x - y = 3$

b. $x + 2y = -3$

$4x + 12 = -8y$

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Lesson: Solving Systems of Equations by Elimination

In this lesson, we will focus on the third method for solving systems: elimination.

Show how to use elimination with this example:
$3x - y = 8$

$2x + y = -3$

Example Use elimination to solve the system.

$4x + 2y = -6$
$3x - 2y = 6$

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Now let's examine a more complicated case of elimination. Write down how to approach these problems:

a. $3x - 6y = 12$

$5x - 2y = -4$

b. $5x - 10y = 5$

$2x - 3y = 6$

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Now let's investigate what happens in the situation where there is no solution or infinite solutions.

a. $5x - 2y = 1$
$-10x + 4y = 3$

b. $3x + y = 5$
$-9x - 3y = -15$

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Lesson: Solving Systems of Equations that Require Clearing of Fractions

Examples

a. $\frac{1}{6}x - \frac{1}{3}y = \frac{1}{2}$
$\frac{2}{5}x - \frac{3}{5}y = -\frac{6}{5}$

b. $-\frac{1}{2}x - \frac{3}{4}y = 1$
$\frac{1}{3}x + \frac{3}{2}y = -\frac{2}{3}$

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c. $\frac{1}{2}x + \frac{3}{5}y = 6$

$\frac{1}{4}x - \frac{1}{2}y = -9$

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Lesson: Solving Systems of Equations that Require Clearing of Decimals

Examples

a. $0.02x + 0.1y = 0.05$
$4x + 20y = 10$

b. $0.1x - 0.05y = 0.2$
$0.3x - 0.25y = 0.2$

c. $-0.4x + 1.3y = 1.$
$1.6x - 0.9y = 6.9$

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Lesson: Applications of Systems

The top of a rectangular desk is twice as long as it is wide. The perimeter of the desk is 180 inches. What are the dimensions of the desk?

Sarah bought 3 shirts and a scarf for $53. Marie bought 2 shirts and 3 scarves for $54. Find the cost of each item.

How many milliliters of a 5% acid solution and how many milliliters of a 17% acid solution must be mixed to obtain a 60 mL solution that is 15% acid?

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Lesson: Systems of Equations with 3 Equations

Show how to solve the system:

$x + 2y + 3z = -11$

$3x - y + z = 0$

$-2x + 3y - z = 4$

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Sometimes you will have missing variables. Show how to solve the following system that is missing some variables:

$x - 2y = -3$

$x + 6z = 3$

$2y - 4z = 6$

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Now let's look at an example where there is no solution versus infinite solutions:

$8x + 20y - 4z = 16$

$-2x - 5y + z = -4$

$10x + 25y - 5z = 20$

$x + 4y - 3z = 2$

$2x - 3y + 2z = -5$

$3x + 12y - 9z = 2$

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Examples: More Examples of Systems of 3 Equations

Example Solve the system:

$x + 3y - z = -2$

$-x + 3y - 2z = 5$

$2x + 4y + 2z = 4$

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Example Solve the following system:

$x - 2z = 2$

$-4y + 2z = -2$

$-x + 3y = -3$

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Example Solve the following system:

$x - 4y + 2z = -7$

$3x - 8y + z = 7$

$6x - 12y + 3z = 12$

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Lesson: Basic Exponent Rules – Product, Power, and Quotient Rules

When working with rules of exponents, it is very important to be able to identify the base of each exponent. To begin, we will practice this.

Example 1

State the base for each exponent in the problems below.

Expression	Exponent	Base for this Exponent
5^3		
-5^4		
$-5x^6$		
$3y^7$		

Product Rule

First we want to remember what the actual definition of an exponent means.
We want to find: $2^3 * 2^2$. We will calculate this a few ways to understand the rules.

First we will work towards finding the exact answer. Evaluate each exponent below as you normally would with the **order of operations**, and then multiply the numbers together:

Now we will re-examine this result solely from the viewpoint of exponents. Write out each exponent as a multiplication problem, and notice the results you get from this exercise.

What do you notice about the exponents?

Product Rule (formal mathematical definition)

Let x be a real number, and let a and b be integers. Then:

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Example 2

Use the product rule to evaluate the following.

a. $x^2 * x^4$

c. $x^3 + x^7$

b. $3x^3 * -1x^4 * 2x$

d. $\frac{2}{3}x^4y^2z^5 * 6x^3y^2 * \frac{1}{2}y^3z^4$

Power Rule

Using the order of operations, first rewrite the expression inside the parentheses without exponents:
$(2^3)^2$ =

Now rewrite this entire result just utilizing what we know about the definition of exponents:

What is the relationship between the exponents in this case?

The Power Rule (formal mathematical definition)

Let x be a real number, and let a and b be integers. Then:

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Example 3

Use the power rule to evaluate the following:

a. $(x^3)^5$

c. $(3x^4y^5)^3$

b. $(x^2y^7z)^4$

d. $\left(\frac{2}{3}x^5y^2\right)^3$

Is there a difference between $(x+y)^n$ and $(xy)^n$? Explain.

The Quotient Rule

Using the order of operations, first rewrite the expression inside the parentheses without exponents:

$$\frac{2^5}{2^3}$$

First we will evaluate and find the final answer by using the order of operations below.

Notice the final answer of this question. How can we rewrite this number in its exponent form?

What relationship do you see between the exponents in the original question and the final answer?

The Quotient Rule (formal mathematical definition)

Let x be a real number, and let a and b be integers. Then:

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Example 4

Use the quotient rule to simplify the following:

a. $\frac{x^7}{x^3}$

c. $\frac{15x^4y^3z^{10}}{2x^2y^2z^5}$

b. $\frac{x^5y^8z^{10}}{xyz}$

d. $\frac{12x^7y^3}{8x^3z^4}$

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Examples: Exponent Rules, Basic Examples

Simplify the following using only the product, power, and quotient rules. Your final answer should not contain negative exponents.

a. $x^2y^7 * 5x^3y^5z$

b. $(4x^7y^3)^2$

c. $\frac{15x^7y^3}{10x^2y}$

d. $\frac{2}{5}x^2y^5 * \frac{1}{3}x^7yz^3$

e. $\left(\frac{5}{3}x^3y^2z^3\right)^3$

f. $(-3x^5)^2$

g. $(-3x^5)^3$

h. $(2x^4y^2z^6)^2 * 3x^2z^3$

i. $\frac{(x^3y^7)^2}{x^4y}$

j. $\frac{(x^4y^2)^3}{(x^2y)^2}$

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Examples: Exponent Rules, With Numbers Only

Simplify the following using only the product, power, and quotient rules. Your final answer should be a number.

a. $3^2 * 3$

b. $(-4)^2 * 2$

c. $\frac{2^7}{2^4}$

d. $\left(\frac{2}{3}\right)^3 * 3^2$

e. $\frac{6^2}{2^3}$

f. $\left(\frac{1}{2}\right)^4 * 2^2$

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Examples: Exponent Rules, Trickier and Harder Examples

Simplify the following using only the product, power, and quotient rules. Your final answer should not include negative exponents.

a. $(3x^2y^4)^2 * (x^4y^5)^3$

b. $\frac{(2x^5y^2)^3}{(x^2yz^5)^2}$

c. $\left(\frac{2}{3}x^5y^7z^3\right)^2 * 9x^3y^3$

d. $\left(\frac{3}{5}x^5y^2z^7\right)^2 * \frac{xy^3}{z^5}$

e. $\frac{(3x^5y^2z^4)^3}{(6x^2yz^3)^2}$

f. $\frac{(4x^3y^2z^5)^2}{(2xz^2)^4}$

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Lesson: A Review of Exponent Rules for Algebra Students

Power Rule: $x^m * x^n =$

Product Rule: $(x^m)^n =$

Quotient Rule: $\frac{x^m}{x^n} =$

Negative Exponents

$x^{-n} =$ $\frac{1}{x^{-n}} =$

$\left(\frac{x}{y}\right)^{-n} =$

Simplify the following using exponent rules. Your final answer should not contain negative exponents.

a. $3x^7y^{-4} * 5x^{-2}y^9$

b. $\left(\frac{2x^4y^5}{3z^5}\right)^3$

c. $\frac{10x^4y^7}{2x^2y^3z^{-5}}$

d. $\frac{4x^3y^{-2}}{6x^{-4}y^{-5}}$

e. $\frac{8x^{-3}y^4z^{-3}}{10x^5y^{-2}z^{-7}}$

f. $\left(\frac{3x^{-3}y^{-2}}{6x^2y^4}\right)^{-2}$

g. $\left(\frac{2x^{-7}z^3}{3x^{-2}y^{-3}z^5}\right)^{-3}$

h. $\left(\frac{5x^{-2}y^{-3}z^3}{x^3y^4z^{-7}}\right)^{-2}$

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Lesson: Negative Exponents

Simplify: $\frac{25}{125}$

If we wrote these numbers as exponents, this calculation would look like this:

$$\frac{5^2}{5^3} = \frac{1}{5}$$

Consider what would happen if we used the quotient rule on this problem:

$$\frac{5^2}{5^3} = 5^{2-3} - 5^{-1} = \frac{1}{5}$$

This problem shows us that exponents can also be negative.

Three Rules for Negative Exponents

Let x and y be a real numbers, and let n be an integer. Then:

$$x^{-n} = \frac{1}{x^n}$$

$$\frac{1}{x^{-n}} = x^n$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^{n} = \frac{y^n}{x^n}$$

In general your final answer cannot have a negative exponent, so it is essential that you understand what to do with it.

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Example 1

Rewrite the following expressions without negative exponents.

a. 4^{-2}

b. $\frac{1}{3^{-3}}$

c. $\left(\frac{2}{5}\right)^{-2}$

d. $\left(\frac{1}{3}\right)^{-2}$

One of the biggest challenges with negative exponents is identifying how to apply them to the proper base. As a quick review, remember how we identified the bases in the last section:

Example 2

Identify the base of the following exponents:

Expression	First Base	First Exponent	Second Base	Second Exponent
3^5				
$-2x^5$				
$5x^3y^7$				

This is extremely important to remember when working with negative exponents. In general, our final answer can't have negative exponents so you need to be used to reformatting your answers properly if negative exponents come up.

Example 3

Simplify the following expressions, and write your final answer without negative exponents.

a. $\frac{a^3b^{-2}}{c^{-4}d^6}$

b. $\frac{3a^{-3}b^4c^{-5}}{d^2e^{-2}f^5}$

c. $\frac{a^5b^3c^4}{4d^{-2}e^{-4}f^{-2}}$

d. $\frac{a^{-3}b^{-2}c^{-6}}{d^4e^7f}$

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With this in mind, we should now be able to work on combining all of the rules of exponents we've learned.

Example 4

Simplify the following. Your final answer cannot contain negative exponents.

a. $(3x^{-2}y^4)^3$

b. $\left(\frac{2x^{-4}y^6}{x^5y^{-3}}\right)^4$

c. $(-2x^3y^{-2}z)^{-3}$

d. $\left(\frac{2x^3y^{-4}}{3x^{-2}y^{-5}}\right)^{-3}$

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Examples: Negative Exponents, Basic Examples

Your final answer should not contain negative exponents.

a. $\left(\frac{4}{5}\right)^{-2}$

b. $3x^{-6}y^{4}z^{-2}$

c. $\frac{4x^{-4}y^{7}}{z^{-5}}$

d. $\frac{6x^{3}y^{-5}}{x^{7}y^{2}}$

e. $\frac{14x^{-3}y^{5}z^{3}}{21x^{-2}y^{-3}z^{9}}$

f. $(3x^{-4}y^{-2}z^{4})^{2}$

g. $\left(\frac{5x^{2}y^{-4}}{z^{2}}\right)^{5}$

h. $(x^{-3}y^{2}z^{-4})^{-3}$

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Examples: Negative Exponents, Tricky Examples

Your final answer should not contain negative exponents.

a. $(3x^{-4}y^{5}z^{-2})^{-3}$

b. $(-5x^{2}y^{-3}z^{4})^{-2}$

c. $\left(\frac{4x^{-5}y^{2}}{x^{-3}y^{-5}}\right)^{-2}$

d. $\left(\frac{3x^{3}y^{-3}z^{4}}{2x^{-2}y^{4}z^{-2}}\right)^{-3}$

e. $\frac{(2x^{-4}y^{5})^{4}}{x^{-3}y^{7}}$

f. $\frac{(3x^{-3}y^{2}z^{-3})^{-3}}{(2x^{4}y^{-5}z^{3})^{-4}}$

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Examples: Negative Exponents versus Negative Numbers

Examples Simplify:

a. 6^{-2}

b. -6^2

c. $(-6)^2$

d. $(-6)^{-2}$

Examples Simplify:

a. $5x^{-2}$

b. $(5x)^{-2}$

c. $-5x^2$

d. $(-5x)^{-2}$

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Examples Simplify

a. -7^2

b. $(-7)^{-2}$

c. 7^{-2}

d. -7^{-2}

Examples Simplify

a. $3x^{-2}$

b. $(-3x)^{-2}$

c. $(-3x)^{-3}$

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Lesson: Scientific Notation

Scientific Notation

A number in scientific notation is of the form: $A \times 10^n$ where $1 \leq |A| < 10$ and n is an integer

Before we can write numbers in this form, it is best if we can identify examples that are or are not in this format.

Example 1

Identify if the following examples are in scientific notation. If not, circle and explain why.

a. 5.34×10^7 Yes, it's in scientific notation No, it's not in scientific notation

b. $-2.7832 \times 10^{\frac{1}{2}}$ Yes, it's in scientific notation No, it's not in scientific notation

c. -4.5382×10^{-8} Yes, it's in scientific notation No, it's not in scientific notation

d. 0.8×10^{23} Yes, it's in scientific notation No, it's not in scientific notation

We will now examine how to rewrite the following example in scientific notation.

Example 1

Rewrite the following in scientific notation.

Example 2

Rewrite the following examples in their expanded form.

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Example 3

Use the rules of exponents to multiply the following results together.

a. $(2 \times 10^6) \times (3 \times 10^8)$

b. $(2 \times 10^{17}) \times (2.1 \times 10^{23})$

c. $(8 \times 10^7) \times (2 \times 10^{-3})$

Example 4

Use the rules of exponents to divide the following results.

a. $\frac{8 \times 10^{12}}{4 \times 10^3}$

b. $\frac{6 \times 10^7}{2 \times 10^{19}}$

c. $\frac{3 \times 10^{-12}}{6 \times 10^{-5}}$

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Example 5

Add the following:

a. $(5 \times 10^{78}) + (2 \times 10^{78})$

b. $(8 \times 10^{6}) + (2 \times 10^{6})$

c. $(3 \times 10^{4}) + (2 \times 10^{3})$

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Lesson: An Intro to Polynomials

We can extend our knowledge of exponents to another algebraic topic – polynomials

Before we define what a polynomial is, it is usually easier to see a few examples of what is or is not a polynomial.

The following ARE polynomials:

- $x^2 - 3x + 8$
- $x^7 - 5x^4 + 2x^3 + x$
- 9
- $7x + \pi$
- $x^3 + \frac{2}{3}$

The following ARE NOT polynomials:

- $\sqrt{x} - 4$
- $4x^{-4} - 7x^{-3}$
- $\frac{4}{x} - \frac{2}{x+3}$

Now we can more formally define a polynomial.

Polynomial

An expression with real numbers, variables, and whole number exponents.

Descending Order

Ordering a polynomial from the largest exponent to the smallest exponent, and any freestanding number without a variable is last

Example 1

Put the polynomials in descending order.

a. $x^3 + 4x^5 - 2x^7 + 3x^2 + 5x^8$

b. $x^2 - x^3 + x^4 - x^5 - x^6 + 3$

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Term
Any part in a polynomial that is separated by a "+" or "-" sign is considered to be its own term

Variables
The letters in a polynomial that represent numbers

Coefficients
The numbers attached to the variables

Constant Term
The freestanding number within a polynomial that is not attached to any variables

Example 1
Examine the polynomials, then determine the variables, coefficients, and constant terms

Polynomial	Terms	Variable(s)	Coefficient(s)	Constant
$2x + 3$				
$3x^2 - x + 5$				
5				
$2x^2y + 3y - x$				

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Degree of a Term
The degree of a term is the sum of the exponents of that term.

Example 2
Identify the degree of each term

Term	Degree	Term	Degree
x^4		x^4y^3	
$3y^6$		x^7y^9	
x		$x^4y^2z^5$	
6		xy^4	

Degree of a Polynomial

The degree of a polynomial is the same as the degree of the term of highest degree within that polynomial

Leading Coefficient
The leading coefficient is the coefficient attached to the term of highest degree in a polynomial

Trinomial
A polynomial with 3 terms

Binomial
A polynomial with 2 terms

Monomial
A polynomial with 1 term

Example 3
Identify the degree of each polynomial and the type of polynomial it is.

Polynomial	Degree of Each Term, Respectively	Deg of Polynom	Leading Coefficient	Type of Polynomial
$x^5 + 4x^3 - 2x$				
$-5x^2 + 18x$				
$6x + 2x^4 - 3x^2$				
$x^5 + 2x^7 - 4$				
8				

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Evaluating Polynomials

Evaluating polynomials is not unlike evaluating other expressions we've seen in this course.

Example 4

Evaluate the polynomials for $x = -2$.

a. $x^2 + 3x + 1$

b. $x^3 - x^2 - 3x + 4$

c. $\frac{1}{4}x^3 - \frac{1}{2}x^2 + 5x$

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Lesson: How to Add and Subtract Polynomials

To add or subtract polynomials, combine like terms.

Examples

Perform the following operations.

a. $(2x + 4) + (5x^2 - 3x + 1)$

b. $(5x^2 - 3x + 1) - (x^3 - 2x^2 - x + 1)$

c. $(3x^3 + 2x^2 - 5x) + (4x^3 + 5x^2 + 2x)$

d. $(2x^2 + 5x - 4) - (x^3 - 2x^2 + 3x - 5)$

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Lesson: How to Multiply Polynomials

To begin, let's first focus on the process of distributing a monomial with a polynomial.

Examples Multiply the following:

a. $3x(4x^2 + 5x - 7)$

b. $x^2y^4(3x^2 + 2xy - 8y^4)$

c. $\frac{1}{2}x^3y^4(4xy^3 - 2x^3y + 8xy)$

Now write down the process for how to multiply $(x + 2)(x^2 + 3x)$

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Examples Multiply the following:

a. $(x+3)(x+2)$

b. $(3x+1)(x-6)$

c. $(x^2+4)(x-2y)$

d. $(x+4)(x-4)$

e. $(x+3)(x^2+4x+5)$

f. $(2x^2+3x+1)(3x^2-4x+2)$

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How do you multiply $(x+5)^2$?

Examples Multiply the following:

a. $(x+2)^2$

b. $(2x^2+1)^2$

c. $(3x^2+x+5)$

How do you multiply 3 polynomials together, such as $(x+2)(x+1)(x-3)$?

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Examples Multiply the following together:

a. $(x-3)(x+4)(x-1)$

b. $(2x+1)(x-2)(x-1)$

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Lesson: The Basics of Dividing Polynomials

In this section, we will discuss how to divide polynomials that are in these forms:

$\frac{x^2+3x}{x}$ or $(4x^7y^3 + 8x^4y^2 - 12x^5y^5) \div 2x^2y$

To divide in these cases, you can just use rules of exponents. Sometimes you will want to separate the problem into separate fractions.

Examples

a. $\frac{x^2+3x}{x}$

b. $\frac{5x^4-10x^3+15x^2}{5x}$

c. $(27x^4y^2 + 18x^3y) \div 9x^2y$

d. $(4x^7y^3 + 8x^4y^2 - 12x^5y^5) \div 2x^2y$

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Lesson: Long Division for Polynomials

In this section, we will discuss how to divide polynomials that are in these forms:

$(x^2 + 5x + 6) \div (x + 3)$ or $(2x^2 + 5x + 3) \div (2x + 3)$

In these types of problems, we need to use long division. In another video, we talked about how to divide polynomial problems like these:

$\frac{4x^3+2x^2}{2x}$

But we did not need long division. What is different about these types of problems that require long division?

Show how to use long division to divide $(x^2 + 5x + 6) \div (x + 3)$

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Examples Divide the following.

a. $(x^2 + 7x + 10) \div (x + 5)$

b. $(2x^2 + 5x + 3) \div (2x + 3)$

c. $\frac{x^2-11x+30}{x-5}$

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Now we will look at what to do when there is a remainder.

Divide the following: $(x^2 + 3x + 1) \div (x + 2)$

Now show how to divide when there are "missing" terms, such as with $(x^4 - 81) \div (x^2 - 9)$

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Examples Divide.

a. $(5x^2 - 10x^3 + 8 - 9x) \div (x + 2)$

b. $(x^3 + 1) \div (x + 1)$

c. $(5x^3 - 4x^2 + 1) \div (x + 2)$

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Lesson: The Greatest Common Factor

The Greatest Common Factor (GCF)

The Greatest Common Factor is the biggest common factor between a set of terms.

Examples

Find the greatest common factor in the following polynomials, then factor the polynomial.

a. Polynomial: $3x + 3y$

GCF:

Factor out the GCF:

b. Polynomial: $3xy + 6xz$

GCF:

Factor out the GCF:

c. Polynomial: $10x^2 - 25xy + 15xz$

GCF:

Factor out the GCF:

d. Polynomial: $3x^7y + 8x^3z^5 - 5y^4z^3$

GCF:

Factor out the GCF:

Examples

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Factor the following polynomials.

a. $12xy^2 + 10x^2y^4 - 14xy^5$

b. $5xy - 10yz + 12xz$

c. $3x + 6x^2 - 12x^3$

d. $5x^4y^2z^5 + 10x^2y^4z^6 - 25x^5yz^3$

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Lesson: Factoring by Grouping

Grouping can best be understood with the following example.

Factor $ax + ay + bx + by$ by grouping:

Examples

Finish the following examples where the grouping had been started.

a. $3(x - y) + 2z(x - y)$

b. $2a(3b + 2c) - 5(2c + 3b)$

c. $4x(y - z) - 3(z + y)$

d. $4z(4x - 7) - (4x - 7)$

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Examples

Factor the following.

a. $5x + 10y + xz + 2yz$

b. $10x^2 + 15x^3y - 2 - 3xy$

c. $3xy + 4y - 27x - 36$

d. $3xy + 6x + 21y + 42$

e. $3x^2 - 48y + 8x - 18xy$

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Lesson: The Basics of Factoring Trinomials of the Form $x^2 + bx + c$

Factoring trinomials will require us to multiply polynomials, so first we will review this skill.

Examples

Multiply.

a. $(x + 3)(x - 5)$

b. $(2x - y)(5x + 2y)$

Definition of a Trinomial

A trinomial is a polynomial with three terms.

How to Factor a Trinomial Using Trial and Error

Show how to factor $x^2 + 3x + 2$

Show how to factor $x^2 + 7x + 12$

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Examples

Factor the following.

a. $x^2 + 6x + 5$

b. $x^2 + 12x + 11$

c. $x^2 - 8x + 15$

d. $x^2 - 13x + 30$

e. $x^2 + 10x - 24$

f. $x^2 - 5x - 14$

g. $2x^2 + 16x + 30$

h. $5x^4 + 55x^3 - 45x^2$

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Lesson: The Basics of Factoring Trinomials of the Form $-x^2 + bx + c$

How can you factor $-x^2 - 5x - 6$?

What about $-x^2 - 15x - 36$?

Examples

Factor the following.

a. $-x^2 + 13x - 30$

b. $-x^2 + 16x - 55$

c. $-x^2 - 2x + 3$

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Lesson: The Basics of Factoring Trinomials of the Form $x^2 + bxy + cy^2$

How do you factor $x^2 + 5x + 6$?

How do you factor $x^2 + 5xy + 6y^2$?

Examples Factor the following.

a. $x^2 + xy - 12y^2$

b. $x^2 - 2xy + y^2$

c. $3x^2 + 21xy + 18y^2$

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Lesson: The Basics of Factoring Trinomials of the Form $ax^2 + bx + c$

In this lesson we will focus on trinomials that do not have a leading coefficient.

To begin, first let's get warmed up by factoring the following using trial and error:

a. $x^2 + 7x + 12$

b. $x^2 - 3xy + 10y^2$

How do you factor $2x^2 + 5x + 3$ using trial and error?

How do you factor $3x^2 + 16x + 5$ using trial and error?

Another way to factor $3x^2 + 16x + 5$ is by grouping. Show how to use this technique below:

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Examples Factor the following:

a. $7x^2 - 11x + 4$

b. $3x^2 + 10x + 8$

c. $5x^2 - 11x - 36$

d. $3x^2 - 17xy - 6y^2$

e. $-10x^2 + 19x - 6$

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Lesson: Factoring Special Trinomials

How do you factor $x^2 + 6x + 9$?

What's another way to write this factorization?

What do you call this type of trinomials?

Examples Factor the following:

a. $x^2 + 12x + 36$

b. $9x^2 - 12x + 16$

c. $x^2 - x + \frac{1}{4}$

d. $36x^2 - 60xy + 25y^2$

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How do you factor $x^2 - 4$?

What is this type of factoring called?

Examples Factor the following:

a. $x^2 - 25$

b. $x^2 - 16$

c. $x^2 - 1$

d. $25x^2 - 36$

e. $16x^2 - 81y^2$

f. $16 - x^2$

g. $25y^2 - 64x^2$

h. $\frac{49}{64} - x^2$

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Lesson: How to Factor The Sum and Difference of Cubes

Write down the formula for the sum of cubes

$a^3 + b^3 =$

Write down the formula for the difference of cubes

$a^3 - b^3 =$

Show how to factor $x^3 + 8$

Show how to factor $8x^3 - 27$

Examples Factor:

a. $x^3 - 1$

b. $x^3 - \frac{1}{8}$

c. $x^3 + 64$

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Review: An Overview of All the Types of Factoring

We will review all of the different types of factoring by presenting an example of each type.

Greatest Common Factor

1. $4x^2 + 8xy$

2. $6x^3y^2 - 8x^2y^5 + 10x^5y$

Grouping

3. $3x + 6y - xz^2 - 2yz^2$

4. $12x^2 - 2y + 3x - 8xy$

Basic Trinomials of the Form $x^2 + bx + c$

5. $x^2 + 7x + 10$

6. $x^2 - 2x - 15$

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Trickier or Special Forms of Trinomials of the Form $x^2 + bx + c$

7. $-x^2 - 3x + 28$

8. $6x^2 + 54x + 120$

9. $x^2 - 6xy + 8y^2$

10. $x^2 - 8x + 16$

11. $x^2 - 25$

12. $x^2 - \frac{9}{4}$

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Trinomials of the Form $ax^2 + bx + c$

13. $2x^2 + 11x + 5$

14. $7x^2 - 15x + 8$

15. $-3x^2 - 15x^2 + 198x$

16. $16x^2 - 24xy + 9y^2$

More Practice with the Difference of Squares

17. $x^2 - 81$

18. $25x^2 - 4$

19. $\frac{1}{4}x^2 - 9y^2$

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Sum and Difference of Cubes

20. $x^3 + 8$

21. $x^3 - 27$

22. $27x^3 - 1$

23. $8x^3 + 64y^3$

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Lesson: How to Solve Quadratic Equations with Factoring

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, and a, b, and c are real numbers.

There are several ways that you can solve quadratic equations, but in this lesson we will focus on how to use factoring as a way to solve them.

Solving a quadratic equation by factoring requires a rule called the **Zero Product Rule.**

Write down the Zero Product Rule:

Examples Solve the following equations using the zero product rule:

a. $3x = 0$

b. $3(x + 1) = 0$

c. $(x + 3)(x + 1) = 0$

d. $x(2x + 1)(3x + 5) = 0$

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Show how to use factoring to solve $x^2 + 5x + 6 = 0$.

Examples Solve the following by using factoring:

a. $x^2 + 12x - 45 = 0$

b. $x^2 - 12x + 11 = 0$

c. $2x^2 + 5x + 3 = 0$

d. $55x = -20x^2 - 30$

e. $x^2 = 25$

f. $x^2 = 9x$

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Lesson: Tips and Tricks to Help with Harder Quadratics Solved by Factoring

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, and a, b, and c are real numbers.

In the previous lesson, we talked about the basics of using factoring to solve quadratics.

In this lesson, we will look at some trickier examples. First, let's warm up by solving the following:

a. $4x = 14x^2 - 48$

b. $3x^3 = 12x^2$

Sometimes we will have to do further work to simplify the problem before we factor.

Examples Use factoring to solve.

a. $(x + 3)(x + 4) = 6$

b. $5x(x + 2) - 4x = 4(x^2 - 2)$

c. $(x + 4)^2 - 2x = 11$

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What can be done to solve $\frac{1}{3}(x+1)^2 - \frac{1}{4}x = \frac{1}{6}(x-2)^2 - \frac{1}{3}$?

Examples Use factoring to solve.

a. $\frac{1}{4}(2x-3)^2 + \frac{1}{4}x = \frac{1}{4}(x-5)^2 - \frac{3}{2}$

b. $\frac{1}{4}x(2-x) - \frac{3}{4} = \frac{1}{5}x(x+1) - \frac{7}{10}$

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Lesson: Applications of Quadratic Equations

The area of a rectangle is 15ft squared, and the length of the rectangle is 7ft less than twice the width. Find the dimensions of the rectangle.

An 12ft ladder leans against the side of a house. The bottom of the ladder is 6ft from the side of the house. How high is the top of the ladder from the ground? If necessary, round your answer to the nearest tenth.

A kite flying in the air has a 14ft line attached to it. Its line is pulled taut and casts a 9ft shadow. Find the height of the kite. If necessary, round your answer to the nearest tenth.

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The longer leg of a right triangle is 2in longer than the shorter leg. The hypotenuse is 4in longer than the shorter leg. Find the side lengths of the triangle.

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Rational Expressions
Video 1: The Basics

What are rational expressions?

What do rational expressions look like? Write down some examples:

<u>Simplifying Rational Expressions</u>
Take down some notes on this:

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Examples together:

a. $\frac{25x^7y^9}{15x^3y^2z^7}$

Examples on your own:

b. $\frac{21x^7y^9}{14y^8z^3}$

c. $\frac{16x^4y^7z^3}{15x^3y^4z^8}$

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Examples together:

a. $\frac{x^2+5x+6}{x^2-9}$

Examples on your own:

b. $\frac{x^2+7x+10}{x^2+10x+25}$

c. $\frac{x^2-x-6}{x^2-9x+14}$

d. $\frac{x^2+10x+16}{x^3+2x^2+x+2}$

Play the video and check your work.

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Examples on your own:

e. $\frac{x^3-8}{x^2-9x+14}$

f. $\frac{10x^3+15x^7}{10x^3+40x^2+40x}$

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Rational Expressions
Video 2: Other Things to Know

When it comes to fractions, where are you allowed to have a zero? In the numerator (top) or denominator (bottom)?

Therefore, what do we have to worry about with rational expressions?

Examples on your own:

Evaluate the following rational expressions for $x = 2$.

a. $\frac{x^2+6x+8}{x^2+8x+15}$

b. $\frac{x^2+9x+20}{x^2+3x-10}$

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Example on your own: (evaluate for x=2)

c. $\frac{x^2-4}{x^2+9x+14}$

To summarize…

When a rational expression has a zero in the numerator, what does this mean?

When a rational expression has a zero in the denominator, what does this mean?

Define the term **domain:**

What are we concerned with for the domain of rational expressions?

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A review of interval notation:

Examples:

Write the following in interval notation:

a.

b.

c.

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Example together:

Determine i) where the expression equals zero ii) where the expression is undefined iii) the domain of the expression

a. $\frac{x+3}{x-5}$

b. $\frac{2x+1}{x^2+7x+10}$

Examples on your own:

Determine i) where the expression equals zero ii) where the expression is undefined iii) the domain of the expression

c. $\frac{x-3}{x-1}$

d. $\frac{x^2-81}{x^2+5x+6}$

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Rational Expressions
Video 3: The Negative One Trick with Simplifying

In its current form, can you simplify: $\frac{x-3}{3-x}$? Why or why not?

Show how you can manipulate this expression to simplify it.

There is no formal name for this trick. Some might refer to it as a *multiplicative property of -1,* but I refer to as "that -1 trick." I know, it's not exactly formal sounding!

Let's do one more example of this. Simplify $\frac{5-x}{3x-15}$

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The first thing to get used to it how to rewrite expressions using this trick.

Examples

Rewrite the order of each expression by factoring out -1 first.

a. $4 - x$

b. $3x - 1$

c. $x - 6$

Now let's use this to simplify rational expressions.

Examples Simplify the following.

a. $\frac{6x-12}{2-x}$

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Examples Simplify the following.

b. $\frac{4x-20}{30x-6x^2}$

c. $\frac{x^2-4}{2-x}$

d. $\frac{x-6}{36-x^2}$

e. $\frac{5-x}{x^2-7x+10}$

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Examples Simplify the following.

f. $\frac{x^2-10x+25}{25-x^2}$

g. $\frac{27-x^3}{x^2-5x+6}$

h. $\frac{x^3-3x^2+5x-15}{6-3x}$

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Examples Simplify the following.

i. $\frac{54-9x}{x^2-36}$

j. $\frac{8-x^3}{x^2-9x+14}$

k. $\frac{y^3-x^3}{x^2-y^2}$

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Example/Question: Troubleshooting the -1 Trick

Explain the actual DETAIL behind WHY $\frac{4}{5-x} = -\frac{4}{x-5}$

Examples Write an equivalent fraction for the following:

a. $\frac{5}{x-8}$

b. $\frac{x-3}{x+2}$

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Lesson: Multiplying Rational Expressions

How do you multiply the fractions $\frac{a}{b} \times \frac{c}{d}$? Include a one-sentence explanation.

Examples Multiply the following.

a. $\frac{4}{5} \times \frac{7}{9}$

b. $\frac{2x^3}{7y^2} \times \frac{5x^5}{9y^7}$

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Show how to make cancellations before multiplying $\frac{12}{25} \times \frac{15}{16}$

We can apply this same logic to rational expressions.

Examples Multiply the following and simplify when possible.

a. $\frac{21x^7}{15y^3} \times \frac{5y^8}{42x^4}$

b. $\frac{2x^2+5x+3}{x^2-25} \times \frac{x^3+125}{4x^2+6x}$

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Lesson: Dividing Rational Expressions

How do you divide $\frac{a}{b} \div \frac{c}{d}$? Write a one sentence explanation of how to do this too.

Example Divide the following: $\frac{5x}{9} \div \frac{2}{7x^4}$

When can you make cancellations in the process?

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Examples Divide.

a. $\frac{12x^7}{45} \div \frac{42x^3}{15}$

b. $\frac{x^2+4x+4}{x+5} \div (x+2)^5$

c. $\frac{x^3+1}{6x^3-6x^2+6x} \div \frac{x^2-1}{8}$

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Explain why $\frac{\frac{5x+5y}{24}}{\frac{15x^2+15xy}{12}}$ is both a fraction and a division problem.

Example Divide: $\frac{\frac{16x^2-25}{x^5}}{\frac{36x-45}{9x^7}}$

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Lesson: Finding the Least Common Denominator for Addition and Subtraction – A Quick Review for Fractions (with only numbers)

We are going to work on adding and subtracting rational expressions with different denominators. Before we do that, we are going to remind ourselves how to find common denominators for fractions that only have numbers.

The simplest case: what is the least common denominator between the fractions $\frac{1}{2}$ and $\frac{1}{3}$?

Why is that the common denominator?

Now let's look at something slightly more complicated. What is the least common denominator between $\frac{1}{6}$ and $\frac{1}{10}$?

First, take a guess. It's totally fine if this guess is wrong, but just make a guess.

Why is this your guess? What did you do?

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There is a specific procedure that you want to use when finding least common denominators in this class. Write down instructions that were given in the video, but write them our in a way that makes sense to you.

Repeat this procedure to find the least common denominator $\frac{1}{15}$ and $\frac{1}{10}$.

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Repeat this procedure to find the least common denominator $\frac{1}{12}$ and $\frac{1}{8}$.

Now we will focus on how to write fractions with this desired denominator.

The simplest case: using the least common denominator you found for $\frac{1}{2}$ and $\frac{1}{3}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.

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Using the least common denominator you found for $\frac{1}{6}$ and $\frac{1}{10}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.

Using the least common denominator you found for $\frac{1}{12}$ and $\frac{1}{8}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.

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Lesson: Finding the Least Common Denominator for Rational Expressions

The simplest case: what is the least common denominator between the fractions $\frac{1}{2x}$ and $\frac{1}{3y^2}$?

Examples Find the least common denominator between the following fractions.

a. $\frac{1}{2x^2}, \frac{1}{3xy^2}$

b. $\frac{1}{2x^2y^5}, \frac{1}{3x^7y^2}$

c. $\frac{1}{2x^3y^5z^2}, \frac{1}{3x^5y^7z^8}$

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Examples Find the least common denominator for the following.

a. $\frac{1}{14x^3y^2}, \frac{1}{21x^2y^7}$

b. $\frac{1}{24x^3y}, \frac{1}{12x^2y^5}$

What is the least common denominator between $\frac{1}{x+2}$ and $\frac{1}{x}$?

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What is the least common denominator between $\frac{1}{x^2-4}$ and $\frac{1}{x^2+5x+6}$?

Examples Find the least common denominator between the following fractions.

a. $\frac{3}{5x-10}, \frac{1}{7x-14}$

b. $\frac{1}{x^2-12x+35}, \frac{1}{x^2+x-20}$

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Examples Find the least common denominator between the following fractions.

a. $\frac{1}{x-3}, \frac{1}{3-x}$

b. $\frac{1}{x^2-4}, \frac{1}{2-x}$

c. $\frac{1}{x^2+9x+20}, \frac{1}{x^2-25}, \frac{1}{x^2-x-20}$

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Lesson: Writing Equivalent Rational Expressions with the Same Denominator

Examples Find the least common denominator between the following rational expressions, and then write each rational expression with it's equivalent and the LCD.

a. $\frac{2}{15x^2y^5}, \frac{4}{9x^3y^2}$

b. $\frac{5}{x+3}, \frac{4}{x}$

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Examples Find the least common denominator between the following rational expressions, and then write each rational expression with it's equivalent and the LCD.

a. $\frac{x}{x^2+9x+18}, \frac{2}{x^2+5x+6}$

b. $\frac{x}{5x^2-30x}, \frac{x-2}{2x^2-17x+30}, \frac{2}{2x^2-5x}$

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Lesson: Adding and Subtracting Rational Expressions

Show how to add or subtract the following:

a. $\frac{5}{x+2} - \frac{3}{x+2}$

b. $\frac{5}{x-4} + \frac{7}{x}$

c. $\frac{5}{12xy^2} - \frac{7}{8x^3y}$

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d. $\frac{x}{x+3} - \frac{2}{x^2-9}$

e. $\frac{x}{x^2-25} - \frac{5}{x^2-3x-40}$

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Example/Question: When Are You Allowed to Cancel with Rational Expressions?

What is the wrong way to cancel $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$ and explain why you can't cancel like that? How can you tell it's wrong?

What is the difference between $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$ and $\frac{3(x+1)(x-3)}{2(x-3)(x+1)}$?

How do you simplify $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$?

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Lesson: Complex Fractions

First let's review how to simplify a complex fraction we've already discussed. Show how to simplify:

$$\frac{\frac{2x}{x+5}}{\frac{2}{x^2-25}}$$

Explain what makes the problem below different from the one we just reviewed above.

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{5}{6}+\frac{1}{3}}$$

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Outline the steps required to simplify. Write out the instructions as they were explained in the video.

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{5}{6}+\frac{1}{3}}$$

Example Simplify the following

$$\frac{\frac{7}{9}-\frac{1}{3}}{1+\frac{5}{9}}$$

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Examples Simplify the following.

a. $\dfrac{\frac{5}{x}+\frac{2}{y}}{\frac{1}{y}-\frac{3}{x}}$

b. $\dfrac{\frac{6}{x+3}-\frac{4}{x-1}}{\frac{2}{x-1}+\frac{5}{x+3}}$

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Example Simplify the following.

$$\frac{1-\frac{4}{t+5}}{\frac{4}{t^2-25}+\frac{t}{t-5}}$$

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Complex Fractions
Video 2: More Complicated Examples

Complex fractions look like this:

$$\frac{\frac{4x}{15y^2}}{\frac{6x^4}{25y^8}} \quad \text{or} \quad \frac{\frac{1}{3}-\frac{1}{6}}{\frac{3}{2}+\frac{5}{6}} \quad \text{or} \quad \frac{\frac{4}{x^2}-\frac{4}{y^2}}{\frac{1}{xy}+\frac{5}{y}}$$

First we will discuss the basics.

Examples

a. $\frac{\frac{1}{x^2}-\frac{1}{y}}{\frac{3}{x}-\frac{4}{y^2}}$

b. $\frac{\frac{3}{x}-x}{\frac{2}{x}+x}$

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Examples

a. $\dfrac{\frac{3}{x-1}-\frac{2}{x+3}}{\frac{5}{x+3}-\frac{2}{x-1}}$

b. $\dfrac{\frac{5}{x-2}-\frac{3}{x+5}}{\frac{1}{x+5}+\frac{2}{x-2}}$

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Examples

a. $\dfrac{1-\frac{1}{x+5}}{\frac{x}{x-5}+\frac{4}{x^2-25}}$

b. $\dfrac{1+\frac{12}{x^2-16}}{\frac{x}{x+4}-\frac{6}{x-4}}$

c. $\dfrac{\frac{1}{x+2}+\frac{4}{x^2-4}}{\frac{3}{x-2}+1}$

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Lesson: Solving Rational Equations

How do you know that you need to solve a problem versus simplify it?

Explain how to solve the rational equation. Include some instructions in your notes.

$$\frac{2}{x-1}+\frac{1}{x+4}=\frac{4}{x+4}$$

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What do we have to watch out for when solving rational equations?

Examples For the following, which solutions would NOT work for these rational equations?

a. $\frac{4}{x+1} - \frac{2}{x+3} = \frac{5}{x+7}$

b. $\frac{4}{x^2-9} + \frac{3}{x-3} = \frac{2}{x+3}$

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Examples Solve.

a. $1 - \frac{8}{x} = \frac{10}{x}$

b. $\frac{5}{x-3} - \frac{2}{x^2-9} = \frac{5x+13}{x^2-9}$

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c. $\frac{x^2}{2} = \frac{x^2-6x}{3}$

d. $\frac{4}{x+1} = \frac{10}{x^2-1} - \frac{5}{x-1}$

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Solving Equations with Rational Expression

We will learn how to solve equations that look like this:

$$z = \frac{x+3y}{4w} \quad \text{or} \quad \frac{1}{a} - \frac{2}{b} = \frac{3}{c}$$

Examples

a. Solve for y: $z = \frac{3xy}{5w}$

b. Solve for x: $w = \frac{7+2y}{xz}$

c. Solve for z: $y = \frac{4xw}{z-w}$

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Examples

a. Solve for x: $\frac{3}{x}+\frac{1}{y}=\frac{2}{z}$

b. Solve for b: $\frac{1}{a}-\frac{1}{b}=\frac{3}{c}$

c. Solve for z: $\frac{5}{x}-\frac{3}{y}=\frac{7}{z}$

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Lesson: Three Ways to Use the LCD with Rational Expressions

We have spent quite a bit of learning how to find the LCD between rational expressions and then using it in very different ways. We are going to compare the 3 techniques for how we used the LCD in this video.

First, simplify completely:

$$\frac{9}{x+5} - \frac{4}{x+1}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?

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Simplify completely:

$$\frac{\frac{9}{x+5}-\frac{4}{x+1}}{\frac{10}{x^2+6x+5}-\frac{3}{x+1}}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?

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Solve:

$$\frac{9}{x+5} - \frac{4}{x+1} = \frac{10}{x^2+6x+5}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?

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Example/Question: When Can You Cross Multiply

Explain what a ratio is:

When can you cross-multiply?

Solve: $\frac{8}{x+2} - \frac{12}{x-4} = \frac{2}{x+2}$. Show that you CANNOT cross-multiply in this situation.

Now show why cross-multiplying works with $-\frac{x}{5} = \frac{3}{x+8}$

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Rational Expression Applications
Video 1: Proportion Type Problems

What is a proportion?

A bakery can make 300 cookies in 1.5 hours. How long would it take to make 525 cookies?

The ratio of patrons at a café who purchase coffee versus those who purchase tea is 5 to 3. On Monday, the café sold 45 teas. How many coffees were sold?

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A painter makes a particular shade of orange by using 3 parts red and 4 parts yellow. The painter has four more gallons of yellow paint than he has of red paint. How many gallons of red paint should he use?

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Rational Expression Applications
Video 2: Distance, Rate, and Time Problems

In these types of problems, we need to use the equation:

$$d = rt$$

Where d=distance, r=rate/speed of a vehicle, and t=time

We can also see that we can rearrange this formula in a few ways:

A boat travels 30 miles downstream in the same time that it takes to travel 20 miles upstream. The current of the river is 5mph. How fast would the boat travel in still water?

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A boat travels 16 miles downstream in the same time that it travels 12 miles upstream. In still water, the boat can travel 6mph. What is the speed of the current?

A plane travels 550 miles with the wind. In the same amount of time, it travels 375 miles against the wind. If the wind is blowing at a speed of 25 mph, what is the speed of the plane when there is no wind?

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Rational Expression Applications
Video 3: Work Problems

What is the setup for work problems?

It takes Maria 3 hours to paint a room, while it takes her younger sister Lupe 5 hours to paint the same room. How long would it take if the girls painted the room together?

It takes Max 30 minutes to shovel the driveway, while it takes his brother Darrell 45 minutes to do the same job. How long would it take for the guys to do it together?

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Variation
Video Notes

The term "variation" refers to a particular way of describing how certain things relate to one another.

There are three types of variation we will look at. Write down the definitions for each, you will remember it better!

1. Direct Variation:

2. Inverse Variation

3. Joint Variation

It's also possible to combine different types of variation.

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The first thing we have to do is learn how to interpret and write variation equations.

Examples Write the variation equation:

a. P varies directly with Q

b. J varies inversely with L

c. H varies jointly with A and B

d. Z varies inversely with the square of R

e. M varies directly as the square root of N

f. Y varies jointly with the cube of M and the square root of L

g. M varies directly as L and inversely as N

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Now we can take a look at how to solve general variation equations.

Example Write down the detailed steps / directions for this example.

J varies directly with L. If J = 12 when L = 4, find J when L = 7.

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

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Examples Using the steps outlined from the previous example, try the following:

a. M varies inversely as the square of L. If M = 12 when L = 2, find M when L = 4.

b. A varies jointly with B and C. If A = 30 when B = 2 and C = 5, find A when B = 3 and C = 4.

c. L varies directly as M and inversely as N. If L = 15 when M = 5 and N = 2, find L when M = 7 and N = 3.

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Example Jada's weekly income varies directly with the amount of hours that she works. Last week, she earned $570 for working 38 hours. This week she worked 36 hours, how much will she earn?

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The Basics of Roots
Video 1: Square Roots

What are the square roots of 9? Write down what it means to be a square root.

Examples Find the square roots of the following numbers.

a.

b.

What does the symbol $\sqrt{}$ refer to?

How can we refer to the negative square root?

Draw a square root and label its parts:

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Examples Evaluate the following:

a. $\sqrt{25}$

b. $\sqrt{81}$

c. $\sqrt{36}$

d. $\sqrt{\frac{1}{36}}$

e. $\sqrt{\frac{16}{49}}$

f. $\sqrt{\frac{9}{25}}$

g. $\sqrt{-36}$

h. $-\sqrt{64}$

Finally, how would you estimate a root like $\sqrt{5}$?

Example Estimate $\sqrt{14}$

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The Basics of Roots
Video 2: Higher Roots

How would you evaluate the following? Explain the rationale for how to evaluate these:

$\sqrt[3]{8}$

$\sqrt[4]{81}$

$\sqrt[5]{32}$

The most general symbol for a root is $\sqrt[n]{a}$. Explain what this symbol means and once again, label it.

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Now we can create a list of the most likely roots that we will encounter. You will very likely want to create another list of this on a page you can easily access.

Squares	**Cubes**	**Other**

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Examples Evaluate the following. Be careful when you evaluate.

a. $\sqrt[3]{125}$

b. $\sqrt[3]{-125}$

c. $\sqrt[4]{16}$

d. $\sqrt[4]{-16}$

d. $-\sqrt[3]{8}$

e. $-\sqrt[3]{-8}$

f. $\sqrt[5]{-32}$

g. $\sqrt[6]{-64}$

Now let's generalize. For all roots of the form $\sqrt[n]{a}$, what can we say if….

1. The index (n) is an **odd** number?

2. The index (n) is an **even** number and a is a **negative** number?

3. The index (n) is an **even** number and a is a **positive** number?

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The Basics of Roots
Video 3: The "Ultimate" Test and A Few Other Notes

To begin, we will first see how well we can put everything we've learned so far about roots together.

Examples Evaluate the following, if possible.

a. $\sqrt[3]{-8}$

b. $\sqrt{-16}$

c. $-\sqrt[4]{16}$

d. $\sqrt[3]{-125}$

e. $-\sqrt{100}$

f. $\sqrt[4]{-81}$

g. $-\sqrt{25}$

h. $-\sqrt[3]{-8}$

i. $-\sqrt[4]{-16}$

j. $-\sqrt[5]{-32}$

And now that you have completed this … it's time for my "ultimate" test … drum roll please:

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The "Ultimate" Test for Roots

Evaluate. Note: at this moment in time, we do not know anything about imaginary numbers.

a. $\sqrt{1}$

b. $\sqrt[3]{1}$

c. $\sqrt[5]{1}$

d. $\sqrt[4]{-1}$

e. $\sqrt[3]{-1}$

f. $-\sqrt{1}$

g. $-\sqrt[4]{-1}$

h. $-\sqrt[3]{1}$

i. $-\sqrt{-1}$

j. $-\sqrt[7]{-1}$

k. $-\sqrt[3]{-1}$

l. $\sqrt[8]{-1}$

m. $-\sqrt[10]{-1}$

n. $\sqrt{-1}$

Are you having fun yet???

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We have one last thing to discuss: the general rule for evaluating $\sqrt[n]{a^n}$

If n is even ….

If n is odd ….

Examples Evaluate the following:

a. $\sqrt[3]{y^3}$

b. $\sqrt[4]{(x-4)^4}$

c. $\sqrt[7]{(2y-4)^7}$

d. $\sqrt[6]{(3x+2)^6}$

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Rational Exponents

Video 1: How to work with rules like $x^{\frac{1}{n}}$ or $x^{\frac{m}{n}}$ or $x^{-\frac{m}{n}}$

In this video, we will learn how to evaluate expressions that look like this:

$8^{\frac{1}{3}}$ or $25^{\frac{3}{2}}$

Define $x^{\frac{1}{n}}$ and show how to use this rule to evaluate $25^{\frac{1}{2}}$

Examples Evaluate the following:

a. $125^{\frac{1}{3}}$

b. $32^{\frac{1}{5}}$

c. $9^{\frac{1}{2}}$

d. $(-9)^{\frac{1}{2}}$

e. $-9^{\frac{1}{2}}$

f. $\left(\frac{27}{125}\right)^{\frac{1}{3}}$

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Define $x^{\frac{m}{n}}$ and show how to use this rule to evaluate $25^{\frac{3}{2}}$

Examples Evaluate the following:

a. $27^{\frac{2}{3}}$

b. $4^{\frac{3}{2}}$

c. $16^{\frac{5}{4}}$

d. $-8^{\frac{2}{3}}$

e. $(-8)^{\frac{2}{3}}$

f. $(-16)^{\frac{3}{4}}$

g. $-81^{\frac{3}{4}}$

h. $\left(\frac{125}{27}\right)^{\frac{2}{3}}$

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Define $x^{-\frac{m}{n}}$ and show how to use this rule to evaluate $25^{-\frac{3}{2}}$

Examples Evaluate the following:

a. $25^{-\frac{1}{2}}$

b. $27^{-\frac{1}{3}}$

c. $16^{-\frac{3}{4}}$

d. $4^{-\frac{5}{2}}$

e. $\left(\frac{25}{4}\right)^{-\frac{3}{2}}$

f. $(-25)^{-\frac{3}{2}}$

g. $(-27)^{-\frac{2}{3}}$

h. $-16^{-\frac{5}{4}}$

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Lesson: Using Exponent Rules with Rational Exponents

First, let's review the main exponent rules:

Next, let's do a quick review on operations with fractions. Complete the following operations:

a. $\frac{2}{3} \times \frac{5}{6}$

b. $\frac{2}{5} \times 10$

c. $\frac{2}{3} + \frac{2}{5}$

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Now we will combine all of these ideas into the following examples!

Examples Simplify the following. Your final answer should not include any negative exponents.

a. $x^{\frac{2}{3}} * x^{\frac{1}{5}}$

b. $\left(4x^{\frac{2}{3}}y^{6}\right)^{\frac{3}{2}}$

c. $\frac{x^{\frac{2}{3}}y^{-\frac{1}{5}}}{x^{\frac{1}{6}}y^{\frac{2}{5}}}$

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d. $\left(\frac{125x^4y^{-\frac{1}{3}}}{x^{-\frac{1}{2}}y^{\frac{2}{5}}}\right)^{-\frac{2}{3}}$

e. $\left(\frac{4x^6y^{-\frac{1}{3}}}{x^{-3}y^{\frac{1}{2}}}\right)^{-\frac{3}{2}}$

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Lesson: Simplifying Radicals with Rational Exponents

Convert the following radicals to rational exponents, simplify, and then rewrite in radical form.

a. $\sqrt{7^2}$

b. $\sqrt{9^2}$

c. $\sqrt[3]{x^{15}}$

d. $\sqrt[4]{x^2}$

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Rational Exponents
Video 3: Convert a radical expression to exponential form

Examples Convert the following expressions to exponential form, and then simplify if needed.

a. $\sqrt{7}$

b. $\sqrt[3]{8}$

c. $\sqrt{5^2}$

d. $\sqrt[3]{7^6}$

e. $\sqrt[4]{x^{16}}$

f. $\sqrt[6]{x^2}$

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Simplifying Radicals
The Basics

We will look at how to simplify radicals like this:

$$\sqrt{50} = 5\sqrt{2} \quad \text{or} \quad \sqrt[3]{250} = 5\sqrt[3]{3}$$

You might want to have the list of squares / cubes / quartics ready for this exercise. If you don't, pause the video and write them all down on a separate sheet of paper so that you can have it handy for this entire lesson.

First we look at **the multiplication property of radicals:**

Examples Multiply the following:

a. $\sqrt[3]{2} * \sqrt[3]{3}$

b. $\sqrt{7} * \sqrt{5}$

c. $\sqrt[3]{3} * \sqrt[3]{9}$

d. $\sqrt[3]{6} * \sqrt[3]{9}$

How are examples c and d related?

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Being able to multiply radicals together is powerful, because it also allows us to break apart radicals and simplify them. Using the example $\sqrt{12}$, write down how to simplify:

Examples Simplify the following:

a. $\sqrt{50}$

b. $\sqrt{20}$

c. $\sqrt[3]{54}$

d. $\sqrt[4]{32}$

e. $\sqrt[3]{24}$

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At this point, we should probably define when a radical is actually simplified.

A simplified radical will ……

Examples Explain why the following radicals are not simplified:

a. $\sqrt{24}$

b. $\frac{5}{\sqrt{7}}$

c. $\sqrt[4]{\frac{3}{5}}$

d. $\sqrt[3]{8x^8y^2}$

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Examples Simplify the following:

a. $\sqrt{24}$

b. $\sqrt[3]{32}$

c. $\sqrt[4]{64}$

d. $\sqrt[5]{64}$

e. $\sqrt{250}$

f. $\sqrt[3]{250}$

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Lesson: Simplifying Radicals with Variables

We are going to rely on a skill we've already learned to help with simplifying radicals.

Examples Show you can simplify the following utilizing rational exponents.

a. $\sqrt{x^6}$

b. $\sqrt[3]{x^{15}}$

c. $\sqrt{x^6y^8}$

d. $\sqrt[4]{x^{12}y^{20}}$

How can we simplify $\sqrt{x^5}$? Watch the 2 explanations in the video and summarize the explanation that makes more sense to you.

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Example Simplify the following.

a. $\sqrt[3]{x^{13}}$

b. $\sqrt[4]{x^{22}}$

c. $\sqrt{12x^6y^7}$

d. $\sqrt[3]{250x^8y^{16}}$

e. $\sqrt[4]{\frac{32x^7}{y^8}}$

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Lesson: Multiplying Radicals

Write a one-sentence explanation of how to multiply $\sqrt[3]{3} * \sqrt[3]{4}$

Examples Multiply the following. Simplify if possible.

a. $\sqrt[3]{6} * \sqrt[3]{4}$

b. $\sqrt[4]{4x^3} * \sqrt[4]{12x^2}$

c. $\sqrt{5x} * \sqrt{10xy}$

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Simplifying Radicals
Multiplying and Dividing Radicals with Different Indices

We will look at how to perform operations like:

$$\sqrt{x} * \sqrt[3]{x} \qquad \text{or} \qquad \frac{\sqrt[4]{x}}{\sqrt[3]{x}}$$

Before we begin, you should be comfortable with doing problems like this:

$8^{\frac{1}{3}}$

$25^{\frac{3}{2}}$

Write down the approach for performing this operation: $\sqrt{x} * \sqrt[3]{x}$

Examples Perform the following operations:

a. $\sqrt[3]{x} * \sqrt[4]{x}$

b. $\frac{\sqrt[5]{x^2}}{\sqrt[3]{x}}$

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Examples Perform the following operations:

a. $\sqrt[4]{x^3} * \sqrt[5]{x^3}$

b. $\frac{\sqrt{x}}{\sqrt[3]{x^2}}$

c. $\sqrt[3]{x^2} * \sqrt{x}$

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Adding and Subtracting Radicals

In this video we will look at how to perform operations like:

$\sqrt{3} + \sqrt[3]{5} - 5\sqrt{3} + 7\sqrt[3]{3}$ or $\sqrt{24x^4y^3} - 3x^2y\sqrt{6y}$

Before you begin, you should already be able to simplify radicals like this:

a. $\sqrt{24x^6y^9}$

b. $\sqrt[4]{16x^7y^{11}}$

What are like terms?

Add the like terms in this problem: $3x + 7y - 9x + 7y^2 - 2x - 2y$

What are like radicals?

Add the like radicals in this problem: $5\sqrt{3} - 2\sqrt[3]{3} + 8\sqrt[3]{3} - 2\sqrt{5} + 8\sqrt{3}$

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Examples Perform the following operations:

a. $4\sqrt{7} - 5\sqrt[3]{3} + 2\sqrt{3} - 3\sqrt[3]{7} - 9\sqrt{3} + \sqrt[3]{3}$

b. $2x\sqrt{7} - 3\sqrt{7} + 8x\sqrt{7} - 2\sqrt{7}$

c. $3x\sqrt[3]{5} + 2x\sqrt[3]{4} - 3\sqrt[3]{4} + 5x\sqrt[3]{4} - 7x\sqrt[3]{5} + 5\sqrt[3]{4}$

It is possible that you might have to simplify a radical before you can add it. Note how this works with the example:

$$\sqrt{24} + 2\sqrt{54}$$

Examples Perform the following operations:

a. $5\sqrt{8} - \sqrt{18}$

b. $4\sqrt[3]{54} - 7\sqrt[3]{16}$

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Examples Perform the following operations:

a. $\sqrt{8x^2y^4} - 9xy^2\sqrt{2}$

b. $4x\sqrt[3]{16x^6y^7} + 9x^3y\sqrt[3]{250y^4}$

c. $5x^2y^3\sqrt[4]{32x^7z^{10}} - 2x^3z^2\sqrt[4]{2x^3y^{12}z^2}$

d. $3x\sqrt[3]{24x^5y^7} + 3xy\sqrt[3]{3x^2y} + y^2\sqrt[3]{375x^8y}$

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Lesson: Multiplying Radicals Expressions

First, let's review how to multiply radicals in general. Show how to multiply:

a. $\sqrt[3]{4x^2y} * \sqrt[3]{6x^2y}$

b. $\sqrt{6xy} * \sqrt{10y}$

Now we will apply that logic to larger rational expressions.

Write a one-sentence explanation of how to multiply: $\sqrt{2}\left(\sqrt{3} + \sqrt{10}\right)$, then multiply.

Example Multiply $2\sqrt[3]{3}\left(\sqrt[3]{9} + 5\sqrt[3]{6}\right)$

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Example Multiply $3x\sqrt{5}\left(\sqrt{10x}-5x\sqrt{2}\right)$

Write out an explanation of how to multiply: $\left(\sqrt{3}-\sqrt{2}\right)\left(\sqrt{6}+\sqrt{3}\right)$

Example Multiply: $\left(3\sqrt{5}-\sqrt{2}\right)\left(\sqrt{10}+4\sqrt{2}\right)$

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Example Multiply: $\left(3\sqrt[3]{4}-\sqrt[3]{6}\right)\left(5\sqrt[3]{2}+\sqrt[3]{6}\right)$

How do you multiply $\left(\sqrt{2}+\sqrt{3}\right)^2$

Examples Multiply: $\left(\sqrt[3]{9}+\sqrt[3]{6}\right)^2$

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Lesson: Rationalizing a Denominator with One Square Root

What does it mean to rationalize the denominator?

How would you rationalize $\frac{2}{\sqrt{3}}$? Explain *why* we can do this.

Can every fraction with a radical in the denominator be rationalized like this? Circle: Yes No

What types of problems can be rationalized this way?

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Examples Rationalize the following.

a. $\frac{20}{\sqrt{8}}$

b. $\sqrt{\frac{40}{60}}$

c. $\sqrt{\frac{81x^7}{2y}}$

d. $\frac{\sqrt{22}}{\sqrt{x^9}}$

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Dividing Radicals/Rationalizing the Denominator
Part 2: Higher Roots

In this video we will look at how to simplify expressions like:

$$\frac{5}{\sqrt[3]{3}} \quad \text{or} \quad \frac{\sqrt[4]{2}}{\sqrt[4]{9}}$$

Before you begin, you should already be able to simplify problems like this:

a. $\frac{5}{\sqrt{3}}$

b. $\frac{\sqrt{6}}{\sqrt{18}}$

First, let's observe why higher roots are a different situation. Write down what happens if we use the same methods as above to rationalize the following: $\frac{5}{\sqrt[3]{3}}$

We have to pivot our thinking to "count powers". Note how this works with this examples: $\sqrt[3]{3} * \_\_\_ = 3$

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Examples Determine what to fill in the blank to create a true statement:

a. $\sqrt[3]{25} * ? = 5$

b. $\sqrt[4]{2} * ? = 2$

c. $\sqrt[4]{27} * ? = 3$

d. $\sqrt[5]{4} * ? = 2$

e. $\sqrt[3]{7} * ? = 7$

f. $\sqrt[4]{25} * ? = 5$

g. $\sqrt[5]{3} * ? = 3$

h. $\sqrt[3]{36} * ? = 6$

Now you should have a sense of what number you want this to equal. This is a very strange exercise, but using the pattern above, determine what to multiply by to get to the desired root:

a. $\sqrt[3]{4} * ? =$

b. $\sqrt[4]{5} * ? =$

c. $\sqrt[5]{9} * ? =$

d. $\sqrt[4]{36} * ? =$

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Now we should have a decent sense of how to rationalize and what the answer should equal. Explain how to rationalize the following:

a. $\frac{5}{\sqrt[3]{2}}$

b. $\frac{7}{\sqrt[4]{5}}$

Examples Rationalize the following denominators:

a. $\frac{6}{\sqrt[3]{5}}$

b. $\frac{2}{\sqrt[4]{9}}$

c. $\frac{2}{\sqrt[5]{125}}$

d. $\frac{7}{\sqrt[4]{8}}$

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Examples Rationalize the following denominators:

a. $\frac{\sqrt[3]{5}}{\sqrt[3]{4}}$

b. $\frac{2\sqrt[3]{3}}{\sqrt[3]{4}}$

c. $\frac{5\sqrt[4]{2}}{\sqrt[4]{27}}$

d. $\frac{6\sqrt[3]{5}}{\sqrt[3]{3}}$

e. $\frac{3}{\sqrt[5]{x^3}}$

f. $\frac{\sqrt[5]{25}}{\sqrt[5]{16}}$

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Lesson: Rationalizing a Denominator with Two Square Roots

How do you find the conjugate of $4 + \sqrt{2}$? Write one sentence to explain what you need to do.

Examples What is the conjugate of the following?

a. $3 - \sqrt{5}$

b. $-4 - \sqrt{2}$

Show how to rationalize: $\frac{5}{\sqrt{2}-\sqrt{3}}$

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Examples Rationalize the following.

a. $\frac{\sqrt{2}}{\sqrt{3}-\sqrt{6}}$

b. $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}+\sqrt{5}}$

c. $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

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Solving Equations with Radicals
Part 1: One Square Root in the Equation

In this video, we will look at how to solving equations like this:

$\sqrt{3x+1}=2$ or $\sqrt{4x-2}=5-x$

Use the example $\sqrt{3x+1}=4$ to show the general procedure for solving these types of equations:

Why must we check the solutions?

Example Solve $\sqrt{3x+1}=8$

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Examples Solve

a. $\sqrt{5x-4}+2=8$

b. $\sqrt{1+3x}+5=2$

Explain what is different about solving $\sqrt{5x+4}=x+2$

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Examples Solve the following.

a. $\sqrt{3x-6} = x-2$

b. $\sqrt{2x+4} = x-2$

c. $\sqrt{5x+1} + 5 = 3x$

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Solving Equations with Radicals
Part 2: Two Square Roots in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt{4x-2} = 3\sqrt{2x+6} \quad \text{or} \quad \sqrt{3x-1} + 2 = \sqrt{4x-2}$$

Let's begin by explaining how to solve: $\sqrt{5x+2} = \sqrt{3x+6}$

Examples Solve:

a. $\sqrt{3-2x} = \sqrt{2x+7}$

b. $\sqrt{5+3x} = \sqrt{3+2x}$

c. $4\sqrt{3-2x} = 2\sqrt{1+3x}$

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Example Solve: $6\sqrt{x} = 4\sqrt{2x+1}$

Now write out the detailed explanation of how to solve $\sqrt{4x+1} - 2 = \sqrt{2x+1}$

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Examples Solve:

a. $\sqrt{3x+6} = 2 + \sqrt{2x-4}$

b. $\sqrt{3x+1} = 2 + \sqrt{2x-2}$

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Examples Solve:

a. $\sqrt{2x+4}+\sqrt{1-2x}=3$

b. $\sqrt{2x+4}=2+\sqrt{2x+3}$

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Solving Equations with Radicals
Part 3: Two Cube Roots of Higher in the Equation

In this video, we will look at how to solving equations like this:

$$\sqrt[3]{3x+1}=2 \qquad \text{or} \qquad \sqrt[4]{2x-1}=5$$

Explain how to solve: $\sqrt[3]{3x+1}=2$

Explain how to solve: $\sqrt[4]{3x+4}=2$

When is it more likely a solution will not work?

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Examples Solve the following:

a. $\sqrt[3]{4x+7} = 3$

b. $\sqrt[3]{3-5x} = 2$

c. $\sqrt[4]{3-2x} = 3$

d. $\sqrt[4]{4x-1} = -7$

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How to Evaluate Square Roots that Have Imaginary Number Answers and Dealing with Negatives in Square Roots

In this video, we will look at how to evaluate things like this:

$\sqrt{-4}$ or $\sqrt{-12}$ or $\sqrt{-3} * \sqrt{-6}$

What is an imaginary number?

What is a complex number?

How do you evaluate something like $\sqrt{-9}$?

Examples Compute the following:

a. $\sqrt{-25}$

b. $\sqrt{-81}$

c. $\sqrt{-12}$

d. $\sqrt{-7}$

e. $\sqrt{-\frac{25}{16}}$

f. $\sqrt[3]{-8}$

g. $\sqrt[4]{-16}$

h. $\sqrt{-32}$

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How do you evaluate problems like $\sqrt{-3} * \sqrt{-2}$?

Examples Evaluate the following:

a. $\sqrt{-6} * \sqrt{-5}$

b. $\sqrt{-6} * \sqrt{-2}$

c. $\sqrt{-10} * \sqrt{-15}$

d. $\frac{\sqrt{-6}}{\sqrt{-2}}$

e. $\frac{\sqrt{-50}}{\sqrt{-2}}$

f. $\frac{\sqrt{-7}}{\sqrt{-2}}$

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Operations with Imaginary Numbers

In this video, we will look at how to evaluate things like this:

$2i(3i-5)$ or $(4i+1)-(5i+3)$ or $\frac{4}{3+2i}$

Adding and Subtracting with Imaginary Numbers:
Use the example $(2i+3)+(5i-6)$ to show how to add or subtract with imaginary numbers.

Examples Simplify:
a. $(3i-2)-(4i-9)$

b. $(5i+3)+(2i-1)$

What can we say about i^2?

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With that in mind, explain what is unique about multiplying: $2i(3i + 1)$

Examples Simplify:

a. $4(2 - 3i)$

b. $6i(3i + 2)$

c. $5i(2 - 8i)$

d. $(5i + 1)(2i - 3)$

e. $(4i - 3)(6i + 7)$

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Examples Simplify:
a. $(5i + 2)^2$

b. $(3i - 5)^2$

Why do you need to rationalize $\frac{3}{2i+1}$? Explain how to do this.

Examples Rationalize the following:
a. $\frac{4}{2i+6}$

b. $\frac{7}{i+3}$

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Examples Rationalize the following:

a. $\frac{3i+1}{5i-2}$

b. $\frac{7i+3}{2i+1}$

c. $\frac{i-3}{5i+3}$

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Evaluating Higher Powers of Imaginary Numbers

In this video, we will look at how to evaluate things like this:

i^{21} or i^{54}

First we need to investigate the powers of i and the patterns with it. Write this down below:

Now that we can see a pattern, explain how to evaluate the following:

i^{15}

i^{22}

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Examples Evaluate the following:

a. i^{36}

b. i^{43}

c. i^{52}

d. i^{33}

e. i^{25}

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Solving Quadratic Equations with Factoring

In this video, we will look at how to solve equations by factoring, which looks like this:

$$x^2 + 5x + 6 = 0 \rightarrow (x + 2)(x + 3) = 0$$

Quadratic equations are equations of the form:

Where a, b, and c are real numbers and $a \neq 0$.

How would you solve the quadratic equation $x^2 + 7x + 6 = 0$ by factoring?

Examples Solve the following by factoring:

a. $x^2 - x - 12 = 0$

b. $x^2 + 9 = -6x$

c. $x^2 = 25$

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Examples Solve the following by factoring:

a. $x^2 + 7x = 0$

b. $\frac{1}{10}x^2 + \frac{1}{2}x + \frac{3}{5} = 0$

c. $2x^3 + 28x^2 + 90x = 0$

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Solving Quadratic Equations using the Square Root Property

In this video, we will look at how to solve equations by using the square root property, which looks like this:

$$(x+2)^2 = 9 \quad \rightarrow \quad x+2 = \pm 3$$

Quadratic equations are equations of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

What are the solutions to $x^2 = 25$?

In general, how do you solve equations of the form $Q^2 = k$, where Q is an expression and k is a constant? In other words, what is the **square root property**?

Examples Solve the following equations using the square root property:

a. $(x+3)^2 = 25$

b. $(x+3)^2 = 12$

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Examples Solve the following equations using the square root property:

a. $(5x-2)^2+5=29$

b. $(6x+7)^2+4=-41$

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Solving Quadratic Equations by Completing the Square

In this video, we will look at how to solve equations by factoring, which looks like this:

$x^2 + 6x = 7 \qquad b = 6, \quad \frac{1}{2}b = 3, \quad \left(\frac{1}{2}b\right)^2 = 9$

Quadratic equations are equations of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

Before we begin, you should be familiar with the square root property. Solve $(2x + 1)^2 = 49$ using the square root property.

First let's talk about how to complete the square. Show how to complete the square on $x^2 + 6x$ below:

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Examples Complete the square on the following:

a. $x^2 + 8x$

b. $x^2 - 10x$

c. $x^2 + 3x$

d. $x^2 - \frac{1}{2}x$

e. $x^2 + \frac{2}{3}x$

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Show how to solve $2x^2 + 12x - 16 = 0$ by completing the square:

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Examples Solve the following by completing the square:

a. $x^2 + 4x - 6 = 0$

b. $x^2 + 5 = 2x$

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Examples Solve the following by completing the square:

a. $3x^2 + 15x = -6$

b. $\frac{1}{2}x^2 + 3x - 2 = 0$

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Examples: The Distance Formula

Find the distance between the following sets of points.

a. (6, 2) and (2, 11)

b. (3, -1) and (-4, -2)

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Solving Quadratic Equations by the Quadratic Formula

Complete the square on: $ax^2 + bx + c = 0$

State the quadratic formula:

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Examples Use the quadratic formula to solve the following:

a. $x^2 + 3x + 2 = 0$

b. $2x^2 + 3x = 5$

c. $\frac{1}{2}x^2 + \frac{1}{3}x + \frac{5}{6} = 0$

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Lesson: The Discriminant

Restate the Quadratic Formula, and then label the discriminant.

Value of the Discriminant	Interpretation

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Examples Find the values of the discriminant for the following equations. Then solve the equations (using the most efficient method) to determine if this makes sense.

a. $x^2 - 4x + 4 = 0$

b. $x^2 + 6x + 5 = 0$

c. $3x^2 + 2x + 5 = 0$

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Choosing the Best Method to Solve a Quadratic

There are 4 ways we can solve quadratics:

1.	3.
2.	4.

Using each of these methods only once, determine the best way to solve these quadratics:

a. $x^2 + 6x = 2$

b. $x^2 + 3 = 2x$

c. $(2x + 1)^2 = 49$

d. $3x^2 + 2 = 3x$

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Examples Using each of these methods only once, determine the best way to solve these quadratics:

a. $(3x-2)^2=-24$

b. $2x^2+5x-1=0$

c. $x^2-12=x$

d. $x^2-2x+5=0$

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Solving Equations that End Up Having Quadratics

In this lesson, we will review some techniques from previous lessons.

Solve $\frac{2}{x} - \frac{1}{x+1} = \frac{2}{3}$

Solve $\sqrt{2x+8} = x+4$

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Examples Solve the following:

a. $\frac{4}{x-2}+\frac{x}{x-3}=\frac{2}{x-3}$

b. $\sqrt{3x+5}=x-3$

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Solving Equations that Are Similar to Quadratics

In this lesson, we will look at how to solve equations that look like this:

$$x^{\frac{2}{3}} - x^{\frac{1}{3}} - 12 = 0 \qquad \text{or} \qquad (2x+3)^2 + 5(2x+3) + 6 = 0$$

Before we begin, what do the following equations have in common if you **look at their exponents**?

a. $x^2 + 5x + 6 = 0$

b. $x^{\frac{2}{3}} + 5x^{\frac{1}{3}} + 6 = 0$

c. $x + 5x^{\frac{1}{2}} + 6 = 0$

d. $x^4 + 5x^2 + 6 = 0$

Explain how to solve $x^4 - 13x^2 + 36 = 0$

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Examples Solve the following:

a. $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 15 = 0$

b. $x - 2x^{\frac{1}{2}} - 8 = 0$

d. $x^{-2} + 8x^{-1} - 9 = 0$

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Examples Solve the following:

a. $(x+3)^2 - 4(x+3) - 5 = 0$

b. $(2x+1)^2 - 4(2x+1) - 21 = 0$

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Solving Formulas with Squares and Square Roots

In this lesson, we will look at how to solve for formulas that look like this:

$$y = 3x^2 \qquad \text{or} \qquad M = \sqrt{\frac{AB}{C}} \qquad \text{or} \qquad kx^2 + cx - n = 0$$

Solving Formulas with Squares

Solve the following for the indicated letters:

a. for x: $y = \frac{4x^2}{z}$

b. for k: $b = \frac{3}{k^2}$

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Solving Formulas with Square Roots

Solve the following for the indicated letters:

a. for b: $a = \sqrt{\frac{b}{c}}$

b. for z: $x = \sqrt{\frac{3y}{z}}$

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Solving Formulas in other forms

Solve the following for the indicated letters:

a. for x: $kx^2 + cx - d = 0$

b. for b: $ab^2 - 5b + m = 0$

c. for k: $\frac{1}{2}k^2 - 5nk + m = 0$

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Solving Applications of Quadratic Equations

An object is thrown upwards off a cliff that is 48 feet above the ground. The height of the object t seconds after the object is released is given by:

$$h = -16t^2 + 32t + 48$$

How long does it take the object to hit the ground?

How long does it take the object to reach a height of 20 feet?

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A 12 ft by 16 ft pond is going to be surrounded by a border of grass of uniform width all the way around. If the area of the enclosure must be 480 feet squared, what is the width of the grass border?

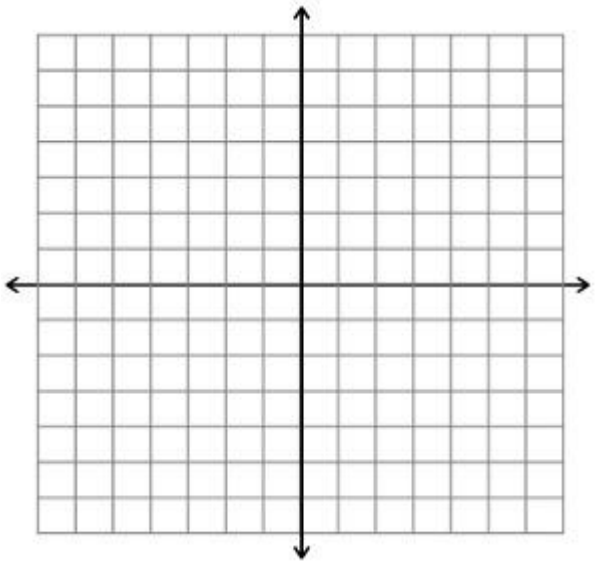
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Lesson: Understanding the Graphs of Quadratic Functions

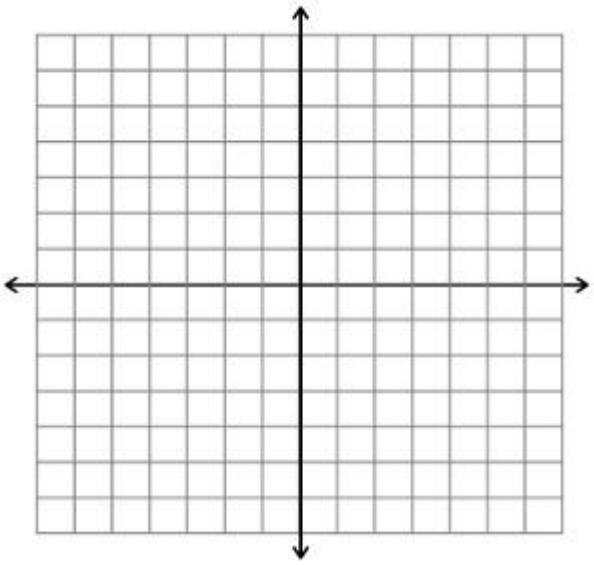
Quadratic functions are functions of the form _____, where $a \neq 0$. But in previous lessons we worked on graphing quadratic functions, ie parabolas in another way. Let's review that first.

Examples Graph the following:

a. $f(x) = (x+2)^2 - 3$



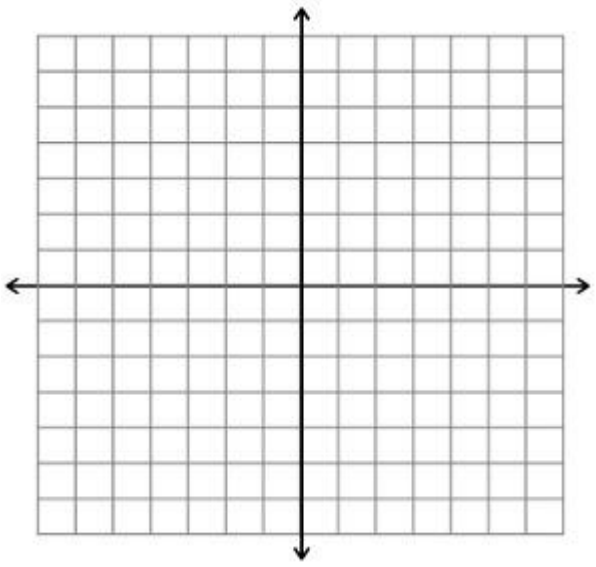
b. $g(x) = -(x-3)^2 + 1$



To begin, we will add on one more way to manipulate the graph of a parabola. Explain how to graph the follow:

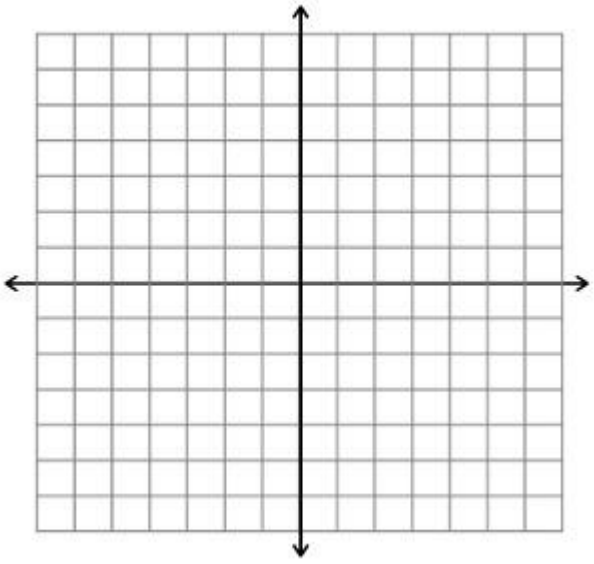
a. $f(x) = 3x^2$

How do you interpret the 3?



b. $g(x) = -\frac{1}{2}(x-2)^2 + 1$

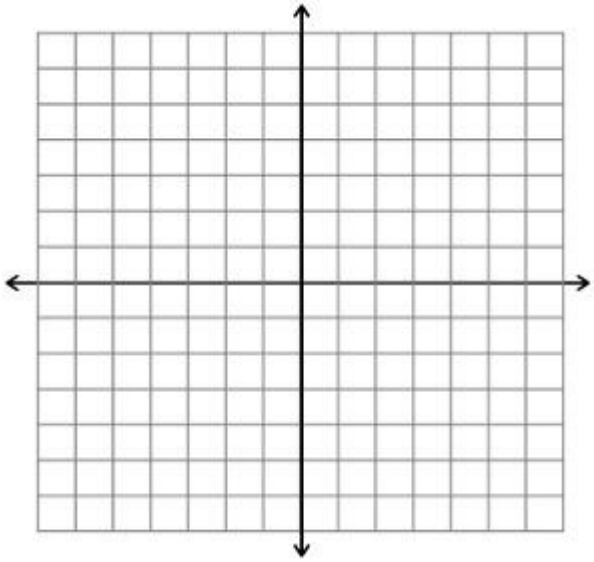
How do you interpret the $-\frac{1}{2}$?



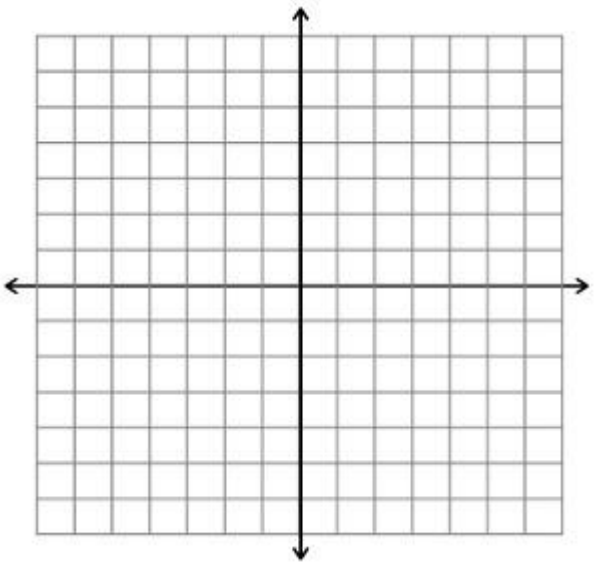
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Examples Graph the following:

a. $f(x) = -2(x+1)^2 + 3$



b. $g(x) = \frac{1}{3}(x-2)^2 - 2$



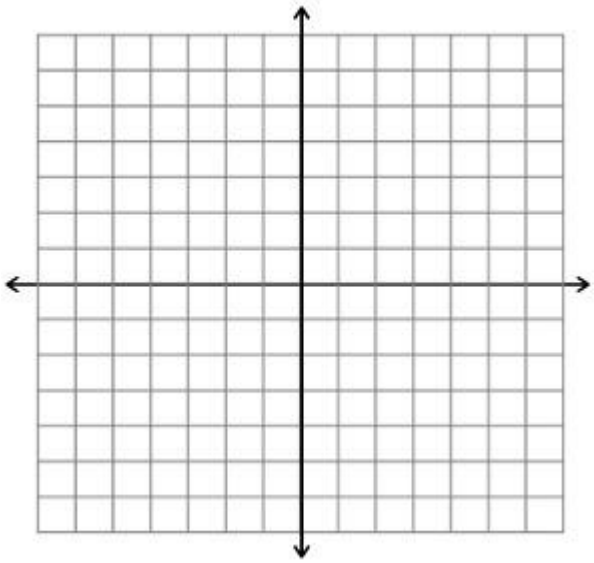
Using all of the examples we have discussed, what is another equation form for the parabola?

Explain some of the characteristics of a parabola that we get from this format:

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Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x-3)^2 - 4$



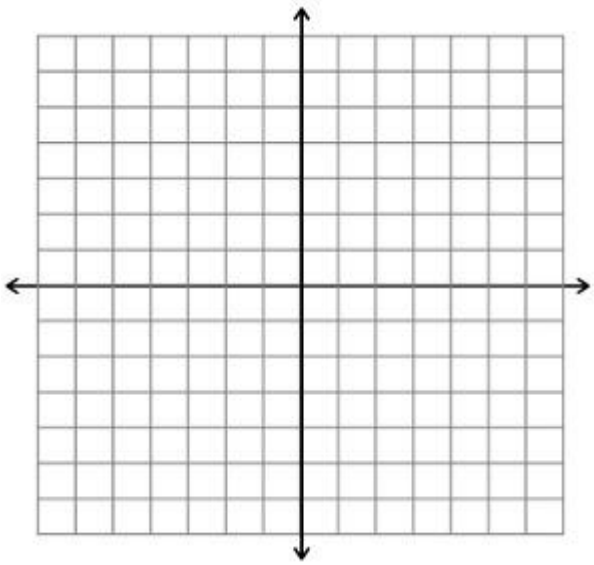
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2 + 8$



Vertex:

Axis of Symmetry:

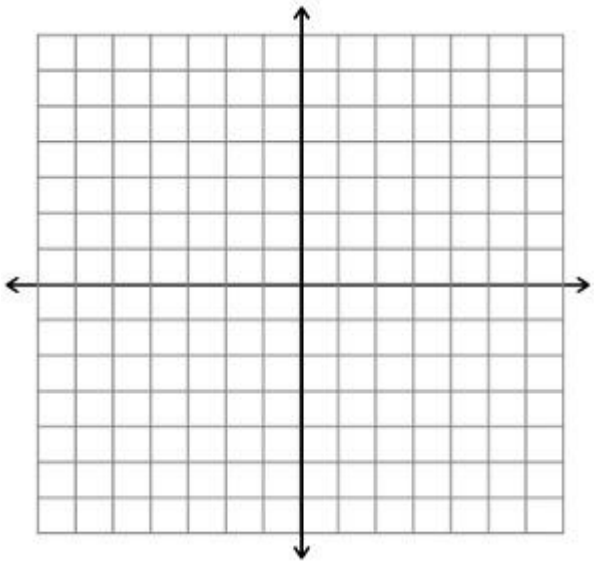
X-intercepts:

Y-intercepts:

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Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{4}(x+2)^2 - 3$



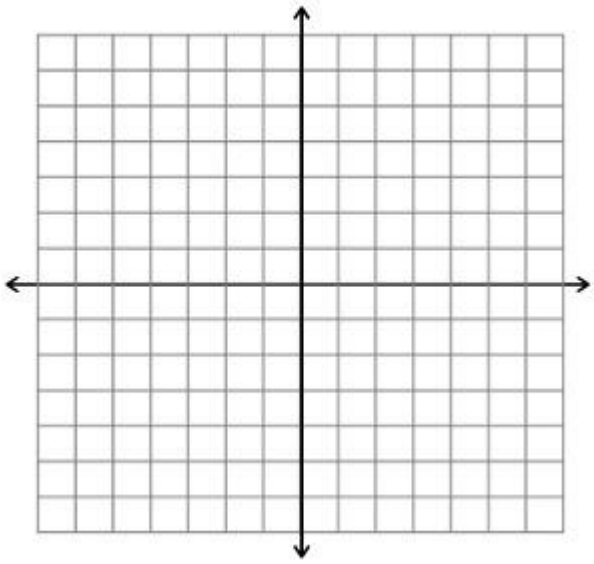
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -\frac{1}{3}(x-1)^2 + 3$



Vertex:

Axis of Symmetry:

X-intercepts:

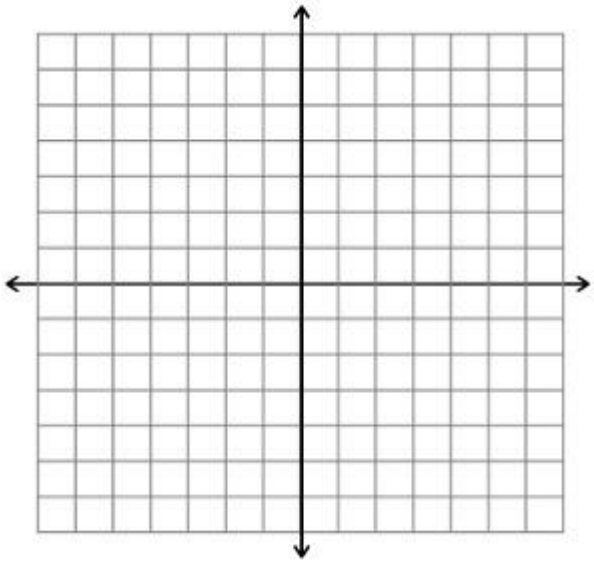
Y-intercepts:

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Examples: Understanding How Many X-Intercepts There Are

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{2}(x+1)^2 - 8$



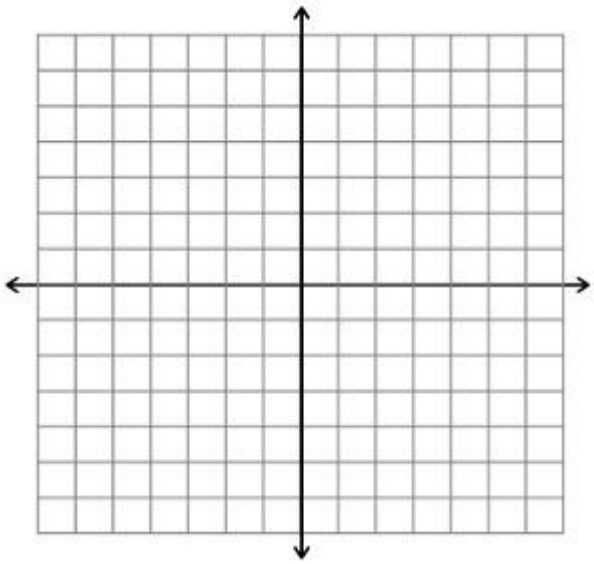
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2$



Vertex:

Axis of Symmetry:

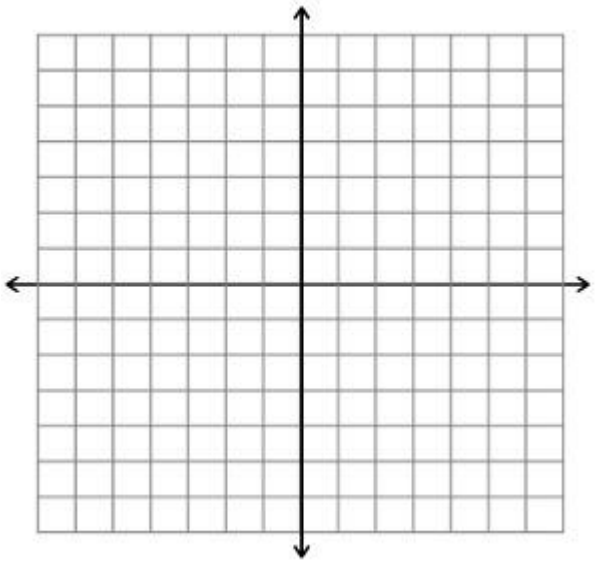
X-intercepts:

Y-intercepts:

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Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x-2)^2 + 1$



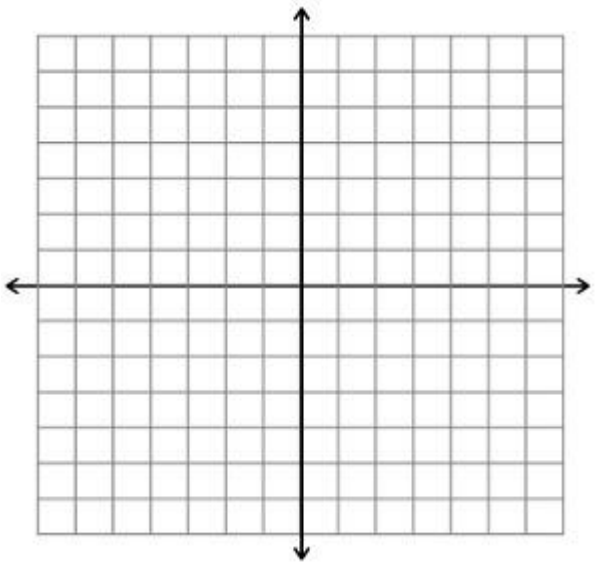
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -(x+3)^2 - 1$



Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

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Lesson: Finding the Vertex of Quadratic Equations in the form $\boldsymbol{f(x) = ax^2 + bx + c}$

What is the vertex formula?

Show how to use the vertex formula for $f(x) = 3x^2 + 6x - 2$

Examples Find the vertex of the following:

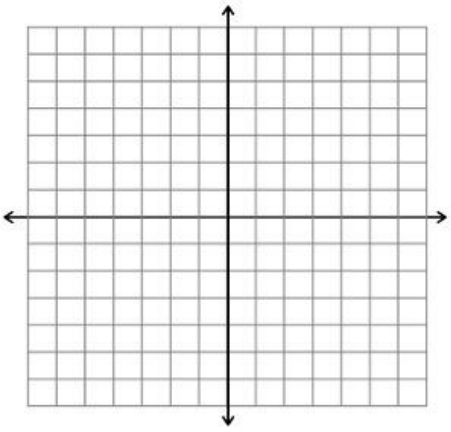
a. $f(x) = -x^2 + 4x - 2$

b. $f(x) = -\frac{1}{2}x^2 + 3x - 1$

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Lesson: Putting Quadratics into the $f(x) = a(x-h)^2 + k$ Form

Before you begin this lesson, you should already feel comfortable graphing an equation like $f(x) = 2(x-1)^2 + 3$. Graph this below:



Now show all the steps to put $f(x) = 2x^2 - 8x + 2$ into $f(x) = a(x-h)^2 + k$ form:

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Examples Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = x^2 + 4x - 2$

b. $g(x) = x^2 + 6x + 10$

c. $h(x) = -x^2 + 2x - 4$

d. $y = 3x^2 + 6x - 12$

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Examples: Putting Quadratics into the $f(x) = a(x-h)^2 + k$ Form

Examples Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = \frac{1}{2}x^2 + x - 3$

b. $g(x) = -\frac{1}{3}x^2 + 4x - 2$

c. $y = x^2 + 3x + 2$

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Lesson: Determining Whether a Quadratic Function will have a Minimum or a Maximum

Using the form $f(x) = ax^2 + bx + c$, explain what determines whether the parabola opens up or down:

Examples Determine whether the following quadratics open upwards or downwards, and then determine their vertex:

a. $f(x) = -2x^2 + 4x - 2$ upwards downwards

b. $g(x) = -\frac{1}{2}x^2 - 2x + 2$ upwards downwards

c. $h(x) = x^2 + 6x + 1$ upwards downwards

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Lesson: Applications of Quadratic Functions 1

A ball is thrown upwards from a cliff 24 feet about the ground. The height h of the ball, in feet, t seconds after it is thrown is given by:

$$h(t) = -16t^2 + 32t + 24$$

How long does it take the ball to reach its maximum height?

What is the maximum height attained by the ball?

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Lesson: Determining if Functions are One-to-One

Before we begin, you should already be familiar with the concept of functions and the vertical line test. First let's do some review.

Review Determine whether the following relations are functions, then determine the domain and range.

a. $\{(2,3),(5,7),(2,4)\}$

b.

Now we will discuss a specific type of function: a one-to-one function. Define this below:

How can we visualize this?

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Example Determine if any of the following are i) functions and ii) one-to-one functions

a. $\{(2,4),(-1,7),(3,7)\}$

b. $\{(3,6),(-1,2),(-3,-4)\}$

Similar to how we have a test to determine if a graph represents a function, we have another test to determine if a function is one-to-one.

Describe the horizontal line test below:

Examples Determine whether the following graphs represent i) functions and ii) one-to-one functions (draw your own):

a. b. c.

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Lesson: Finding the Inverse of Functions

One-to-one functions have inverses. Describe what the inverse of a function is:

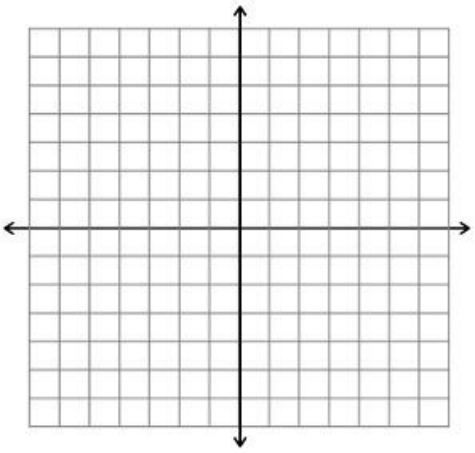
Example Find the inverse of the following function: $\{(1,4),(7,8),(3,-2)\}$

Describe the steps to find the inverse of a function:

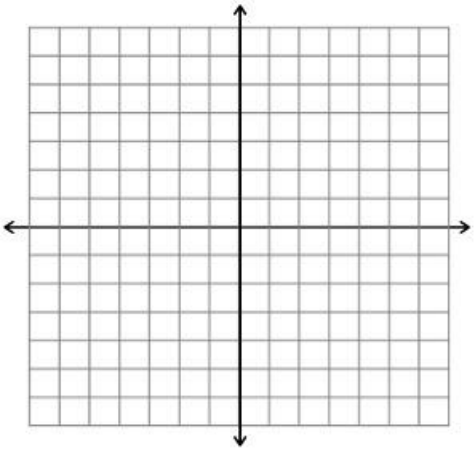
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Examples Find the inverse of the following one-to-one functions. Then plot the original function on the graph with its inverse.

a. $f(x) = 3x + 1$



b. $g(x) = -\frac{1}{2}x + 4$



What is the relationship of a graph with its inverse?

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Examples Find the inverse of the following one-to-one functions.

a. $h(x) = \sqrt[3]{x}$

b. $f(x) = x^2, x \geq 0$

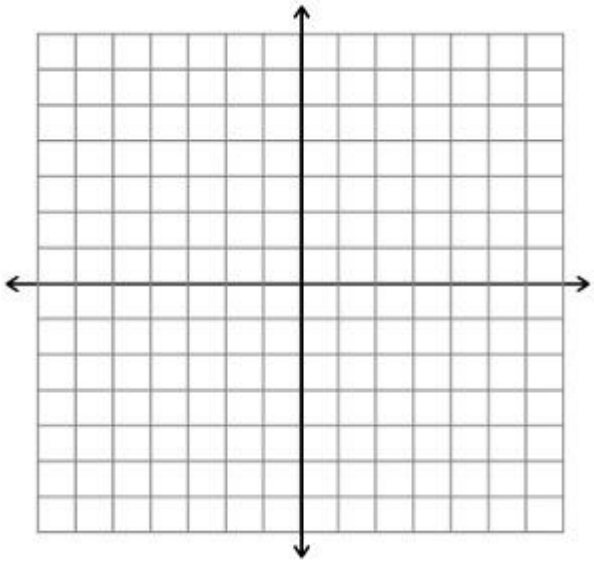
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Lesson: Exponential Functions

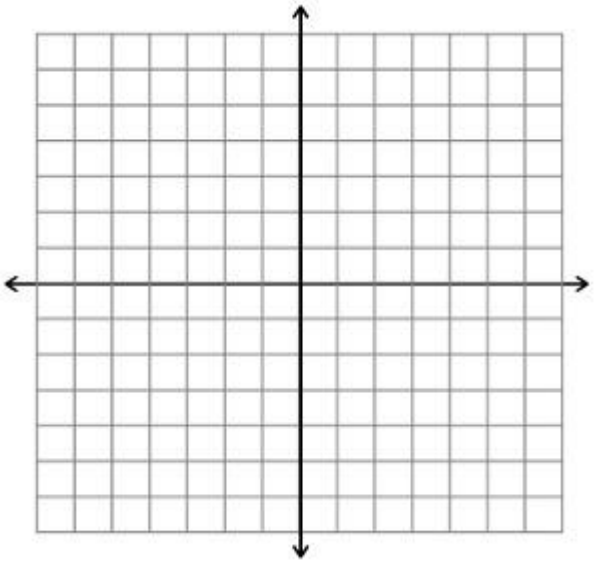
An exponential function is a function of the form _____ where $a > 0, a \neq 1$.

Examples Graph the following functions, then determine their domain and range.

a. $f(x) = 2^x$



b. $g(x) = \left(\frac{1}{2}\right)^x$

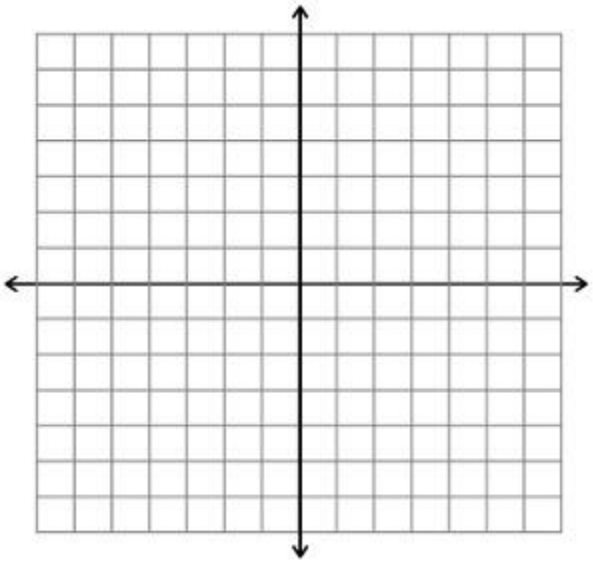


Using the graphs above, summarize some of the characteristics of these graphs:

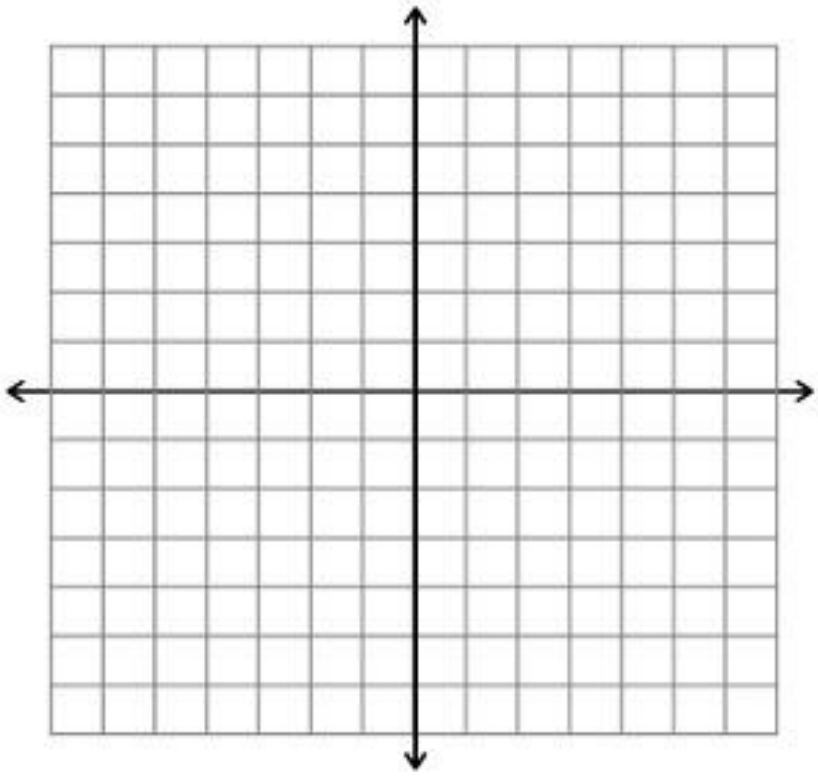
The number e is an irrational number that is frequently used in math. What is an approximation of e?

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Example Graph $f(x) = 2^x, g(x) = e^x, h(x) = 3^x$ all on the same graph below for comparison:



Translations work similarly to other graphs we have discussed. Using the graph of $f(x) = 2^x$ as a "base graph", draw the following graphs together on the same graph: $f(x) = 2^x, g(x) = 2^x + 3, h(x) = 2^{x-1}$



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Lesson: Solving Exponential Equations

In this lesson, we will solve equations that look like this:

$$3^x = 27 \qquad \text{or} \qquad 4^{x+3} = 8^{x-2}$$

To do this, we will write directions around the following example: $2^{3x} = 8^{x+1}$

Examples Solve:

a. $49^{x-3} = 7^{2x}$

b. $9^{4x} = 27^{x+4}$

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Examples Solve:

a. $5^{x-2} = \frac{1}{25}$

b. $\left(\frac{3}{5}\right)^{2x} = \left(\frac{27}{125}\right)^{x+2}$

c. $27^{3x+1} = 9^{4x+2}$

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Lesson: A Review of Exponential Functions and their Graphs

What is the form of a general exponential equation?

Example Let $f(x) = 3^x$ and $g(x) = \left(\frac{1}{2}\right)^x$. Find the following.

a. $f(-2)$

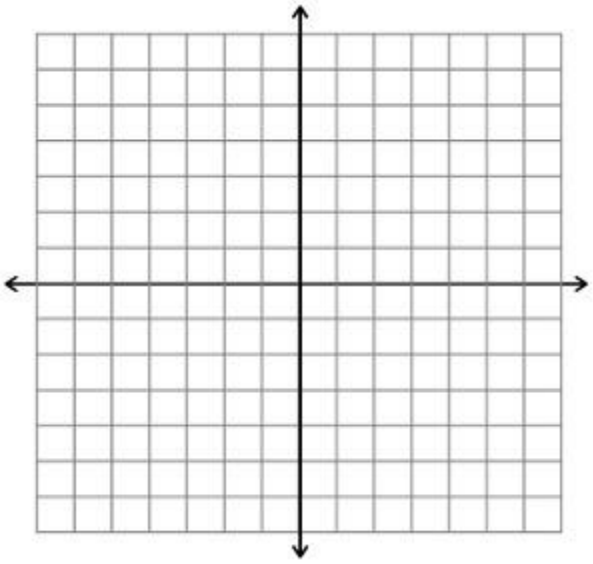
b. $f\left(\frac{3}{2}\right)$

c. $g(-3)$

d. $g(3.2)$

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Sketch the graph of $f(x) = 2^x$ and create a table to go with it.



When will an exponential function $f(x) = a^x$ take this shape?

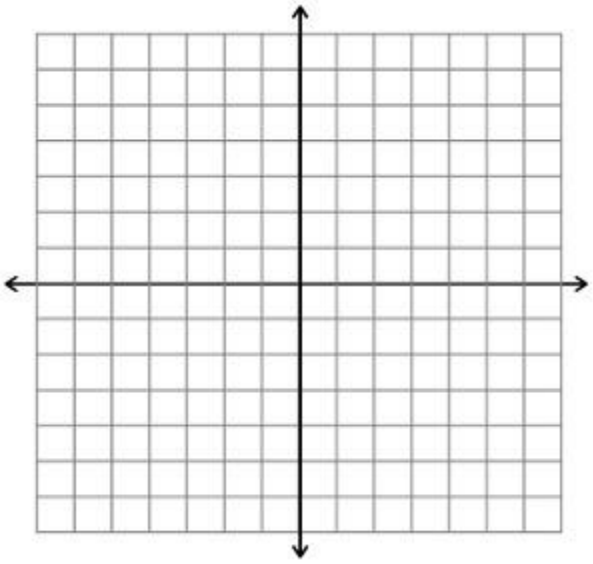
What is the x-axis for this graph?

What are 3 standard points to note here?

What is the domain and range?

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Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^x$ and create a table to go with it.



When will an exponential function $f(x) = a^x$ take this shape?

What is the x-axis for this graph?

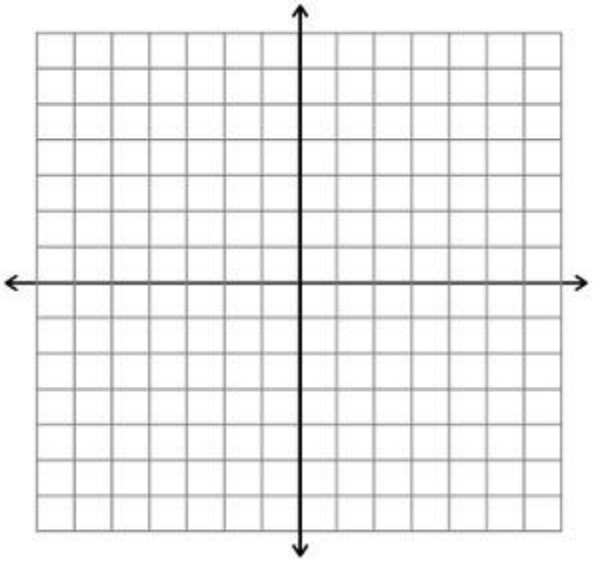
What are 3 standard points to note here?

What is the domain and range?

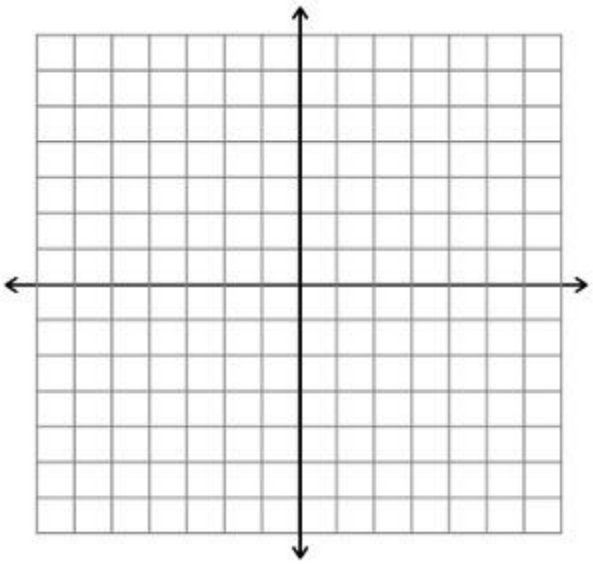
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Examples Describe the translations and sketch the following graphs.

a. $f(x) = -3^x + 1$



b. $f(x) = \left(\frac{1}{2}\right)^{x+1} - 2$



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Lesson: Solving Basic Exponential Equations

Outline the general steps to solve $3^{x+1} = 81$.

Outline the general steps to solve $4^{x-2} = 8^{x+1}$.

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Examples Solve the following.

a. $27^{2x} = 9^{x-1}$

b. $e^{3-x} = (e^2)^{-x}$

c. $\left(\sqrt{3}\right)^{x+3} = 9^x$

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Examples Solve the following.

a. $x^{\frac{3}{2}} = 8$

b. $x^{\frac{5}{2}} = 4$

c. $x^{-4} = \frac{1}{16}$

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Lesson: Compound Interest

What is the formula for compound interest and what does it mean?

Example Suppose $2000 is invested into an account paying 2.5% interest per year compounded monthly. How much is in the account after 10 years, and how much interest is earned in that time?

What else do we call the A from the compound interest formula?

What else do we call the P from the compound interest formula?

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Example In 5 years, you want to have $8000 in cash to buy a car (including taxes). Let's use savings to help us reach this goal.

a. You find an account with a 3.25% interest rate compounded biannually (2x a year). What amount would you have to deposit today to ensure you have $8000 in 5 years?

b. Let's pretend that as of today, you have $5000. What interested rate would you need to ensure that this will grow to $8000 in 5 years?

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What is the number e?

What is the formula for continuous compounding?

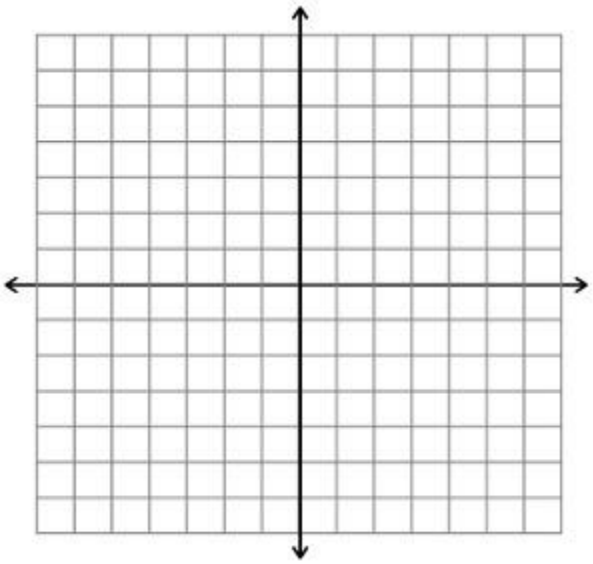
Example Suppose $2000 is invested into an account paying 2.5% interest per year compounded continuously. How much is in the account after 10 years, and how much interest is earned in that time?

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Lesson: Intro to Logarithmic Functions

First, we want to review how to function inverses of functions. Find the inverse of: $f(x) = 2x + 8$

Graph the exponential function $f(x) = 3^x$ below:



Recall the exponential functions are one-to-one, which means that they have inverses. Attempt to find the inverse of $f(x) = 3^x$ below:

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As we can see, there is an issue with finding the inverse – we have no way to solve for y! As such, we need new mathematics to help solve this.

Write down the definition of a logarithm:

Examples Write the following in exponential form:

a. $\log_3 81 = 4$

b. $\log_2 \frac{1}{8} = -3$

c. $\log_5 125 = 3$

d. $\log_9 9 = 1$

Examples Write the following in logarithmic form:

a. $2^4 = 16$

b. $3^0 = 1$

c. $6^{-2} = \frac{1}{36}$

d. $8^{\frac{1}{3}} = 2$

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Now we can finally determine what is the inverse of $f(x) = 3^x$:

There are a few common logarithms:

- $\log x$ means:

- $\ln x$ means:

There are also 2 key properties of logarithms:

1. $\log_a a = 1$: explain why this makes sense

2. $\log_a 1 = 0$: explain why this makes sense:

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Lesson: Basics of Solving Logarithmic Equations and Evaluating Logarithms

We want to solve equations that look like:

$$\log_3 x = 4 \qquad \text{or} \qquad \log_x 36 = 2$$

Before you begin, you should already be able to do the following:

a. Put 3^5 in logarithmic form:

b. Put $\log_5 5 = 1$ in exponential form:

Now we can explain how to solve logarithmic equations using the example $\log_3(2x + 1) = 3$

Example Solve: $\log_x 16 = 4$

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Example Solve:

a. $\log_9 x = 2$

b. $\log_6(2x + 4) = 2$

Now we can also discuss how to evaluate logarithms. Explain how to evaluate: $\log_3 27$:

Examples Evaluate:

a. $\log_5 \frac{1}{25}$

b. $\log 100$

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Examples Evaluate:

a. $\ln e$

b. $\log_3 27$

c. $\log_{25} 5$

d. $\log \frac{1}{10}$

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Lesson: How to Graph a Logarithm

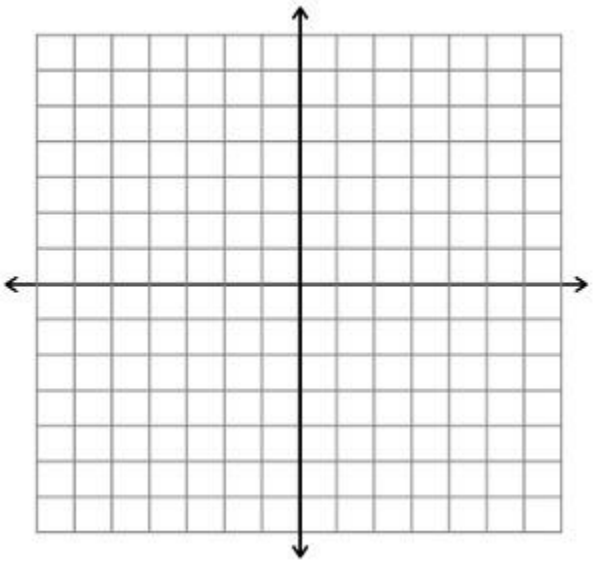
Before we begin, you should be able to do the following:

a. Convert to logarithmic form: $25 = 5^2$

b. Convert to exponential form: $\log_2 8 = 3$

c. Find the inverse of $y = 2^{\mathrm{x}}$

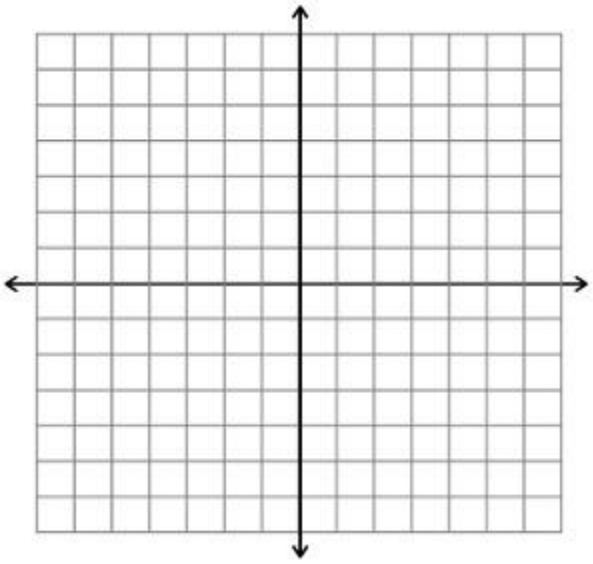
Now we can explain how to graph $y = \log_3 x$



For comparison, also graph $f(x) = 3^x$ on the same graph above.

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Example Graph $y = \log_{\frac{1}{2}} x$ below:



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Lesson: Review of the Basics of Logarithmic Functions

Explain how to find the inverse of $f(x) = 3^x$, and explain how this relates to logarithms.

Define a logarithm:

Example Write the following in logarithmic form.

a. $3^2 = 9$ b. $5^0 = 1$ c. $4^{-2} = \frac{1}{16}$

Example Write the following in exponential form.

a. $\log_3 1 = 0$ b. $\log_5 125 = 3$ c. $\log_3 81 = 4$

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Example Show how to solve the following basic logarithmic equations.

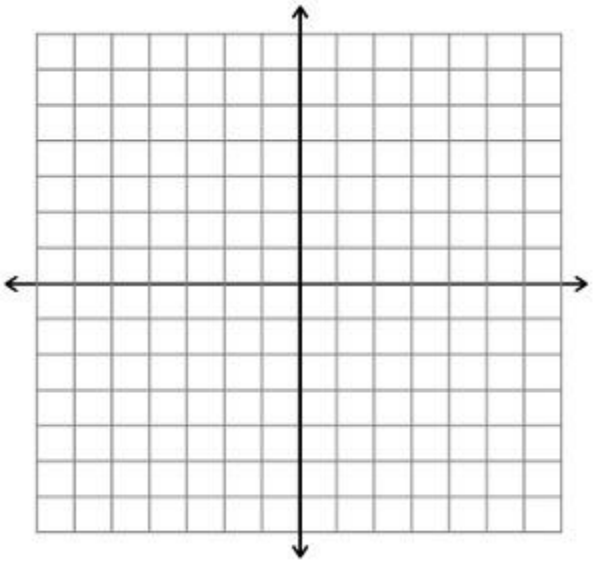
a. $\log_x 8 = 3$

b. $\log_3 x = -2$

c. $\log_{49} 7 = x$

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Create a table and show how to graph $f(x) = \log_2 x$



What 3 points can you easily find on the graph?

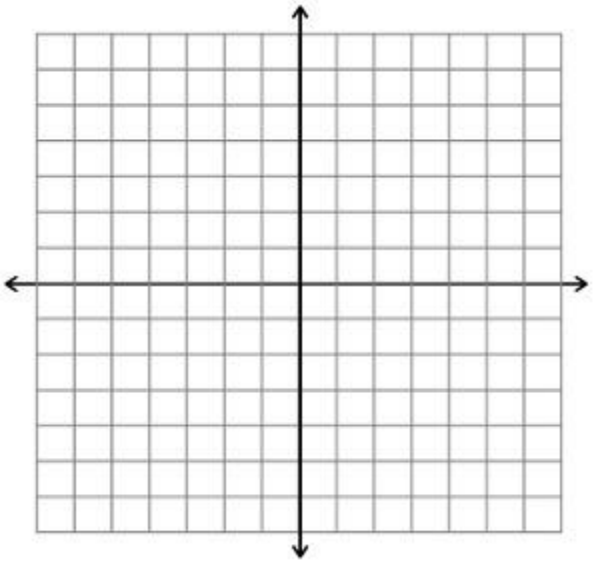
When does the graph take a shape like this for $f(x) = \log_a x$?

What is the y-axis for this graph?

What are the domain and range?

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Create a table and show how to graph $f(x) = \log_{\frac{1}{2}} x$



What 3 points can you easily find on the graph?

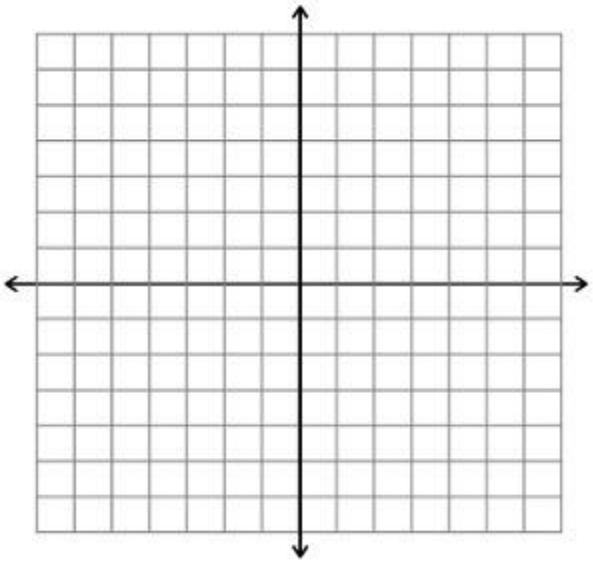
When does the graph take a shape like this for $f(x) = \log_a x$?

What is the y-axis for this graph?

What are the domain and range?

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Examples Graph the following: $g(x) = \log_2(x - 1) + 1$



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Lesson: Review of the Basics of Properties of Logarithsm

List the major properties of logarithms.

Explain what the common logarithms mean:

$\log x =$

$\ln x =$

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Examples Expand the following as much as possible.

a. $\log_3\left(\frac{3x^3y^7}{\sqrt{z}}\right)$

b. $\ln\left(\sqrt[3]{x^2y}\right)$

c. $\log_3\sqrt{6}$

d. $\log_5\frac{25x^3y^2}{p^7r^3}$

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Example Write as a single logarithm.

a. $\log(x+2)+\log(x-2)-\log(x+3)$

b. $3\log_2 3+2\log_2 y-5\log_2 x$

c. $\frac{1}{2}\log_5 16+3\log_5 y^2-\log_5 x^7-\log_5 y$

d. $4\ln 2+\ln x^6-2\ln y^3-3\ln x^2$

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Lesson: Change of Base Theorem

State the change of base theorem:

Example Show how to evaluate the following:

a. $\log_5 17$

b. $\log_2 3$

c. $\log_{\frac{1}{2}} 5$

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Lesson: Solving Exponential Equations

Outline the steps to solve: $2^{x+3} = 7^{3x}$

Examples Solve the following.

a. $e^{x^2} = 100$

b. $e^{2x-3} * e^{3x} = 5e$

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Examples Solve.

a. $3(2)^{x+2} + 6 = 105$

b. $e^{2x} + 8e^{x} + 15 = 0$

c. $3^{2x} + 2(3^{x}) = 24$

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Lesson: Solving Logarithmic Equations

Outline the steps to solve: $\log(3 - x) = 0.75$

Compare these steps to: $\log(x + 20) = \log(x + 10) + \log(6)$. How are these problems different?

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Examples Solve the following.

a. $\log_4(x^3 + 37) = 3$

b. $\log x + \log x^3 = 4$

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Examples Solve the following.

a. $\log(x^2 - 4) - \log(x - 2) = \log(8)$

b. $\log(5 + 4x) - \log(3 + x) = \log 3$

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Lesson: Solving Exponential and Logarithmic Formulas

Examples Solve for the desired variable.

a. for b: $p = a + \frac{c}{\ln b}$

b. for n: $A = \frac{Pr}{1-(1+r)^{-n}}$

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c. for A: $\log A = \log B - C \log D$

d. for x: $D = 100 + 5 \log x$

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Lesson: Models of Exponential Growth and Decay

Explain the equation for the general model:

Some examples of growth can come from populations or investments. Some examples of decay can come from half-life/radioactive decay, or carbon dating.

Define the following:

Doubling time:

Half-life:

What is the formula for continuous compounding?

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Example A sample of 400g of radioactive lead decays to polonium according to the function: $A(t) = 400e^{-0.03t}$.

a. Find the amount of lead remaining after 5 years.

b. Find the half-life.

Example The amount of carbon-14 present in a paint after t years if given by $y = y_0 e^{-0.0002t}$. The paint contains 37% of its carbon-14. How old are the paintings?

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Example Find the doubling time of an investment earning 5% interest if interest is compounded continuously.

Example During an epidemic, the number of people who have never had a particular virus and are not immune decreases exponential, where this decline is modeled by the function: $f(x) = 20{,}000e^{-0.025t}$, where t is time in days.

a. How many people were susceptible at the beginning of the epidemic?

b. How many people are susceptible after 2 weeks?

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Lesson: An Intro to Conic Sections

What are the 4 conic sections we will discuss? Draw an illustration with it.

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Lesson: Parabolas as Conic Sections

What is the previous equation we used for parabolas opening upwards or downwards? Draw a illustration to go with this.

How can we represent parabolas in similar notation for parabolas that open sideways?

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Examples Give the domain and range of the following parabolas, then graph the parabola.

a. $x + 1 = (y + 1)^2$

b. $-x = 3y^2 + 6y + 2$

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When dealing with conic sections, we shift how we think about things.

What is the geometric definition of a parabola? Draw an illustration to accompany this definition.

What equations do we use that align with the geometric definition?

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Examples Give the focus, directrix, vertex, and axis of symmetry for each parabola. Then use this information to graph the parabola.

a. $x^2 = 4y$

b. $(y-2)^2 = 24(x-3)$

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Lesson: Writing Equations of Parabolas

Example Write the equation of a parabola with the following characteristics.

a. focus $\left(0, -\frac{1}{2}\right)$ with the vertex at the origin

b. vertex (4, 3) and focus is (4, 5)

c. vertex (1, 2), directrix x=4

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Lesson: Ellipses as Conic Sections

Give the geometric definition of an ellipse. Draw an illustration to accompany this.

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What are the standard forms of an ellipse? Describe the parts and draw VERY CLEAR illustrations.

What is important to note with this equation?

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Example Graph each ellipse. Give the domain, range, center, vertices, endpoints of the minor axis, and focus.

a. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

b. $9x^2 + y^2 = 81$

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Example Graph each ellipse. Give the domain, range, center, vertices, endpoints of the minor axis, and focus.

c. $\frac{(x-1)^2}{9}+\frac{(y+2)^2}{4}=1$

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Lesson: Writing Equations of Ellipses

Example Write the equation of an ellipse with the following characteristics.

a. major axis 6, focus at (0,2) and (0, -2)

b. center at (-2, 7), major axis vertical with length 10, and c=2

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Lesson: Eccentricity with Ellipses

To discuss the concept of eccentricity, we need to discuss what the formal mathematical definition of conic sections is.

Define a conic:

The intuitive idea behind eccentricity can change slightly with each conic section, but it is generally a ratio of different measures within the conic section.

What does the eccentricity of an ellipse tell us?

How do we represent eccentricity?

How do you calculate the eccentricity of an ellipse?

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How does the eccentricity of a parabola compare to the eccentricity of an ellipse? Summarize the explanation from the video.

Examples Find the eccentricity of each ellipse.

a. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

b. $x^2 + 25y^2 = 25$

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Example Find the equation of an ellipse with $e = \frac{3}{4}$ with foci at $(-2,0), (2,0)$

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Examples: Using Complete the Square to Write an Ellipse Equation in Standard Form

Rewrite the following equations in standard form.

a. $x^2 + 12y^2 - 14x - 48y = -85$

b. $9x^2 + 36x + 16y^2 - 160y + 292 = 0$

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Lesson: Hyperbolas as Conic Sections

What is the geometric definition of a hyperbola? Draw an illustration with your definition.

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State the equations for the standard form of the hyperbola. Draw illustrations with it.

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Examples Graph each hyperbola. Give the domain, range, center, vertices, foci, and equations of the asymptotes.

a. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

b. $9x^2 - 25y^2 = 225$

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Examples Graph each hyperbola. Give the domain, range, center, vertices, foci, and equations of the asymptotes.

c. $\frac{(y-1)^2}{9} - \frac{(x+3)^2}{4} = 1$

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Lesson: Writing Equations of Hyperbolas

Examples Write the equation of the hyperbola with the following characteristics.

a. vertices at (0, 6) and (0, -6) with asymptotes $y = \pm\frac{1}{2}x$

b. foci at $\left(-3\sqrt{5}, 0\right), (3\sqrt{5}, 0)$ with asymptotes $y = \pm 2x$

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Examples Write the equation of the hyperbola with the following characteristics.

c. center at (1, -2) and focus at (-2, -2) and vertex at (-1, -2)

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Lesson: Eccentricity of Hyperbolas

How do you calculate the eccentricity of a hyperbola?

What can we say about the eccentricity of the hyperbola? What does it mean?

Example Find the eccentricity of the hyperbolas.

a. $\frac{x^2}{36} - \frac{y^2}{9} = 1$

b. $8y^2 - 16x^2 = 16$

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Example Find the equation of a hyperbola with $e = \frac{25}{9}$ and foci at (9, -1) and (-11, -1)

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Examples: Writing the Standard Form of a Hyperbola

Write each hyperbola in standard form.

a. $x^2 - 6x - y^2 + 4y = 1$

b. $y^2 - 14y - x^2 + 14x = 16$

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Lesson: Sequences

Define a sequence.

Examples Write the first five terms of each sequence.

a. $a_n = \frac{1}{n}$

b. $a_n = \frac{n \mp 1}{n+3}$

c. $a_n = (-1)^{n+1} * (2n + 1)$

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What does it mean for a sequence to converge?

What does it mean for a sequence to diverge?

What is a recursively defined sequence?

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Examples Find the first four terms in the sequence.

a. $a_1 = 3; a_n = 2a_{n-1} - 4$

b. $a_1 = 2; a_n = a_{n-1}^2 + n - 2$

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Lesson: Series

What is the summation sign and what does it mean? What do we called the “i” in this formula?

Define a finite series.

Define an infinite series.

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Examples Evaluate the following sums.

a. $\Sigma_{i=1}^{4}(3^i + 1)$

b. $\Sigma_{j=3}^{6} a_j$

c. $\Sigma_{k=1}^{4}(4x_k - 1)$ if $x_1 = 2, x_2 = 3, x_3 = 5, x_4 = 8$

d. $\Sigma_{i=1}^{3} f(x_i)\Delta x$, if $f(x) = 3x, x_1 = 1, x_2 = 4, x_3 = 7, \Delta x = 3$

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What are the summation properties?

What are the summation rules for summing up terms?

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Examples Use the summation properties and rules to evaluate each series.

a. $\Sigma_{i=1}^{50} 3$

b. $\Sigma_{k=1}^{25} 3k$

c. $\Sigma_{j=1}^{33} (4j^2 + 1)$

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Lesson: Arithmetic Sequences

What is an arithmetic sequence?

Give some examples of an arithmetic sequence.

Example Determine the common difference, d, for the arithmetic sequence:

$$-7, -5, -3, -1, \ldots$$

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Example Find the first five terms of each arithmetic sequence.

a. The first term is 3, and the common difference is -2.

b. $a_1 = -5, d = 7$

How do we define the general nth term of an arithmetic sequence?

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Example Determine a_{11} and a_n for the arithmetic sequence $-2, 2, 6, 10, \ldots$

Example Determine a_n and a_{20} for the arithmetic sequence having $a_3 = 9$, and $a_4 = 16$.

Example An arithmetic sequence has $a_7 = -15$ and $a_{14} = -35$. Determine a_1.

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Lesson: Arithmetic Series

When we sum up the terms of an arithmetic sequence, how can we define that sum? (sum of an arithmetic sequence)

Example Consider the arithmetic sequence $-7, -3, 1, 5, \ldots$

a. Evaluate S_{10}

b. Evaluate the sum of the first 50 positive integers.

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Example The sum of the first 15 terms of an arithmetic sequence is 345. If $a_{15} = 65$, find a_1 and d.

Examples Evaluate each sum.

a. $\Sigma_{i=1}^{19}(-3 - 5i)$

b. $\Sigma_{k=4}^{15}(2 + 4k)$

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Lesson: Geometric Sequences and Series

Define a geometric sequence:

Example Determine a_6 and a_n for the geometric sequence 4, 12, 36, 108, ….

Example Determine r and a_1 for the geometric sequence with $a_2 = -18$ and $a_5 = 486$

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How do we define the sum of the first n terms of a geometric sequence?

Example Evaluate: $\Sigma_{i=1}^{8} 3 * 2^i$

If we start with 2 and use a common ratio of $\frac{1}{2}$, what eventually happens as the series continues infinitely? Summarize the explanation below.

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Examples Evaluate the following infinite sums.

a. The first term is 1 and the common ratio is $\frac{1}{3}$

b. $\Sigma_{i=1}^{\infty}\left(-\frac{1}{2}\right)\left(-\frac{3}{4}\right)^{i-1}$

c. $\Sigma_{i=1}^{\infty}\left(\frac{3}{7}\right)^{i}$

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Lesson: The Binomial Theorem

Below we can observe the pattern for binomial expansion:

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2+2xy+y^2$

$(x+y)^3 = x^3+3x^2y+3xy^2+y^3$

$(x+y)^4 = x^4+4x^3y+6x^2y^2+4xy^3+y^4$

$(x+y)^5 = x^5+5x^4y+10x^3y^2+10x^2y^3+5xy^4+y^5$

Focusing on the VARIABLES x and y only, how can we generalize the exponents for the variables?

Pascal's Triangle can explain how to find the coefficient. Draw out Pascal's triangle below.

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Having to draw out this pattern would be very difficult for larger expansions, so we will explore a few ways to simplify how to expand this pattern.

First, we need to understand factorials.

n-factorial: for any positive integer n,

$$n! = n(n-1)(n-2)\dots(3)(2)(1)$$

By definition: $0! = 1$

Examples Evaluate:

a. $5!$

b. $8!$

Now that we understand factorials, we can discuss the formula to find binomial coefficients.

For nonnegative integers n and r, $r \leq n$, the binomial coefficient is defined as follows:

Examples Evaluate each binomial coefficient:

a. $\binom{5}{2}$

b. $\binom{6}{0}$

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We can summarize how to expand $(x + y)^n$ as follows:

1. There are n+1 terms in the expansion

2. The first and last terms are x^n and y^n respectively

3. In each succeeding term, the exponent on x decreases by 1 and the exponent on y increase by 1

4. The sum of the exponents on x and y in any term is n.

5. The coefficient of the term with $x^r y^{n-r}$ or $x^{n-r} y^r$ is $\binom{n}{r}$

These observations are captured in the binomial theorem.

State **The Binomial Theorem:**

Example Find the first, last, and seventh term of the binomial expansion: $(3x + 2y)^{10}$

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Example Use the binomial theorem to expand: $(a-b)^5$

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Not included (no notes file):

037 (Examples of Solving Absolute Value Equations)

042 (Examples on Absolute Value Inequalities)

878 (Examples: Putting Everything Together and Another Explanation of Special Cases, Slower-Paced)

013 (Examples of Simplifying Rational Expressions)

014 (Examples of How to Use the Multiplicative Property of -1 to Simplify Rational Expressions)

015 (Examples of Finding Domains of Rational Expressions)

018 (Examples of Multiplying and Dividing Rational Expressions)

022 (Examples of Finding Common Denominators for Rational Expressions)