**Lesson: A Few Basic Graphs**

There are a few graphs that are helpful to immediately know as we move forward. Let's break these down.

The Parabola: $f(x) = x^2$

Create table of relevant points and then sketch your own graph.

Absolute Value: $f(x) = |x|$

Create table of relevant points and then sketch your own graph.



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X cubed: $f(x) = x^3$

Create table of relevant points and then sketch your own graph.

Square Root: $f(x) = \sqrt{x}$

Create table of relevant points and then sketch your own graph.



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Cube Root: $f(x) = \sqrt[3]{x}$

Create table of relevant points and then sketch your own graph.



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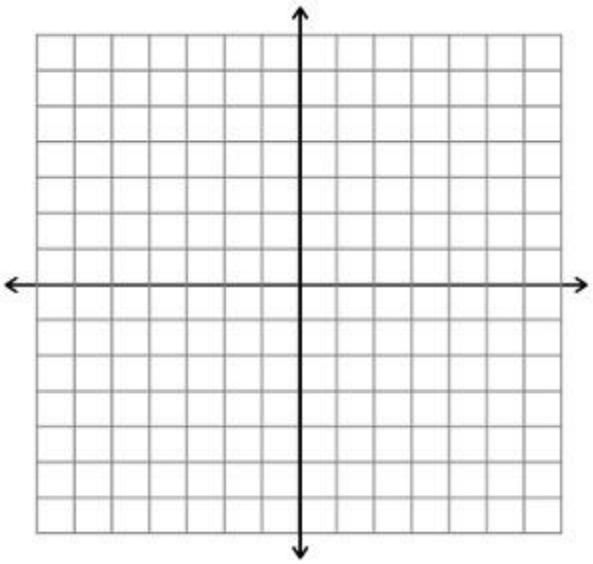
**Lesson: An Intro to Translations and Reflections of Graphs**

We are going to develop the intuition for this concept before we actually graph anything.

**Vertical Translations**

Draw the base graph of $f(x) = x^2$ in one color and then use some other colors to show how the translations work. Label the graphs.



Explain the general way that vertical shifts work:

**<u>Example</u>** How does $f(x) = \sqrt{x} + 6$ shift?



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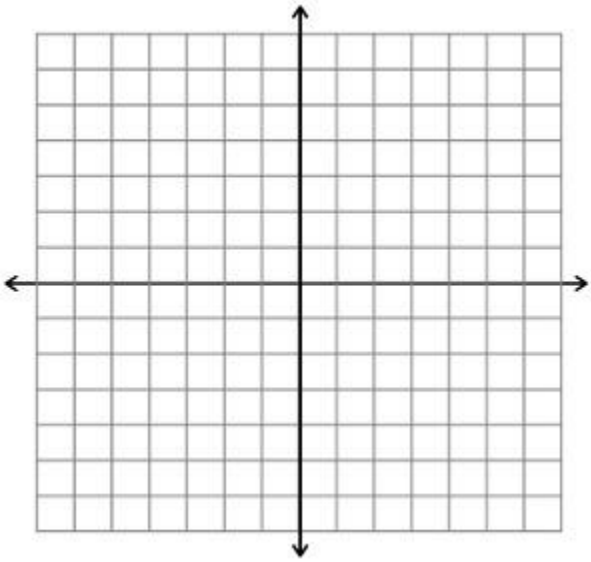
**Horizontal Translations**

Draw the base graph of $f(x) = |x|$ in one color and then use some other colors to show how the translations work. Label the graphs.



Explain the general way that horizontal shifts work:

**<u>Example</u>** How does $f(x) = \sqrt[3]{x+3}$ shift?



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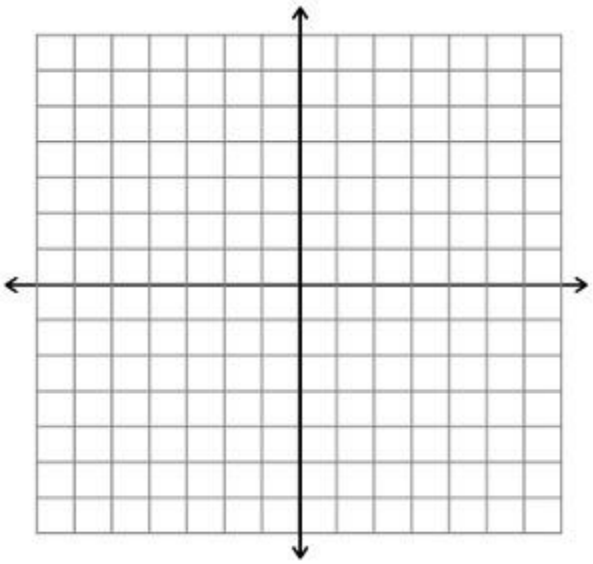
**Reflections Over the X-Axis**

Draw the base graph of $f(x) = \sqrt{x}$ and then graph the reflection in another color. Label the graph of the reflection. Label the graphs.



Explain the general way that reflections over the x-axis work:

**<u>Example</u>** How does $f(x) = -x^2$ look?



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**Example** Describe the translations and reflections of the following.

a. $f(x) = -x^2 + 3$

b. $g(x) = \sqrt{x+2} - 1$

c. $h(x) = (x-5)^3 + 2$

d. $f(x) = -|x+2| + 5$

e. $g(x) = -\sqrt[3]{x-1} - 4$



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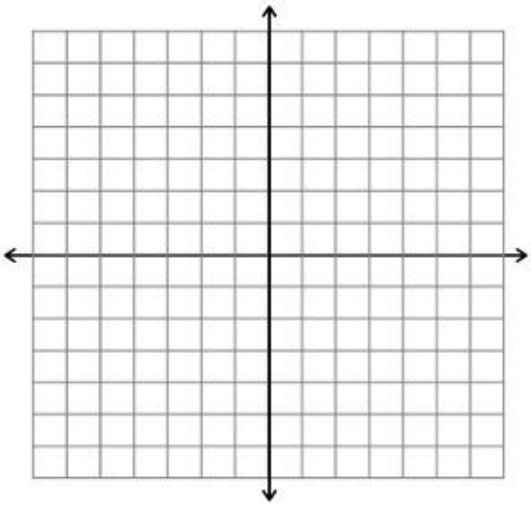
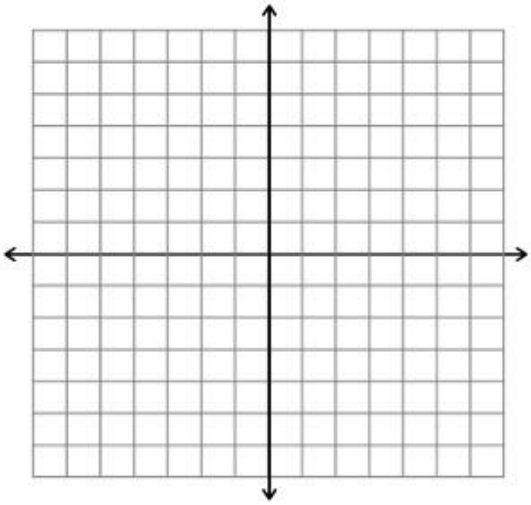
**Lesson: Graphing with Translations and Reflections**

**<u>Examples</u>** For each of the following, describe the translations and reflections. Then graph with 2-3 points.

a. $f(x) = x^2 + 1$
   Describe any translations or reflections:



b. $g(x) = \sqrt{x - 2}$
   Describe any translations or reflections:





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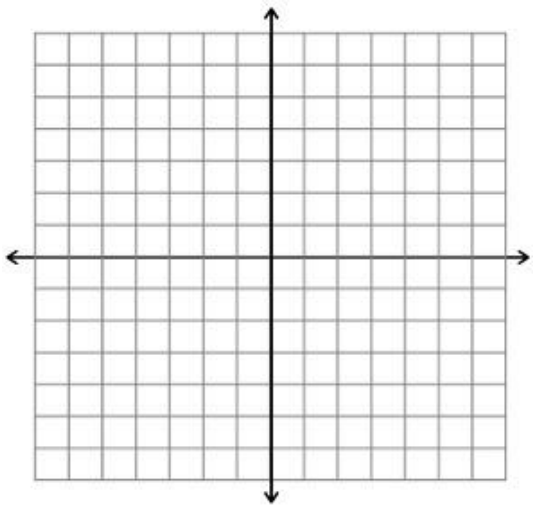
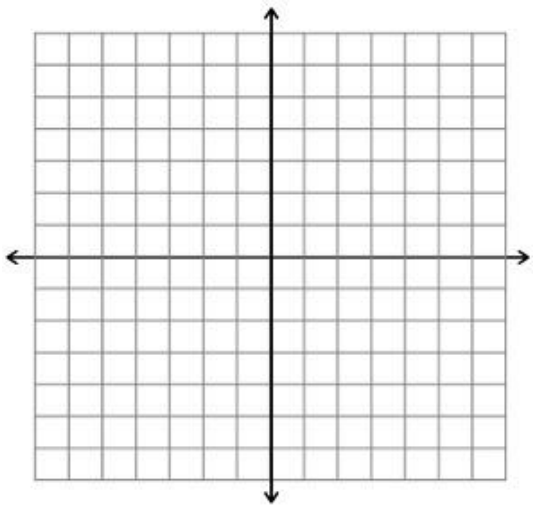
c. $f(x) = (x + 2)^3 - 1$
Describe any translations or reflections:



d. $h(x) = |x - 3| + 2$
Describe any translations or reflections:





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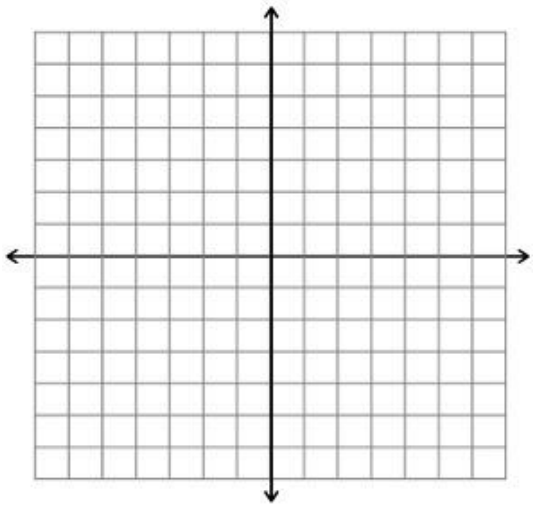
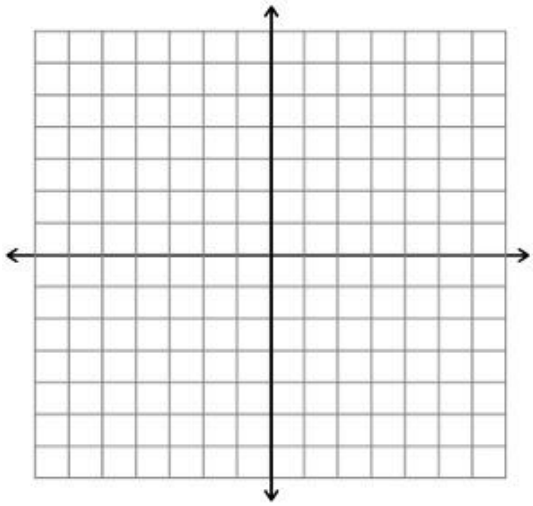
e. $f(x) = -|x - 3| - 2$
Describe any translations or reflections:



f. $f(x) = -\sqrt{x + 3} + 1$
Describe any translations or reflections:





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**Lesson: Writing Equations of Functions with Translations and Reflections**

**<u>Example</u>** Write the equation of a function $g(x)$ where the following transformations are performed on the graph of $f(x)$

a. $f(x) = \sqrt[3]{x}$ is shifted 3 units down and 2 units right

b. $f(x) = x^2$ is shifted 2.5 units up and $\pi$ units left

c. $f(x) = e^x$ is reflected about the x-axis, shifted 4 units down and 1 unit left



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**Lesson: The Greatest Integer Function**

Show how to graph the greatest integer function $f(x) = [\![x]\!]$

**<u>Example</u>** Graph $f(x) = \left[\!\left[\frac{1}{2}x - 2\right]\!\right]$



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**Lesson: Review of Lines and Linear Functions**

What makes a function **linear?** What are 2 forms of a linear function? How do you interpret this equation?

**Examples** What is the domain and range of $f(x) = 4x + 1$. Show how to graph it.

**Examples** What is the domain and range of $f(x) = 5$. Show how to graph it.

**Examples** What is the domain and range of $x = -4$. Show how to graph it.



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What is the **standard form** of a line?

**<u>Examples</u>** Give the domain and range of $3x - y = 6$. Show how to graph it.

How do you find the **slope** of a line?

What is the slope of a vertical line?

What is the slope of a horizontal line?



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**Examples** Find the slopes between the following sets of points.

a. $(5, 6)$ and $(-3, 2)$

b. $(-4, 7)$ and $(2, 7)$

c. $(6, 2)$ and $(6, -1)$

**Example** Find the slope of the following lines:

a. $4x - 2y = 8$

b. $-5x + 3y = -15$



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**Lesson: Review of Writing Equations of Lines**

What is the point-slope form of a line?

**<u>Example</u>** Write the equation of a line through the point $(4, 3)$ with slope $m = -2$.

**<u>Example</u>** Write the equation of a line that goes through the points $(2, 4)$ and $(-4, 10)$. Put your answer in standard form.

What is the slope-intercept form of a line?



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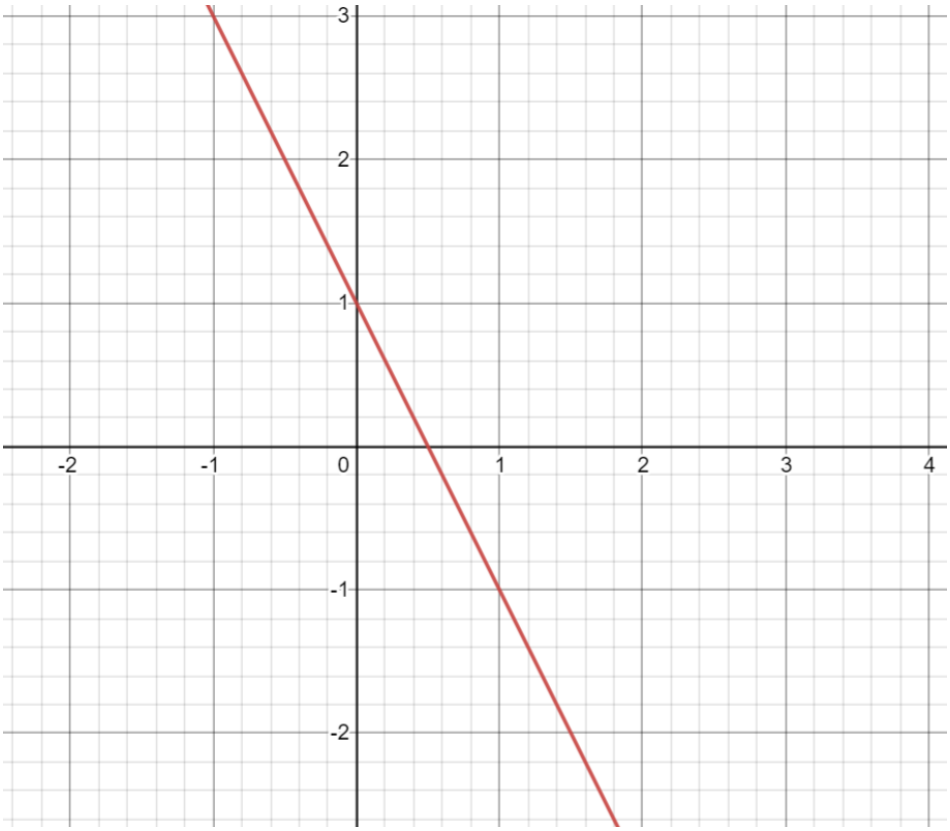
**Example** Show how to write $4x - 6y = 12$ in slope-intercept form.

**Example** Find the slope, x- and y-intercept, and equation of the line:



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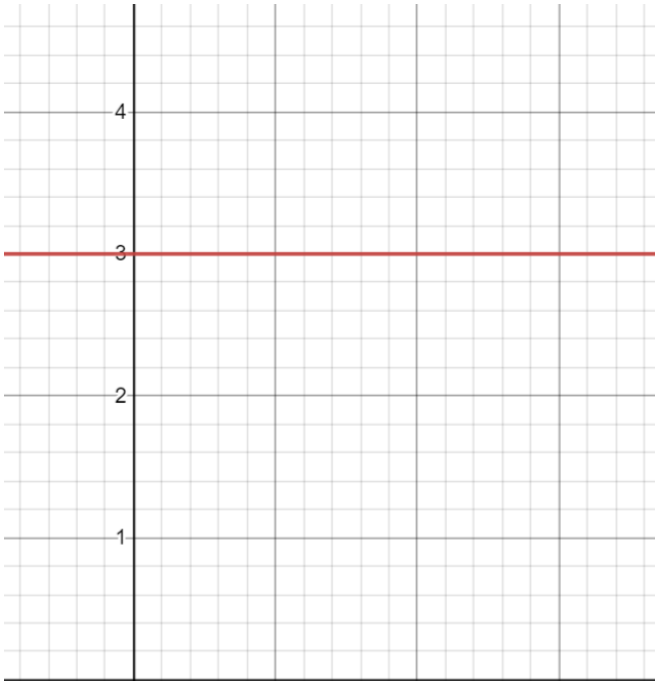
What is the form of a vertical line?

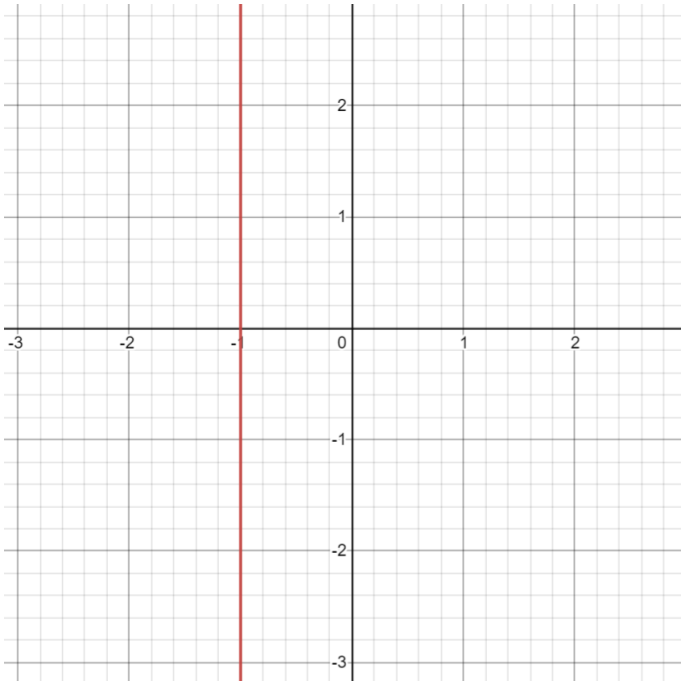
What is the form of a horizontal line?

**<u>Example</u>** Determine the equation of the following lines.

a.




b.




What makes 2 lines parallel?

What makes 2 lines perpendicular?



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**Examples** Write the equation of a line that goes through $(-2, -5)$ that satisfies the conditions:

a. parallel to the line: $3x - 2y = -6$

b. perpendicular to the line: $x - 2y = -4$



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**Quick Reference of Line Formulas**

Slope-Intercept Form

Standard Form

Point-Slope Form

Slope Formula

Form of a Horizontal Line

Form of a Vertical Line

What makes 2 lines parallel?

What makes 2 lines perpendicular?



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**Examples: Writing the Equation of a Line Through 2 Points**

**<u>Example</u>** Write the equation of a line through the given 2 points. Put your answer in standard form, and your standard form should not include any fractions or decimals.

a. $(4, 5)$ and $(2, 9)$

b. $(2, 7)$ and $(-4, 7)$

c. $(-4, 2)$ and $(2, 5)$

d. $(3, 5)$ and $(3, 8)$



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**Examples: Writing the Equation of a Line Through 2 Points**

**<u>Example</u>** Write the equation of a line through the point $(2, 3)$ that satisfies the given conditions. Your final answer should be in standard form with no fractions or decimals.

a. Parallel to $5x - 2y = 10$

b. Perpendicular to $4x - 2y = 1$

c. Parallel to $y = 5$

d. Perpendicular to $x = 7$



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**Lesson: A Quick Review of the Basics of Solving Systems of Linear Equations**

Systems of equations are 2 or more equations in 2 or more variables. They can look like:

$\begin{cases} 2x + 3y = 6 \\ x - 3y = 3 \end{cases}$ or $\begin{cases} 2x - y + z = 8 \\ 5x + 2y - z = 9 \\ x - 4y + z = 10 \end{cases}$ or $\begin{cases} 3x + 2y - z = 7 \\ x - 4y + 2z = 8 \end{cases}$

A solution of a system is an ordered pair that is true in each equation of the system. Some systems have no solutions, and some systems have infinite solutions.

In this lesson, we will briefly review the main techniques to solve systems. We will focus on systems that have a solution.

**<u>Example</u>** Use the substitution method to solve: $\begin{cases} 3x - y = -4 \\ x + 3y = 12 \end{cases}$

**<u>Example</u>** Use the elimination method to solve: $\begin{cases} 3x - y = -4 \\ x + 3y = 12 \end{cases}$



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**<u>Example</u>** Solve: $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 4 \\ \frac{3x}{2} + \frac{3y}{2} = 15 \end{cases}$



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**<u>Example</u>** Solve: $\begin{cases} 2x + y + z = 9 \\ -x - y + z = 1 \\ 3x - y + z = 9 \end{cases}$



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**Examples: Systems of 3 Linear Equations with 3 Variables**

Solve: $\begin{cases} x + y + z = 2 \\ 2x + y - z = 5 \\ x - y + z = -2 \end{cases}$



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Solve: $\begin{cases} x + 2y + 3z = -11 \\ 3x - y + z = 0 \\ -2x + 3y - z = 4 \end{cases}$



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**Lesson: Systems with No Solutions or Infinite Solutions**

**<u>Example</u>** Solve: $\begin{cases} x + 4y - 3z = 2 \\ 2x - 5y + 2z = -8 \\ -3x - 12y + 9z = 7 \end{cases}$

**<u>Example</u>** Solve: $\begin{cases} 7x + 2y = 6 \\ 14x + 4y = 12 \end{cases}$



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**<u>Example</u>** Solve: $\begin{cases} 5x - 4y + z = 9 \\ y + z = 15 \end{cases}$

**<u>Example</u>** $\begin{cases} 3x - 2y + z = 15 \\ x + 4y - z = 11 \end{cases}$



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**<u>Example</u>** Solve: $\begin{cases} 2x + y - 3z = 0 \\ 4x + 2y - 6z = 0 \\ x - y + z = 0 \end{cases}$



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**Lesson: Using Matrices to Solve Systems of Linear Equations**

What is a matrix?

Show how to setup the associated **augmented matrix** for the system: $\begin{cases} 3x + 4y = 4 \\ x - y = 13 \end{cases}$

What are the dimensions of this matrix?

In general, how do we state dimensions of a matrix (circle): rows x columns columns x rows

**<u>Example</u>** What is the dimension of

a.

$$\begin{bmatrix} 3 & 2 & 4 & 1 \\ 7 & 8 & 1 & 2 \\ 9 & 5 & -2 & -3 \end{bmatrix}$$

b.

$$\begin{bmatrix} 2 & 4 \\ 5 & 3 \\ 8 & 3 \\ -1 & -4 \end{bmatrix}$$



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With matrices, there are 3 **row transformations** that we can make. We are not allowed to do anything else to a matrix. Write down these 3 transformations.

Now, recall, our goal in this lesson is to use a matrix to solve systems of linear equations. What do we call this specific solving process?

What do we call the “ideal form” of a matrix in its solved form?



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The Gauss-Jordan method has a goal of transforming a matrix into reduced-row echelon form or diagonal form. Draw a few examples of how the FINAL matrix should look after we transform it:

When we use the Gauss-Jordan method, which direction do we want to work on the matrix?

What is the general strategy to remember with this method?



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**Example** Outline the steps for the Gauss-Jordan method for the system: $\begin{cases} -2x + 4y = -4 \\ x - y = 4 \end{cases}$

**Example** Use Gauss-Jordan method for the system: $\begin{cases} 3x + 4y = 4 \\ x - y = 13 \end{cases}$



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**Lesson: Using The Gauss-Jordan Method for a System with 3 Equations**

**<u>Example</u>** Use the Gauss-Jordan method for the system: $\begin{cases} x + y + z = 2 \\ 2x + y - z = 5 \\ x - y + z = -2 \end{cases}$



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Another page for you to show your work …



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**Lesson: Using Matrices to Solve Systems of Linear Equations when There is No Solution or Infinite Solutions**

**<u>Example</u>** Use the Gauss-Jordan method to solve: $\begin{cases} 3x + 5y - z = -2 \\ 4x - y + 2z = 1 \\ -6x - 10y + 2z = 0 \end{cases}$

**<u>Example</u>** Use the Gauss-Jordan method to solve: $\begin{cases} 3x + 5y = -2 \\ 9x + 15y = -6 \end{cases}$



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**<u>Example</u>** Use the Gauss-Jordan method to solve: $\begin{cases} x - 2y + 3z = 6 \\ 2x - y + 2z = 5 \end{cases}$



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**<u>Example</u>** Use the Gauss-Jordan method to solve: $\begin{cases} 5x - 4y + z = 0 \\ x + y = 0 \\ -10x + 8y - 2z = 0 \end{cases}$



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**Examples: Using Matrices to Solve Systems of Linear Equations (any type of solutions)**

Use the Gauss-Jordan method to solve the following.

a. $\begin{cases} x + 4y - z = 6 \\ 2x - y + z = 3 \\ 3x + 2y + 3z = 9 \end{cases}$



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b. $\begin{cases} 2x + y - 3z = 0 \\ 4x + 2y - 6z = 0 \\ x - y + z = 0 \end{cases}$



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c. $\begin{cases} 3x - 2y + z = 15 \\ x + 4y - z = 11 \end{cases}$



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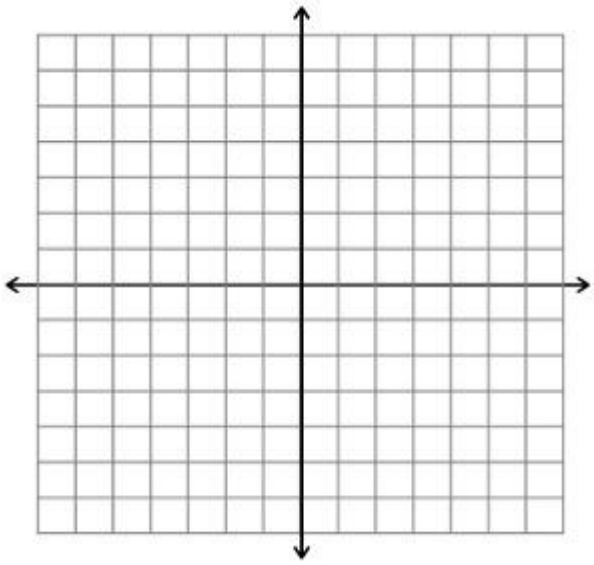
**Lesson: Understanding the Graphs of Quadratic Functions**

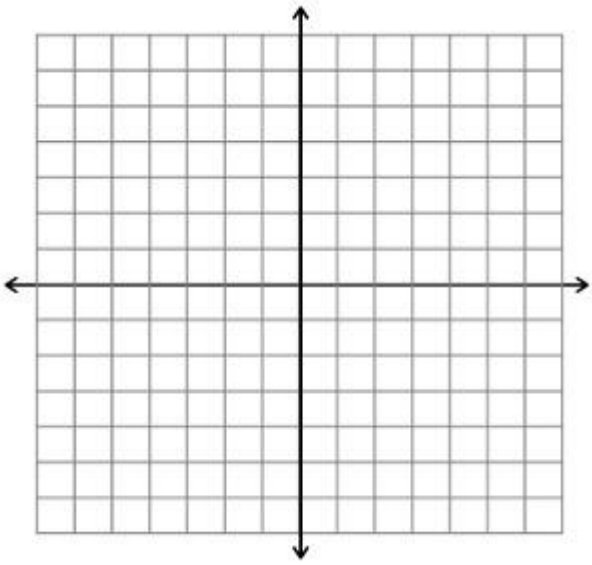
Quadratic functions are functions of the form _____, where $a \neq 0$. But in previous lessons we worked on graphing quadratic functions, ie parabolas in another way. Let's review that first.

**<u>Examples</u>** Graph the following:

a. $f(x) = (x+2)^2 - 3$

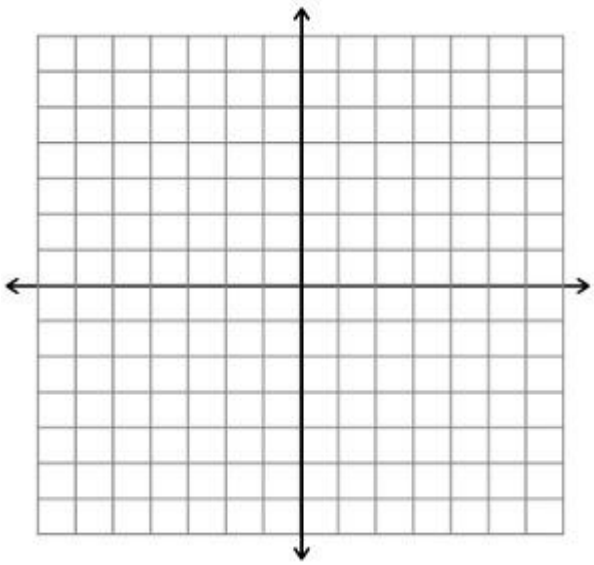


b. $g(x) = -(x-3)^2 + 1$

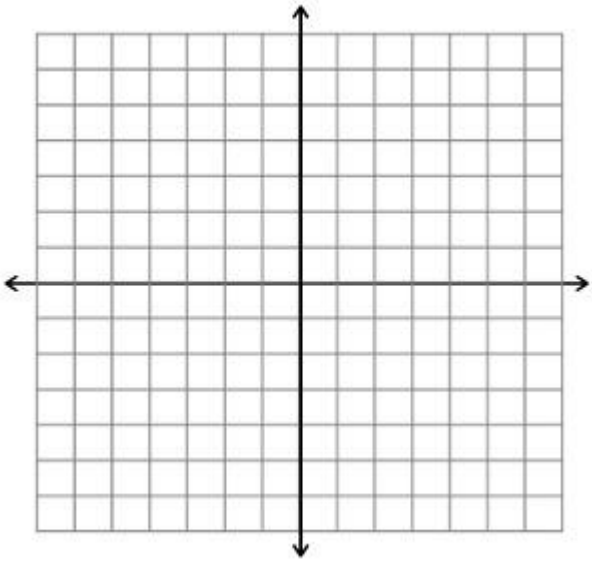


To begin, we will add on one more way to manipulate the graph of a parabola. Explain how to graph the follow:

a. $f(x) = 3x^2$
How do you interpret the 3?



b. $g(x) = -\frac{1}{2}(x-2)^2 + 1$
How do you interpret the $-\frac{1}{2}$?





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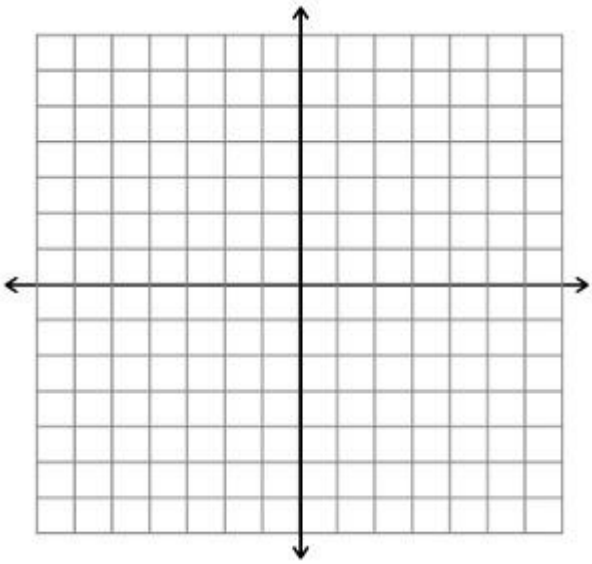
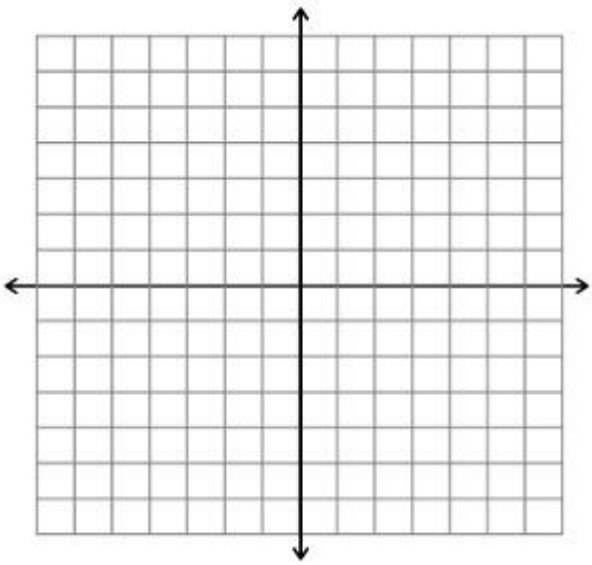
**Examples** Graph the following:

a. $f(x) = -2(x+1)^2 + 3$



b. $g(x) = \frac{1}{3}(x-2)^2 - 2$



Using all of the examples we have discussed, what is another equation form for the parabola?

Explain some of the characteristics of a parabola that we get from this format:



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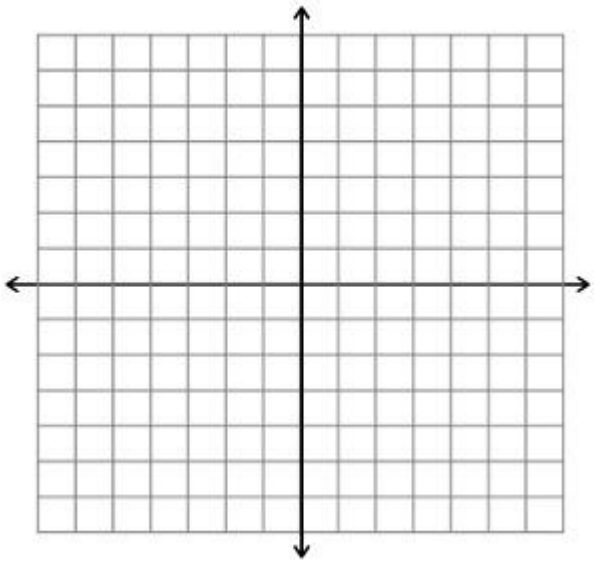
**Examples** For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x-3)^2 - 4$



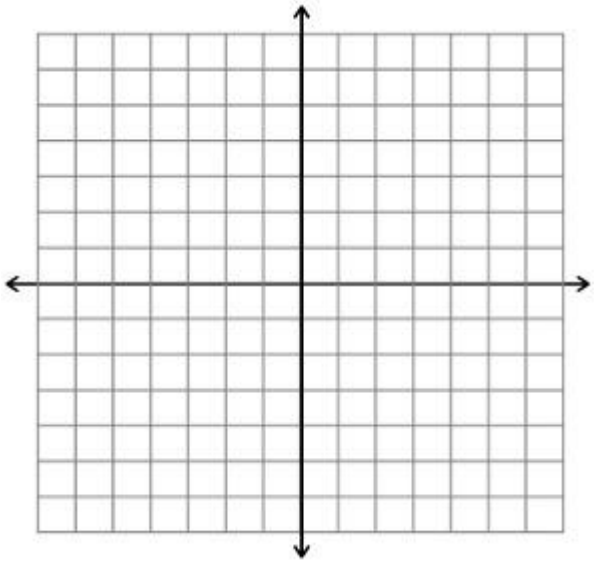
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2 + 8$



Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:



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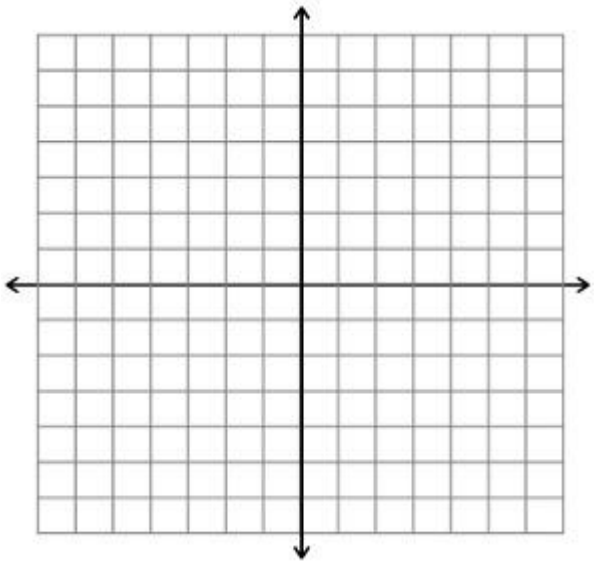
**Examples** For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{4}(x+2)^2 - 3$



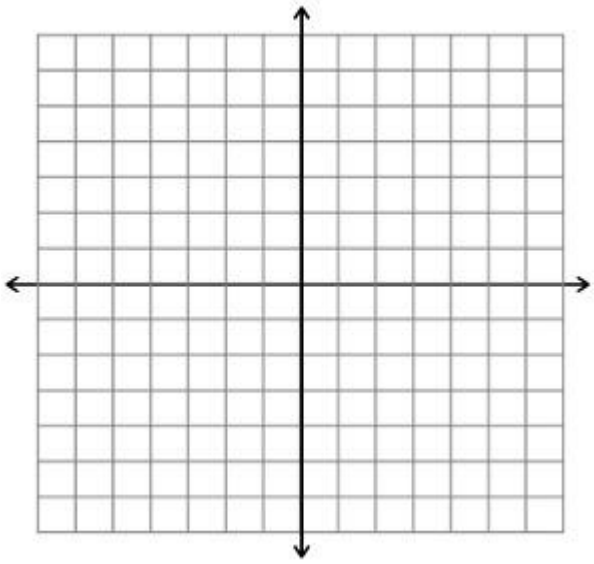
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -\frac{1}{3}(x-1)^2 + 3$



Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:



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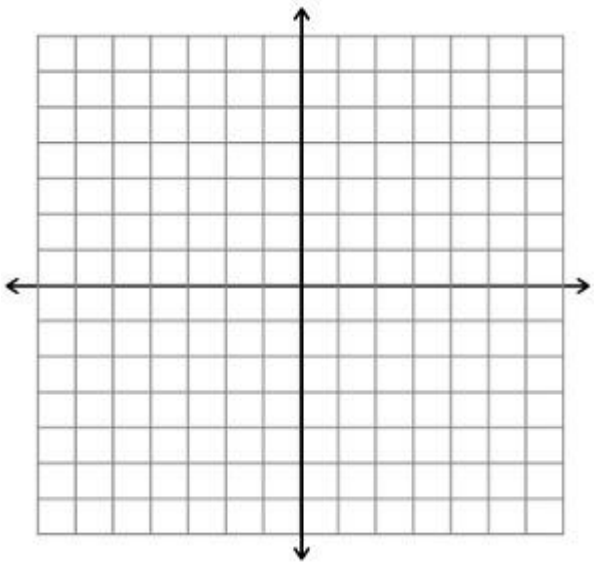
**Examples: Understanding How Many X-Intercepts There Are**

**<u>Examples</u>** For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{2}(x+1)^2 - 8$



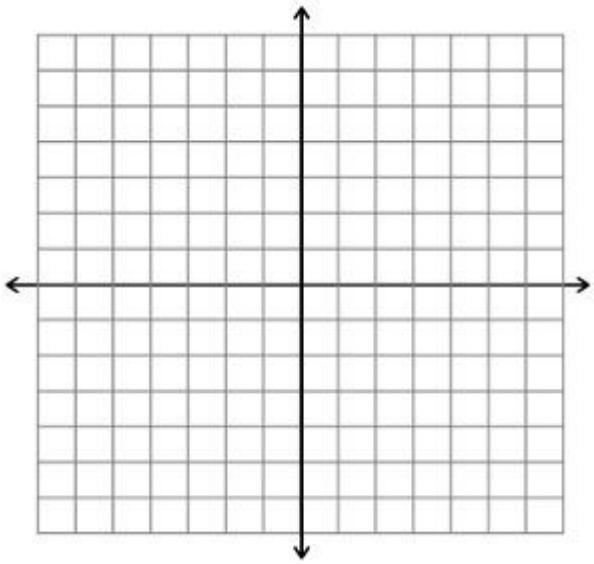
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2$



Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:



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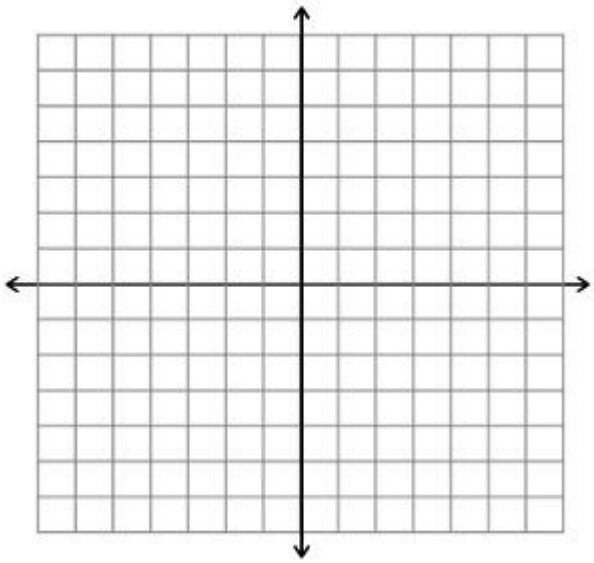
**Examples** For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x-2)^2 + 1$



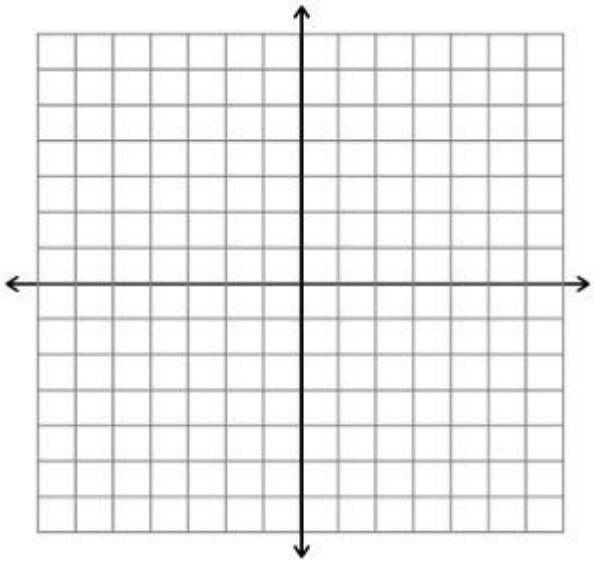
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -(x+3)^2 - 1$



Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:



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**Lesson: Finding the Vertex of Quadratic Equations in the form $\boldsymbol{f(x) = ax^2 + bx + c}$**

What is the vertex formula?

Show how to use the vertex formula for $f(x) = 3x^2 + 6x - 2$

**<u>Examples</u>** Find the vertex of the following:

a. $f(x) = -x^2 + 4x - 2$

b. $f(x) = -\frac{1}{2}x^2 + 3x - 1$



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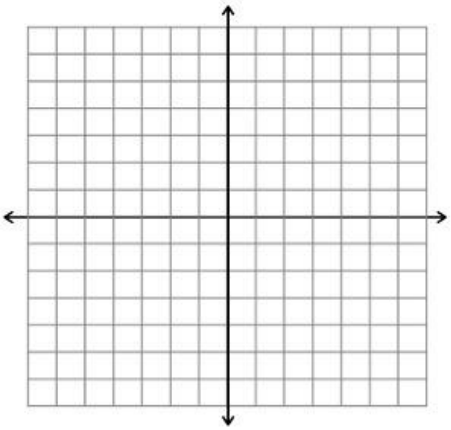
**Lesson: Putting Quadratics into the $f(x) = a(x-h)^2 + k$ Form**

Before you begin this lesson, you should already feel comfortable graphing an equation like $f(x) = 2(x-1)^2 + 3$. Graph this below:



Now show all the steps to put $f(x) = 2x^2 - 8x + 2$ into $f(x) = a(x-h)^2 + k$ form:



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**Examples** Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = x^2 + 4x - 2$

b. $g(x) = x^2 + 6x + 10$

c. $h(x) = -x^2 + 2x - 4$

d. $y = 3x^2 + 6x - 12$



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**Examples: Putting Quadratics into the $f(x) = a(x-h)^2 + k$ Form**

**<u>Examples</u>** Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = \frac{1}{2}x^2 + x - 3$

b. $g(x) = -\frac{1}{3}x^2 + 4x - 2$

c. $y = x^2 + 3x + 2$



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**Lesson: Review of Quadratic Functions**

A polynomial function of degree n is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Where $a_n, a_{n-1}, \ldots, a_1, a_0$ are any complex or real number, $a_n \neq 0$, and $n$ is a nonnegative integer.

What do we call $n$ in this equation?

What do we call $a_n$ in this equation?

What do we call $a_n x^n$ in this equation?

**<u>Example</u>** What is the degree, leading coefficient, and dominating term in the following?

a. $f(x) = 4x^4 - 5x^2 + x^7 + 100$

b. $g(x) = x\sqrt{3} - 5$

c. $h(x) = 7$



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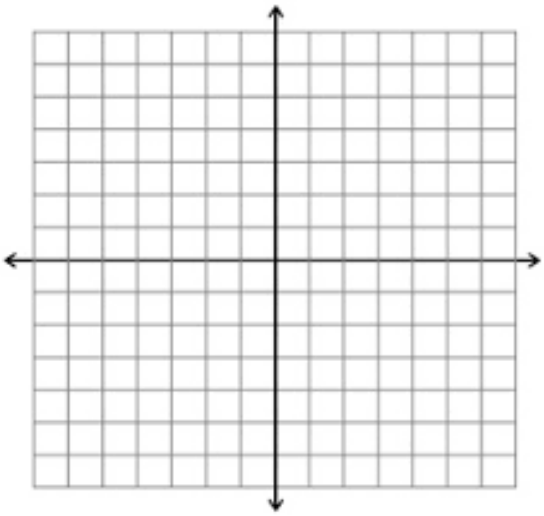
What is a quadratic function?

What do we call the graph of a quadratic function?

Draw and label the parts of a parabola.

**<u>Example</u>** Sketch the graph of $f(x) = -(x+4)^2 - 2$



What is the **vertex form** of the equation of the parabola?



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**Example** Show how to use completing the square to put $f(x) = x^2 + 6x - 4$ in vertex form. Write out clear directions.

**Example** Use completing the square to write $f(x) = 3x^2 + 5x + 2$ in vertex form, then sketch the graph.



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**<u>Example</u>** Find the intercepts of $f(x) = 3x^2 + 5x + 2$

How many x-intercepts can a parabola have? Draw a little sketch of each situation.

How can you tell how many intercepts a parabola will have if the parabola is in the form: $f(x) = ax^2 + bx + c$



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What is the vertex formula for a parabola in the form $f(x) = ax^2 + bx + c$?

**<u>Example</u>** Use the vertex formula to find the vertex of the following: $f(x) = 2x^2 - 4x + 1$



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**Examples: Finding Intercepts, Vertices, Etc of Quadratic Functions**

Find the vertex, axis of symmetry, domain, range, intercepts, and intervals of increasing and decreasing. If the function is in $f(x) = ax^2 + bx + c$ form, put it in vertex form. Sketch the graph. If the function is in $f(x) = a(x-h)^2 + k$ form, put it into standard form.

a. $f(x) = -\frac{1}{2}x^2 - 3x - \frac{1}{2}$



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b. $f(x) = 2(x-1)^2 + 3$



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**Lesson: Determining Whether a Quadratic Function will have a Minimum or a Maximum**

Using the form $f(x) = ax^2 + bx + c$, explain what determines whether the parabola opens up or down:

**<u>Examples</u>** Determine whether the following quadratics open upwards or downwards, and then determine their vertex:

a. $f(x) = -2x^2 + 4x - 2$ upwards downwards

b. $g(x) = -\frac{1}{2}x^2 - 2x + 2$ upwards downwards

c. $h(x) = x^2 + 6x + 1$ upwards downwards



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**Lesson: Applications of Quadratic Functions 1**

A ball is thrown upwards from a cliff 24 feet about the ground. The height h of the ball, in feet, t seconds after it is thrown is given by:

$$h(t) = -16t^2 + 32t + 24$$

How long does it take the ball to reach its maximum height?

What is the maximum height attained by the ball?



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**Solving Quadratic Equations with Factoring**

In this video, we will look at how to solve equations by factoring, which looks like this:

$$x^2 + 5x + 6 = 0 \rightarrow (x + 2)(x + 3) = 0$$

Quadratic equations are equations of the form:

Where a, b, and c are real numbers and $a \neq 0$.

How would you solve the quadratic equation $x^2 + 7x + 6 = 0$ by factoring?

**<u>Examples</u>** Solve the following by factoring:

a. $x^2 - x - 12 = 0$

b. $x^2 + 9 = -6x$

c. $x^2 = 25$



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**<u>Examples</u>** Solve the following by factoring:

a. $x^2 + 7x = 0$

b. $\frac{1}{10}x^2 + \frac{1}{2}x + \frac{3}{5} = 0$

c. $2x^3 + 28x^2 + 90x = 0$



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**Solving Quadratic Equations using the Square Root Property**

In this video, we will look at how to solve equations by using the square root property, which looks like this:

$$(x+2)^2 = 9 \quad \rightarrow \quad x+2 = \pm 3$$

Quadratic equations are equations of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

What are the solutions to $x^2 = 25$?

In general, how do you solve equations of the form $Q^2 = k$, where Q is an expression and k is a constant? In other words, what is the **square root property**?

**<u>Examples</u>** Solve the following equations using the square root property:

a. $(x+3)^2 = 25$

b. $(x+3)^2 = 12$



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**Examples** Solve the following equations using the square root property:

a. $(5x-2)^2+5=29$

b. $(6x+7)^2+4=-41$



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**Solving Quadratic Equations by Completing the Square**

In this video, we will look at how to solve equations by factoring, which looks like this:

$x^2 + 6x = 7 \qquad b = 6, \quad \frac{1}{2}b = 3, \quad \left(\frac{1}{2}b\right)^2 = 9$

Quadratic equations are equations of the form $ax^2 + bx + c = 0$ where a, b, and c are real numbers and $a \neq 0$.

Before we begin, you should be familiar with the square root property. Solve $(2x + 1)^2 = 49$ using the square root property.

First let's talk about how to complete the square. Show how to complete the square on $x^2 + 6x$ below:



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**Examples** Complete the square on the following:

a. $x^2 + 8x$

b. $x^2 - 10x$

c. $x^2 + 3x$

d. $x^2 - \frac{1}{2}x$

e. $x^2 + \frac{2}{3}x$



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Show how to solve $2x^2 + 12x - 16 = 0$ by completing the square:



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**Examples** Solve the following by completing the square:

a. $x^2 + 4x - 6 = 0$

b. $x^2 + 5 = 2x$



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**Examples** Solve the following by completing the square:

a. $3x^2 + 15x = -6$

b. $\frac{1}{2}x^2 + 3x - 2 = 0$



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**Lesson: How to Solve Quadratic Equations with Factoring**

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, and a, b, and c are real numbers.

There are several ways that you can solve quadratic equations, but in this lesson we will focus on how to use factoring as a way to solve them.

Solving a quadratic equation by factoring requires a rule called the **Zero Product Rule.**

Write down the Zero Product Rule:

**<u>Examples</u>** Solve the following equations using the zero product rule:

a. $3x = 0$

b. $3(x + 1) = 0$

c. $(x + 3)(x + 1) = 0$

d. $x(2x + 1)(3x + 5) = 0$



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Show how to use factoring to solve $x^2 + 5x + 6 = 0$.

**<u>Examples</u>** Solve the following by using factoring:

a. $x^2 + 12x - 45 = 0$

b. $x^2 - 12x + 11 = 0$

c. $2x^2 + 5x + 3 = 0$

d. $55x = -20x^2 - 30$

e. $x^2 = 25$

f. $x^2 = 9x$



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**Lesson: Tips and Tricks to Help with Harder Quadratics Solved by Factoring**

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, and a, b, and c are real numbers.

In the previous lesson, we talked about the basics of using factoring to solve quadratics.

In this lesson, we will look at some trickier examples. First, let's warm up by solving the following:

a. $4x = 14x^2 - 48$

b. $3x^3 = 12x^2$

Sometimes we will have to do further work to simplify the problem before we factor.

**<u>Examples</u>** Use factoring to solve.

a. $(x + 3)(x + 4) = 6$

b. $5x(x + 2) - 4x = 4(x^2 - 2)$

c. $(x + 4)^2 - 2x = 11$



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What can be done to solve $\frac{1}{3}(x+1)^2 - \frac{1}{4}x = \frac{1}{6}(x-2)^2 - \frac{1}{3}$?

**<u>Examples</u>** Use factoring to solve.

a. $\frac{1}{4}(2x-3)^2 + \frac{1}{4}x = \frac{1}{4}(x-5)^2 - \frac{3}{2}$

b. $\frac{1}{4}x(2-x) - \frac{3}{4} = \frac{1}{5}x(x+1) - \frac{7}{10}$



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**How to Evaluate Square Roots that Have Imaginary Number Answers and Dealing with Negatives in Square Roots**

In this video, we will look at how to evaluate things like this:

$\sqrt{-4}$ or $\sqrt{-12}$ or $\sqrt{-3} * \sqrt{-6}$

What is an imaginary number?

What is a complex number?

How do you evaluate something like $\sqrt{-9}$?

**<u>Examples</u>** Compute the following:

a. $\sqrt{-25}$

b. $\sqrt{-81}$

c. $\sqrt{-12}$

d. $\sqrt{-7}$

e. $\sqrt{-\frac{25}{16}}$

f. $\sqrt[3]{-8}$

g. $\sqrt[4]{-16}$

h. $\sqrt{-32}$



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How do you evaluate problems like $\sqrt{-3} * \sqrt{-2}$?

**<u>Examples</u>** Evaluate the following:

a. $\sqrt{-6} * \sqrt{-5}$

b. $\sqrt{-6} * \sqrt{-2}$

c. $\sqrt{-10} * \sqrt{-15}$

d. $\frac{\sqrt{-6}}{\sqrt{-2}}$

e. $\frac{\sqrt{-50}}{\sqrt{-2}}$

f. $\frac{\sqrt{-7}}{\sqrt{-2}}$



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**Operations with Imaginary Numbers**

In this video, we will look at how to evaluate things like this:

$2i(3i-5)$ or $(4i+1)-(5i+3)$ or $\frac{4}{3+2i}$

**Adding and Subtracting with Imaginary Numbers:**
Use the example $(2i+3)+(5i-6)$ to show how to add or subtract with imaginary numbers.

**<u>Examples</u>** Simplify:
a. $(3i-2)-(4i-9)$

b. $(5i+3)+(2i-1)$

What can we say about $i^2$?



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With that in mind, explain what is unique about multiplying: $2i(3i+1)$

**<u>Examples</u>** Simplify:

a. $4(2-3i)$

b. $6i(3i+2)$

c. $5i(2-8i)$

d. $(5i+1)(2i-3)$

e. $(4i-3)(6i+7)$



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**Examples** Simplify:

a. $(5i + 2)^2$

b. $(3i - 5)^2$

Why do you need to rationalize $\frac{3}{2i+1}$? Explain how to do this.

**Examples** Rationalize the following:

a. $\frac{4}{2i+6}$

b. $\frac{7}{i+3}$



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**Examples** Rationalize the following:

a. $\frac{3i+1}{5i-2}$

b. $\frac{7i+3}{2i+1}$

c. $\frac{i-3}{5i+3}$



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**Evaluating Higher Powers of Imaginary Numbers**

In this video, we will look at how to evaluate things like this:

$i^{21}$ or $i^{54}$

First we need to investigate the powers of $i$ and the patterns with it. Write this down below:

Now that we can see a pattern, explain how to evaluate the following:

$i^{15}$

$i^{22}$



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**<u>Examples</u>** Evaluate the following:

a. $i^{36}$

b. $i^{43}$

c. $i^{52}$

d. $i^{33}$

e. $i^{25}$



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**Lesson: A Review of Imaginary Numbers**

*Please note:* Knowing imaginary numbers if one of the "prerequisite" topics for this course. Ie – you should already know this (though I also realize you probably need a review!). If you have *never* seen imaginary numbers before, then I strongly suggest that you watch my series on imaginary numbers (because, yes, I have one). So if you felt like this went too fast and you want more details, please take responsibility for your education and go back to review this!

How do we represent an imaginary number?

What is $i^2$?

What is a complex number?

When are 2 complex numbers equal?

What does $\sqrt{-a}$ mean?

**Examples** Write using the definition of $\sqrt{-a}$

a. $\sqrt{-12}$

b. $\sqrt{-25}$



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**Examples** Complete the following operations.

a. $\sqrt{-5} * \sqrt{-3}$

b. $\frac{\sqrt{-12}}{\sqrt{-18}}$

c. $\frac{\sqrt{-50}}{\sqrt{9}}$

To add or subtract complex numbers, we can add/subtract the real parts and the imaginary parts similar to working with like terms.

**Examples** Find the sum or difference.

a. $(2 - 6i) + (8 + 2i)$

b. $(5 - 7i) - (-4 + 2i)$



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When multiplying complex numbers, recall that $i * i = i^2 = -1$!

**<u>Examples</u>** Multiply.

a. $(2 + 4i)(3 - 5i)$

b. $(4 - 5i)(3 - 2i)$

c. $(3 + 2i)(3 - 2i)$

Recall, the conjugate is found by using the opposite side for the second term!

**<u>Examples</u>** What is the conjugate of the following?

a. $4 + 2i$

b. $-3 - 4i$

c. $i$



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To find the quotient of 2 complex numbers, we multiply by the conjugate of the denominator.

**Examples** Find each quotient. Write your answer in *standard form,* ie $a + bi$ form

a. $\frac{3-4i}{2+i}$

b. $\frac{5}{i}$

How do the powers of $i$ cycle through?

**Examples** Show how to simplify the following powers of i. Write your work in great detail so that you remember how to do this!

| | |
|---|---|
| a. $i^{17}$ | c. $i^{-14}$ |
| b. $i^{28}$ | d. $i^{-5}$ |



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**<u>Examples</u>** The following quadratic equations have imaginary solutions. Show how to solve them.

a. $x^2 = -25$ using the **square root property**

b. $x^2 + 75 = 0$ using the **square root property**

c. $2x^2 - 3x = -6$ using the **quadratic formula**

d. $x^4 - 16$ using **factoring** and also the **square root property**



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**Examples: The Distance Formula**

Find the distance between the following sets of points.

a. (6, 2) and (2, 11)

b. (3, -1) and (-4, -2)



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**Solving Quadratic Equations by the Quadratic Formula**

Complete the square on: $ax^2 + bx + c = 0$

State the quadratic formula:



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**Examples** Use the quadratic formula to solve the following:

a. $x^2 + 3x + 2 = 0$

b. $2x^2 + 3x = 5$

c. $\frac{1}{2}x^2 + \frac{1}{3}x + \frac{5}{6} = 0$



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**Lesson: The Discriminant**

Restate the Quadratic Formula, and then label the discriminant.

| Value of the Discriminant | Interpretation |
|---|---|
| | |
| | |
| | |
| | |



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**Examples** Find the values of the discriminant for the following equations. Then solve the equations (using the most efficient method) to determine if this makes sense.

a. $x^2 - 4x + 4 = 0$

b. $x^2 + 6x + 5 = 0$

c. $3x^2 + 2x + 5 = 0$



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**Choosing the Best Method to Solve a Quadratic**

There are 4 ways we can solve quadratics:

1.

2.

3.

4.

Using each of these methods only once, determine the best way to solve these quadratics:

a. $x^2 + 6x = 2$
b. $x^2 + 3 = 2x$
c. $(2x + 1)^2 = 49$
d. $3x^2 + 2 = 3x$



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**<u>Examples</u>** Using each of these methods only once, determine the best way to solve these quadratics:

a. $(3x-2)^2=-24$

b. $2x^2+5x-1=0$

c. $x^2-12=x$

d. $x^2-2x+5=0$



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**Solving Equations that End Up Having Quadratics**

In this lesson, we will review some techniques from previous lessons.

Solve $\frac{2}{x} - \frac{1}{x+1} = \frac{2}{3}$

Solve $\sqrt{2x+8} = x + 4$



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**<u>Examples</u>** Solve the following:

a. $\frac{4}{x-2}+\frac{x}{x-3}=\frac{2}{x-3}$

b. $\sqrt{3x+5}=x-3$



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**Solving Equations that Are Similar to Quadratics**

In this lesson, we will look at how to solve equations that look like this:

$$x^{\frac{2}{3}} - x^{\frac{1}{3}} - 12 = 0 \qquad \text{or} \qquad (2x+3)^2 + 5(2x+3) + 6 = 0$$

Before we begin, what do the following equations have in common if you **look at their exponents**?

a. $x^2 + 5x + 6 = 0$

b. $x^{\frac{2}{3}} + 5x^{\frac{1}{3}} + 6 = 0$

c. $x + 5x^{\frac{1}{2}} + 6 = 0$

d. $x^4 + 5x^2 + 6 = 0$

Explain how to solve $x^4 - 13x^2 + 36 = 0$



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**<u>Examples</u>** Solve the following:

a. $x^{\frac{2}{3}} - 2x^{\frac{1}{3}} - 15 = 0$

b. $x - 2x^{\frac{1}{2}} - 8 = 0$

d. $x^{-2} + 8x^{-1} - 9 = 0$



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**Examples** Solve the following:

a. $(x+3)^2 - 4(x+3) - 5 = 0$

b. $(2x+1)^2 - 4(2x+1) - 21 = 0$



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**Lesson: A Review of Basic Inequalities**

Please check out the examples video on how to graph on a number line, use interval notation, or use set notation in case you forgot!

**The Inequality Signs**

$<$ Less than

$\leq$ Less than or equal to

$>$ Greater than

$\geq$ Greater than or equal to

**The Properties of Inequalities**

If a, b, and c are real numbers then:

1. If $a < b$, then $a + c < b + c$

2. If $a < b$ and $c > 0$ then $ac < bc$

3. If $a < b$ and $c < 0$, then $ac > bc$

These properties also hold if you switch to $>, \leq, \geq$

**<u>Examples</u>** Solve the following. Put your final answer in interval notation.

$-2x + 8 < 10$



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**<u>Examples</u>** Solve the following. Put your final answer in interval notation.

a. $-\frac{1}{3}(x+3)+\frac{1}{3}x+\frac{2}{5}x \leq \frac{1}{10}$

b. $-5 \leq 1+2x < 13$

c. $2 \leq -6x-3 < 9$



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**Lesson: Quadratic Inequalities**

A quality inequality is an inequality in the form: $ax^2 + bx + c < 0$ where a, b, and c are real numbers. You can interchange $<$ with $\leq, >, \geq$.

There are 3 steps to solving a quadratic inequality. Write out each step as you solve the next example.

**<u>Example</u>** Write out the steps that you use to solve the following: $x^2 - x - 6 > 0$. Write your answer in interval notation.

Explain Step 1:

Explain Step 2:

Explain Step 3:



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**Example** Solve: $3x^2 - 4 \leq x$. Write your answer in interval notation.

**Example** Solve $x^2 \geq 25$. Write your answer in interval notation.



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**Lesson: Rational Inequalities**

Rational inequalities look like this:

$\frac{5}{x+3} > 3$ or $\frac{2x+5}{4x-1} \leq 2$

We will out the steps to solve a rational inequality by using the following example.

**<u>Example</u>** Solve $\frac{4}{x+2} \geq 6$ Put your final answer in interval notation.

Explain Step 1 and show how to do it:

Explain Step 2 and show how to do it:

Explain Step 3 and show how to do it:



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What do you have to watch out for with rational inequalities?

**<u>Example</u>** Solve $\frac{3}{x+6} \leq 2$ Put your final answer in interval notation.

**<u>Example</u>** Solve $\frac{x+3}{x-5} \geq 1$ Put your final answer in interval notation.



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**Lesson: Graphs of Basic Polynomial Functions and Translations of the Basic Graphs**

Explain the graphs of $f(x) = x^2, f(x) = x^4, f(x) = x^6$, etc.

Explain the graphs of $f(x) = x^3, f(x) = x^5, f(x) = x^7$, etc.



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If we multiply the polynomial like so: $f(x) = ax^n$, what happens to the graph?

What happens if we have $f(x) = a(x-h)^n + k$?

**<u>Example</u>** Sketch the graphs:

a. $f(x) = (x-5)^5 + 3$

b. $f(x) = -2(x+1)^6 - 4$



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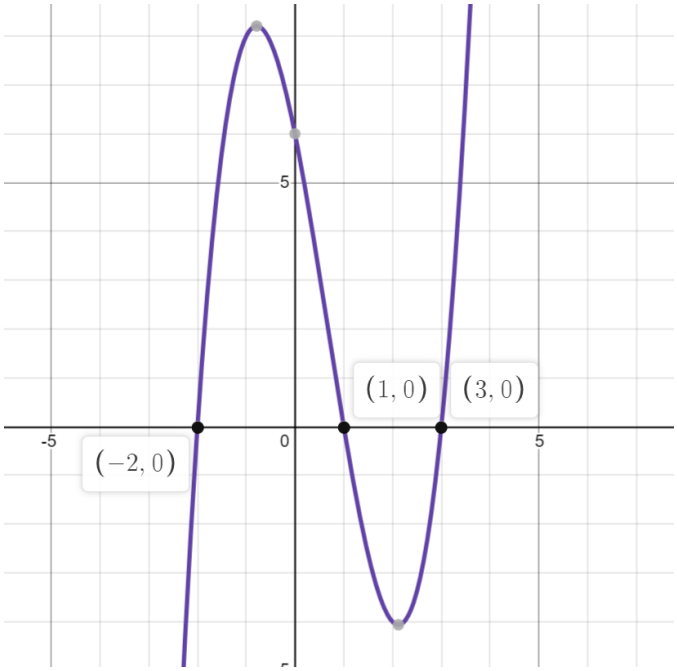
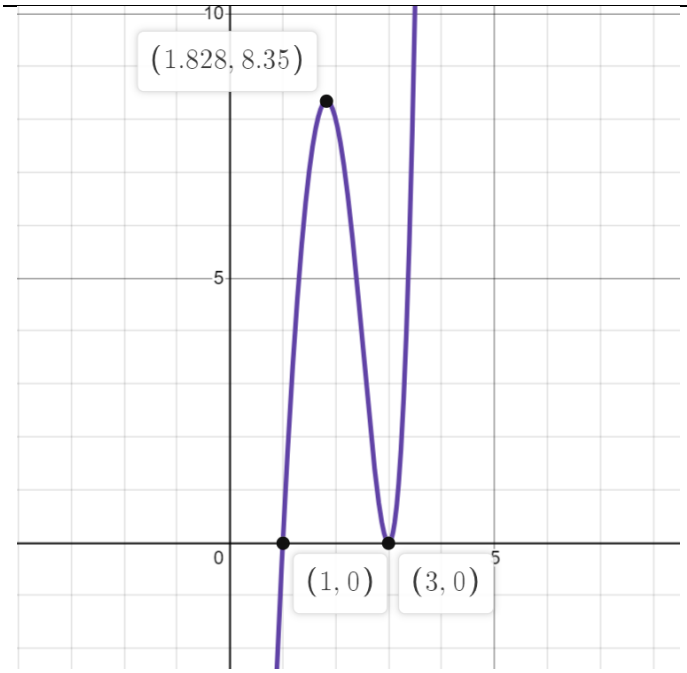
**Lesson: Graphs of Polynomial Functions, Turning Points, and End Behavior**

In the last video, we discuss the graphs of basic polynomials and translations. In this video, we will look at more complicated possibilities.

Below are the graphs of a few polynomials. What patterns do you notice?




$$f(x) = x^3 - 2x^2 - 5x + 6 = (x-1)(x+2)(x-3)$$




$$f(x) = x^5 - 7x^4 + 19x^3 - 37x^2 + 60x - 36$$
$$= (x-3)^2(x-1)(x-2i)(x+2i)$$



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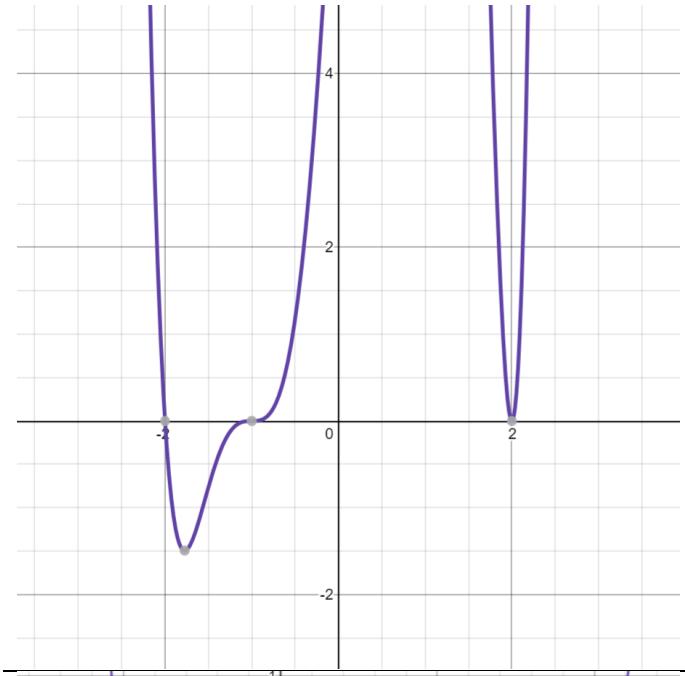
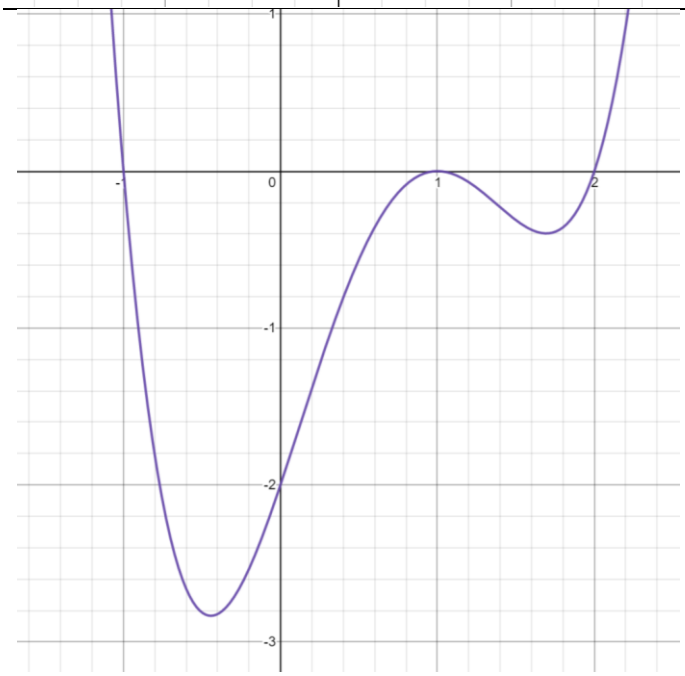



$f(x) = (x-2)^2(x+1)^3(x+2)$




$f(x) = (x-1)^2(x+1)(x-2)$

What do you notice about the behavior of these graphs at zeros? Summarize this in your own words, it was discussed as a general pattern in the video.



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What are turning points and how many will a degree n polynomial have?

What is the end behavior of a polynomial and how do you determine it?

**<u>Example</u>** What are the number of turning points of the following and what is the end behavior of each?

a. $f(x) = x^2 + 3x + 2$

b. $f(x) = (x+1)^2(x-2)$

c. $f(x) = -\frac{1}{2}x(x-3)^4(x+1)^3(x+2)$



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List the steps required to graph a polynomial:

**<u>Example</u>** Graph $f(x) = x^3 - x^2 - 4x + 4$



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**<u>Example</u>** Graph $f(x) = -(x+1)(x-2)^2(x-3)$



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What is the relationship between x-intercepts, zeros, solutions, and factors of polynomials????



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**Examples: Graphs of Polynomial Functions, Turning Points, and End Behavior**

Graph the following polynomials:

a. $f(x) = x^3 + 5x^2 + 2x - 8$



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b. $f(x) = 3x^4 + 5x^3 - 2x^2$



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c. $f(x) = (x-1)^3(x+2)^2$



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**Lesson: The Intermediate Value Theorem and the Boundedness Theorem**

State the Intermediate Value Theorem

**<u>Example</u>** Using synthetic division to show $f(x) = -x^3 + 2x^2 - 4x + 5$ has a real zero between 1 and 2.

**<u>Example</u>** Using synthetic division to show $f(x) = x^4 - 4x^3 - x + 3$has a real zero between 0.5 and 1.



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State the Boundedness Theorem

**<u>Example</u>** Show that $f(x) = x^4 + 5x^2 + 3x - 7$ has no real zero less than -3.

**<u>Example</u>** Show that $f(x) = 2x^5 - x^4 + 2x^3 - 2x^2 + 4x - 4$ has no real zero greater than 2



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**Lesson: An Intro to Polynomials**

We can extend our knowledge of exponents to another algebraic topic – polynomials

Before we define what a polynomial is, it is usually easier to see a few examples of what is or is not a polynomial.

The following ARE polynomials:

- $x^2 - 3x + 8$
- $x^7 - 5x^4 + 2x^3 + x$
- $9$
- $7x + \pi$
- $x^3 + \frac{2}{3}$

The following ARE NOT polynomials:

- $\sqrt{x} - 4$
- $4x^{-4} - 7x^{-3}$
- $\frac{4}{x} - \frac{2}{x+3}$

Now we can more formally define a polynomial.

## Polynomial

An expression with real numbers, variables, and whole number exponents.

## Descending Order

Ordering a polynomial from the largest exponent to the smallest exponent, and any freestanding number without a variable is last

## Example 1

Put the polynomials in descending order.

a. $x^3 + 4x^5 - 2x^7 + 3x^2 + 5x^8$

b. $x^2 - x^3 + x^4 - x^5 - x^6 + 3$



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**Term**
Any part in a polynomial that is separated by a "+" or "-" sign is considered to be its own term

**Variables**
The letters in a polynomial that represent numbers

**Coefficients**
The numbers attached to the variables

**Constant Term**
The freestanding number within a polynomial that is not attached to any variables

**Example 1**
Examine the polynomials, then determine the variables, coefficients, and constant terms

| Polynomial | Terms | Variable(s) | Coefficient(s) | Constant |
|---|---|---|---|---|
| $2x + 3$ | | | | |
| $3x^2 - x + 5$ | | | | |
| $5$ | | | | |
| $2x^2y + 3y - x$ | | | | |



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## Degree of a Term

The degree of a term is the sum of the exponents of that term.

## Example 2

Identify the degree of each term

| Term | Degree | Term | Degree |
|---|---|---|---|
| $x^4$ | | $x^4y^3$ | |
| $3y^6$ | | $x^7y^9$ | |
| $x$ | | $x^4y^2z^5$ | |
| $6$ | | $xy^4$ | |

## Degree of a Polynomial

The degree of a polynomial is the same as the degree of the term of highest degree within that polynomial

## Leading Coefficient

The leading coefficient is the coefficient attached to the term of highest degree in a polynomial

## Trinomial

A polynomial with 3 terms

## Binomial

A polynomial with 2 terms

## Monomial

A polynomial with 1 term

## Example 3

Identify the degree of each polynomial and the type of polynomial it is.

| Polynomial | Degree of Each Term, Respectively | Deg of Polynom | Leading Coefficient | Type of Polynomial |
|---|---|---|---|---|
| $x^5 + 4x^3 - 2x$ | | | | |
| $-5x^2 + 18x$ | | | | |
| $6x + 2x^4 - 3x^2$ | | | | |
| $x^5 + 2x^7 - 4$ | | | | |
| $8$ | | | | |



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**Evaluating Polynomials**

Evaluating polynomials is not unlike evaluating other expressions we've seen in this course.

**Example 4**

Evaluate the polynomials for $x = -2$.

a. $x^2 + 3x + 1$

b. $x^3 - x^2 - 3x + 4$

c. $\frac{1}{4}x^3 - \frac{1}{2}x^2 + 5x$



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**Lesson: How to Add and Subtract Polynomials**

To add or subtract polynomials, combine like terms.

**<u>Examples</u>**

Perform the following operations.

a. $(2x + 4) + (5x^2 - 3x + 1)$

b. $(5x^2 - 3x + 1) - (x^3 - 2x^2 - x + 1)$

c. $(3x^3 + 2x^2 - 5x) + (4x^3 + 5x^2 + 2x)$

d. $(2x^2 + 5x - 4) - (x^3 - 2x^2 + 3x - 5)$



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**Lesson: How to Multiply Polynomials**

To begin, let's first focus on the process of distributing a monomial with a polynomial.

**<u>Examples</u>** Multiply the following:

a. $3x(4x^2 + 5x - 7)$

b. $x^2y^4(3x^2 + 2xy - 8y^4)$

c. $\frac{1}{2}x^3y^4(4xy^3 - 2x^3y + 8xy)$

Now write down the process for how to multiply $(x + 2)(x^2 + 3x)$



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**Examples** Multiply the following:

a. $(x+3)(x+2)$

b. $(3x+1)(x-6)$

c. $(x^2+4)(x-2y)$

d. $(x+4)(x-4)$

e. $(x+3)(x^2+4x+5)$

f. $(2x^2+3x+1)(3x^2-4x+2)$



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How do you multiply $(x+5)^2$?

**<u>Examples</u>** Multiply the following:

a. $(x+2)^2$

b. $(2x^2+1)^2$

c. $(3x^2+x+5)$

How do you multiply 3 polynomials together, such as $(x+2)(x+1)(x-3)$?



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**Examples** Multiply the following together:

a. $(x-3)(x+4)(x-1)$

b. $(2x+1)(x-2)(x-1)$



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**Lesson: Synthetic Division and the Remainder Theorem**

The outcome of division can be restated using the division algorithm.

State the division algorithm:

Show how we use polynomial long division with $(x^2 - 3x - 1) \div (x + 2)$, and then restate the result using the format of the division algorithm.



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Now show how to do the operation $(x^2 - 3x - 1) \div (x + 2)$ with synthetic division:

Restate this result using the division algorithm:

**<u>Example</u>** Use synthetic division to perform: $\frac{4x^3-3x^2-7}{x+1}$. Rewrite the result using the division algorithm.



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So what are 3 keys about using synthetic division to remember?

What is the remainder theorem?



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**<u>Example</u>** Use the remainder theorem to find the desired functional values: $f(x) = -3x^4 + 5x^3 - 30x + 20, f(2)$. What is the equivalent division problem associated with this? Write the result of your division using the division algorithm.



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**Example** Use the remainder theorem to find the desired functional values: $f(x) = 3x^2 + 5, k = 3 - i$



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**Lesson: Synthetic Division and Potential Zeros of Polynomials**

What is a “zero” of a polynomial?

How does the remainder theorem help us to determine if we have a zero?

What are some other terms that we use for “zero” of a polynomial?



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**<u>Example</u>** Determine if the given k is a zero of the polynomial. If it is not, then what is the value of $f(k)$?

a. $f(x) = x^2 - 4x - 5; k = 5$

b. $f(x) = 2x^3 - 3x^2 + 5; k = 1$

c. $f(x) = x^2 - 2x + 2; k = 1 + i$



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**Lesson: The Basics of Dividing Polynomials**

In this section, we will discuss how to divide polynomials that are in these forms:

$\frac{x^2+3x}{x}$ or $(4x^7y^3 + 8x^4y^2 - 12x^5y^5) \div 2x^2y$

To divide in these cases, you can just use rules of exponents. Sometimes you will want to separate the problem into separate fractions.

**<u>Examples</u>**

a. $\frac{x^2+3x}{x}$

b. $\frac{5x^4-10x^3+15x^2}{5x}$

c. $(27x^4y^2 + 18x^3y) \div 9x^2y$

d. $(4x^7y^3 + 8x^4y^2 - 12x^5y^5) \div 2x^2y$



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**Lesson: Long Division for Polynomials**

In this section, we will discuss how to divide polynomials that are in these forms:

$(x^2 + 5x + 6) \div (x + 3)$ or $(2x^2 + 5x + 3) \div (2x + 3)$

In these types of problems, we need to use long division. In another video, we talked about how to divide polynomial problems like these:

$\frac{4x^3+2x^2}{2x}$

But we did not need long division. What is different about these types of problems that require long division?

Show how to use long division to divide $(x^2 + 5x + 6) \div (x + 3)$



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**Examples** Divide the following.

a. $(x^2 + 7x + 10) \div (x + 5)$

b. $(2x^2 + 5x + 3) \div (2x + 3)$

c. $\frac{x^2-11x+30}{x-5}$



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Now we will look at what to do when there is a remainder.

Divide the following: $(x^2 + 3x + 1) \div (x + 2)$

Now show how to divide when there are "missing" terms, such as with $(x^4 - 81) \div (x^2 - 9)$



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**Examples** Divide.

a. $(5x^2 - 10x^3 + 8 - 9x) \div (x + 2)$

b. $(x^3 + 1) \div (x + 1)$

c. $(5x^3 - 4x^2 + 1) \div (x + 2)$



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**Lesson: The Greatest Common Factor**

## The Greatest Common Factor (GCF)

The Greatest Common Factor is the biggest common factor between a set of terms.

## Examples

Find the greatest common factor in the following polynomials, then factor the polynomial.

a. Polynomial: $3x + 3y$

   GCF:

   Factor out the GCF:

b. Polynomial: $3xy + 6xz$

   GCF:

   Factor out the GCF:

c. Polynomial: $10x^2 - 25xy + 15xz$

   GCF:

   Factor out the GCF:

d. Polynomial: $3x^7y + 8x^3z^5 - 5y^4z^3$

   GCF:

   Factor out the GCF:

## Examples



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Factor the following polynomials.

a. $12xy^2 + 10x^2y^4 - 14xy^5$

b. $5xy - 10yz + 12xz$

c. $3x + 6x^2 - 12x^3$

d. $5x^4y^2z^5 + 10x^2y^4z^6 - 25x^5yz^3$



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**Lesson: Factoring by Grouping**

Grouping can best be understood with the following example.

Factor $ax + ay + bx + by$ by grouping:

## Examples

Finish the following examples where the grouping had been started.

a. $3(x - y) + 2z(x - y)$

b. $2a(3b + 2c) - 5(2c + 3b)$

c. $4x(y - z) - 3(z + y)$

d. $4z(4x - 7) - (4x - 7)$



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**Examples**

Factor the following.

a. $5x + 10y + xz + 2yz$

b. $10x^2 + 15x^3y - 2 - 3xy$

c. $3xy + 4y - 27x - 36$

d. $3xy + 6x + 21y + 42$

e. $3x^2 - 48y + 8x - 18xy$



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**Lesson: Factoring Special Trinomials**

How do you factor $x^2 + 6x + 9$?

What's another way to write this factorization?

What do you call this type of trinomials?

**<u>Examples</u>** Factor the following:

a. $x^2 + 12x + 36$

b. $9x^2 - 12x + 16$

c. $x^2 - x + \frac{1}{4}$

d. $36x^2 - 60xy + 25y^2$



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How do you factor $x^2 - 4$?

What is this type of factoring called?

**<u>Examples</u>** Factor the following:

a. $x^2 - 25$

b. $x^2 - 16$

c. $x^2 - 1$

d. $25x^2 - 36$

e. $16x^2 - 81y^2$

f. $16 - x^2$

g. $25y^2 - 64x^2$

h. $\frac{49}{64} - x^2$



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**Lesson: How to Factor The Sum and Difference of Cubes**

Write down the formula for the sum of cubes

$a^3 + b^3 =$

Write down the formula for the difference of cubes

$a^3 - b^3 =$

Show how to factor $x^3 + 8$

Show how to factor $8x^3 - 27$

**<u>Examples</u>** Factor:

a. $x^3 - 1$

b. $x^3 - \frac{1}{8}$

c. $x^3 + 64$



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**Review: An Overview of All the Types of Factoring**

We will review all of the different types of factoring by presenting an example of each type.

**Greatest Common Factor**

1. $4x^2 + 8xy$

2. $6x^3y^2 - 8x^2y^5 + 10x^5y$

**Grouping**

3. $3x + 6y - xz^2 - 2yz^2$

4. $12x^2 - 2y + 3x - 8xy$

**Basic Trinomials of the Form $\boldsymbol{x^2 + bx + c}$**

5. $x^2 + 7x + 10$

6. $x^2 - 2x - 15$



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**Trickier or Special Forms of Trinomials of the Form $x^2 + bx + c$**

7. $-x^2 - 3x + 28$

8. $6x^2 + 54x + 120$

9. $x^2 - 6xy + 8y^2$

10. $x^2 - 8x + 16$

11. $x^2 - 25$

12. $x^2 - \frac{9}{4}$



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**Trinomials of the Form $\boldsymbol{ax^2 + bx + c}$**

13. $2x^2 + 11x + 5$

14. $7x^2 - 15x + 8$

15. $-3x^2 - 15x^2 + 198x$

16. $16x^2 - 24xy + 9y^2$

**More Practice with the Difference of Squares**

17. $x^2 - 81$

18. $25x^2 - 4$

19. $\frac{1}{4}x^2 - 9y^2$



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**Sum and Difference of Cubes**

20. $x^3 + 8$

21. $x^3 - 27$

22. $27x^3 - 1$

23. $8x^3 + 64y^3$



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**Lesson: The Factor Theorem**

What does the factor theorem say?

**<u>Example</u>** Determine whether $(x-1)$ is a factor of each polynomial:

a. $f(x) = 9x^4 - 13x^2 + 4$

b. $f(x) = 4x^3 + 10x^2 + 2x - 4$

Given the above, what can we say is also a zero for example a?



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**Example** Factor $f(x) = 3x^3 + 4x^2 - 17x - 6$ into linear factors given that $-3$ is a zero.

**Example** Factor $f(x) = 2x^3 - x^2 - 8x + 4$ into linear factors given that $\frac{1}{2}$ is a zero.



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**Lesson: The Rational Zeros Theorem**

The rational zeros theorem is a way to determine *possible* candidates for *rational* zeros of a polynomial function.

State the Rational Zeros Theorem:

**<u>Example</u>** Find all the possible rational zeros for $f(x) = 2x^3 - x^2 - 8x + 4$ and then factor into linear factors.



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**<u>Example</u>** Find all the possible rational zeros for $f(x) = 5x^4 - 19x + x^2 + 31x + 6$ and then factor into linear factors.



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**Lesson: The Fundamental Theorem of Algebra and the Number of Zeros Theorem**

What does the Fundamental Theorem of Algebra state?

What is the number of zeros theorem?

**<u>Example</u>** What is the maximum number of zeros that each polynomial could possibly have?

a. $f(x) = x^2 - 3x + 1$

b. $f(x) = x^{17} - 3x + 1$

c. $f(x) = 9$



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**Example** Find a polynomial $f(x)$ that has zeros $-3, 3$, and $1$, where $f(2) = -10$

**Example** Find a polynomial $f(x)$ that has zeros $6, -3$, and $-2$, where $f(1) = 20$



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**Lesson: Conjugate Zeros Theorem**

Before we can discuss the theorem, let's review a few properties of conjugates.

What is the conjugate of the following:

a. $3 + 2i$ b. $5 - 3i$ c. $-7 + i$

What is another way that we denote conjugates? Summarize the explanation in the video. Give an example of how this notation works.

What are some properties of conjugates?



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What is the conjugate zeroes theorem?

**<u>Example</u>** Find a polynomial function of least degree with only real coefficients and zeros $2$ and $(3 - i)$.



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**Example** Suppose $k = -i$ is one zero of $f(x) = x^4 + 26x^2 + 25$. What are the other zeros?

**Example** Factor $f(x) = 2x^3 + (3 - 2i)x^2 + (-8 - 5i)x + (3 + 3i)$ into linear factors given $k = 1 + i$ is a factor.

Before you begin, what is unique about this problem?



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**Lesson: Finding Zeros of a Polynomial Function**

In previous videos we discussed:

- The factor theorem
- The rational zeros theorem
- The fundamental theorem of algebra
- The number of zeros theorem
- The conjugate zeros theorem

Using all of these theorems, we can combine them to find the zeros of *any* polynomial.

What can we say about a polynomial function of odd degree with real coefficients?

What can we say about a polynomial function of even degree with real coefficients?

What if a polynomial doesn't have only real coefficients?



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**Example** Find all zeroes of the following polynomials.

a. $f(x) = x^3 - 21x - 20$

b. $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$



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**Examples: Finding Zeros of a Polynomial Function**

Find all the zeros of the polynomials.

a. $f(x) = x^4 + 13x^2 + 36$

b. $f(x) = 2x^4 - 11x^3 + 4x^2 + 29x + 12$



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c. $f(x) = x^4 - 7x^3 + 17x^2 - x - 26$



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**Lesson: Descartes' Rule of Signs**

Let $f(x)$ be a polynomial function with real coefficients and a nonzero constant term, with terms in descending powers of x. Explain how to use Descartes' Rule of Signs:

**<u>Example</u>** Determine the number of different possibilities for the number of positive, negative, and nonreal complex zeroes:

a. $f(x) = 3x^5 + 4x^4 - x^3 + 2x - 1$



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b. $f(x) = x^6 + x^5 - x^4 + x^3 + x^2 - x + 1$

c. $f(x) = 3x^5 - 7x^3 + 6x + 8$



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**Lesson: Graphs of Basic Functions**

In this section, we will review some basic graphs that are vital to know for this class and calculus. Knowing the "basic shapes" of certain graphs will make your life much easier and give you greater intuition when graphing.

Show hot to graph the following and highlight the **key points** and **key patterns.** This will come in handy for the next lesson. Take your time to sketch your own NEAT CLEAN CLEAR graphs.

$f(x) = x$

$f(x) = x^2$



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$f(x) = x^3$

$f(x) = \sqrt{x}$



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$f(x) = \sqrt[3]{x}$

$f(x) = |x|$



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**Lesson: Piecewise Functions**

What is a piecewise function? Draw an example of what they look like.

**<u>Example</u>** Given the function: $f(x) = \begin{cases} 3x - 1, & x \geq 2 \\ \frac{1}{2}x - 4, & x < 2 \end{cases}$, evaluate the following:

a. $f(3) =$

b. $f(-1) =$

c. $f(2) =$

**<u>Example</u>** Given the function: $f(x) = \begin{cases} 4, & x > 3 \\ 4x - 2, & -2 < x \leq 3 \\ x^2 - 5, & x \leq -2 \end{cases}$ evaluate the following:

a. $f(4) =$

b. $f(-5) =$

c. $f(-2) =$

d. $f(3) =$



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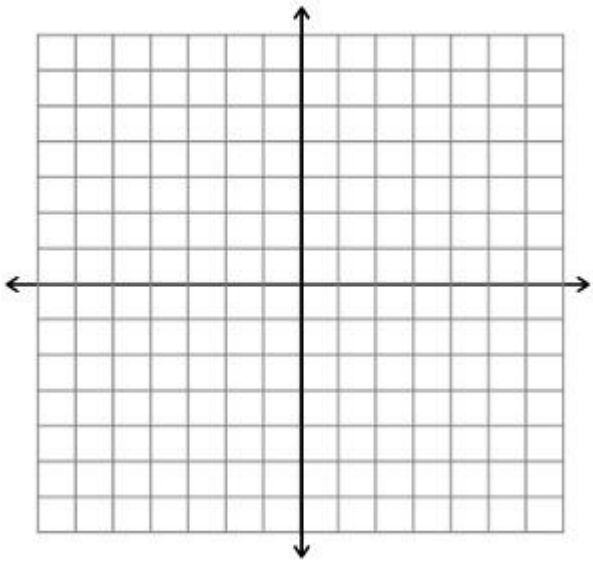
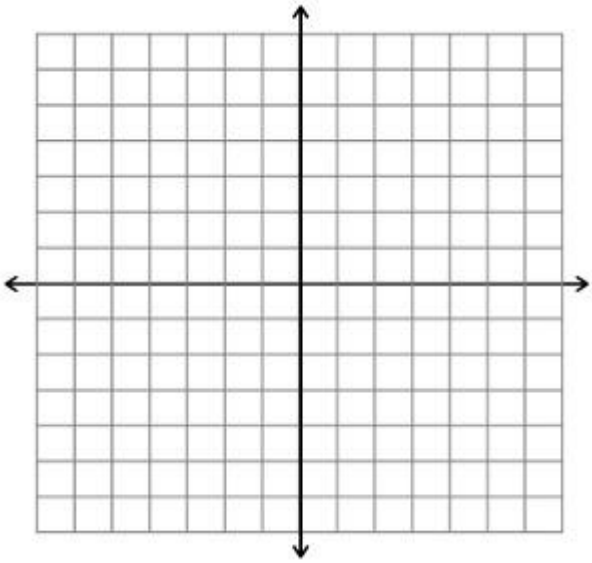
**Example** Show how to graph the following piecewise functions.

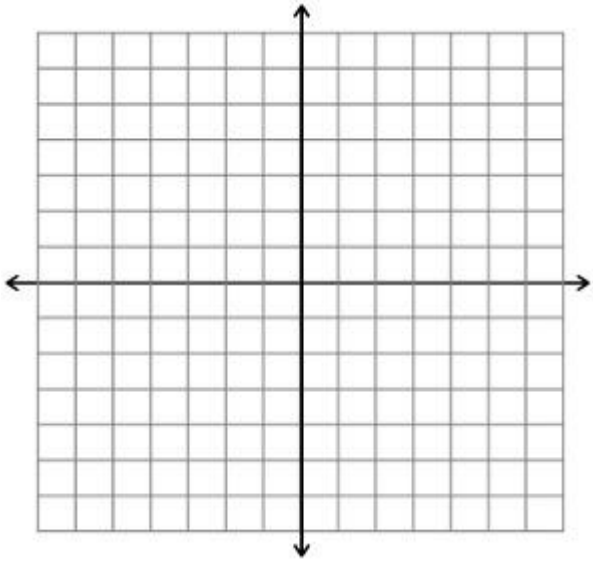
a. $f(x) = \begin{cases} 4, & x > 5 \\ -1, & x \leq 5 \end{cases}$



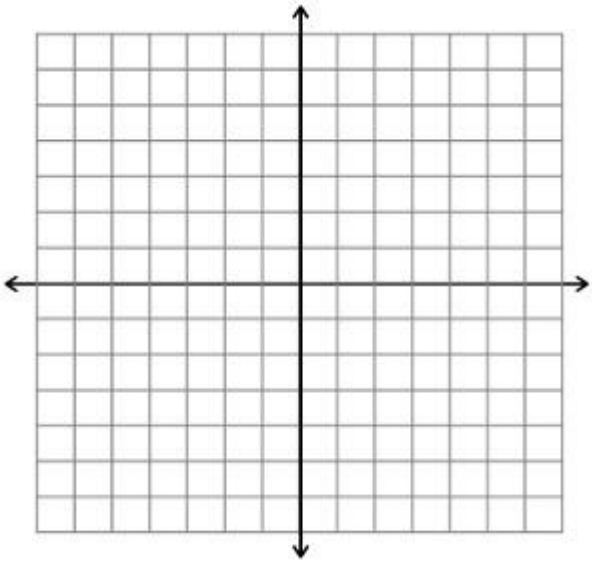
b. $f(x) = \begin{cases} 3x + 1, & x > -3 \\ -2x, & x \leq -3 \end{cases}$



c. $f(x) = \begin{cases} 3, & x > 2 \\ 2x, & -1 < x \leq 2 \\ -x + 3, & x \leq -1 \end{cases}$



d. $f(x) = \begin{cases} 3x + 1, & x \geq 1 \\ 6, & -1 \leq x < 1 \\ x^2, & x < -1 \end{cases}$





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**The Basics of Roots**
**Video 1: Square Roots**

What are the square roots of 9? Write down what it means to be a square root.

**<u>Examples</u>** Find the square roots of the following numbers.

a.

b.

What does the symbol $\sqrt{\phantom{x}}$ refer to?

How can we refer to the negative square root?

Draw a square root and label its parts:



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**Examples** Evaluate the following:

a. $\sqrt{25}$

b. $\sqrt{81}$

c. $\sqrt{36}$

d. $\sqrt{\frac{1}{36}}$

e. $\sqrt{\frac{16}{49}}$

f. $\sqrt{\frac{9}{25}}$

g. $\sqrt{-36}$

h. $-\sqrt{64}$

Finally, how would you estimate a root like $\sqrt{5}$?

**Example** Estimate $\sqrt{14}$



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**The Basics of Roots**
**Video 2: Higher Roots**

How would you evaluate the following? Explain the rationale for how to evaluate these:

$\sqrt[3]{8}$

$\sqrt[4]{81}$

$\sqrt[5]{32}$

The most general symbol for a root is $\sqrt[n]{a}$. Explain what this symbol means and once again, label it.



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Now we can create a list of the most likely roots that we will encounter. You will very likely want to create another list of this on a page you can easily access.

| **Squares** | **Cubes** | **Other** |
| --- | --- | --- |



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**Examples** Evaluate the following. Be careful when you evaluate.

a. $\sqrt[3]{125}$

b. $\sqrt[3]{-125}$

c. $\sqrt[4]{16}$

d. $\sqrt[4]{-16}$

d. $-\sqrt[3]{8}$

e. $-\sqrt[3]{-8}$

f. $\sqrt[5]{-32}$

g. $\sqrt[6]{-64}$

Now let's generalize. For all roots of the form $\sqrt[n]{a}$, what can we say if….

1. The index (n) is an **odd** number?

2. The index (n) is an **even** number and a is a **negative** number?

3. The index (n) is an **even** number and a is a **positive** number?



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**The Basics of Roots**
**Video 3: The "Ultimate" Test and A Few Other Notes**

To begin, we will first see how well we can put everything we've learned so far about roots together.

**<u>Examples</u>** Evaluate the following, if possible.

a. $\sqrt[3]{-8}$

b. $\sqrt{-16}$

c. $-\sqrt[4]{16}$

d. $\sqrt[3]{-125}$

e. $-\sqrt{100}$

f. $\sqrt[4]{-81}$

g. $-\sqrt{25}$

h. $-\sqrt[3]{-8}$

i. $-\sqrt[4]{-16}$

j. $-\sqrt[5]{-32}$

And now that you have completed this … it's time for my "ultimate" test … drum roll please:



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**The "Ultimate" Test for Roots**

Evaluate. Note: at this moment in time, we do not know anything about imaginary numbers.

a. $\sqrt{1}$

b. $\sqrt[3]{1}$

c. $\sqrt[5]{1}$

d. $\sqrt[4]{-1}$

e. $\sqrt[3]{-1}$

f. $-\sqrt{1}$

g. $-\sqrt[4]{-1}$

h. $-\sqrt[3]{1}$

i. $-\sqrt{-1}$

j. $-\sqrt[7]{-1}$

k. $-\sqrt[3]{-1}$

l. $\sqrt[8]{-1}$

m. $-\sqrt[10]{-1}$

n. $\sqrt{-1}$

**Are you having fun yet???**



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We have one last thing to discuss: the general rule for evaluating $\sqrt[n]{a^n}$

If n is even ….

If n is odd ….

**<u>Examples</u>** Evaluate the following:

a. $\sqrt[3]{y^3}$

b. $\sqrt[4]{(x-4)^4}$

c. $\sqrt[7]{(2y-4)^7}$

d. $\sqrt[6]{(3x+2)^6}$



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**Rational Exponents**

**Video 1: How to work with rules like $x^{\frac{1}{n}}$ or $x^{\frac{m}{n}}$ or $x^{-\frac{m}{n}}$**

In this video, we will learn how to evaluate expressions that look like this:

$8^{\frac{1}{3}}$ or $25^{\frac{3}{2}}$

Define $x^{\frac{1}{n}}$ and show how to use this rule to evaluate $25^{\frac{1}{2}}$

**<u>Examples</u>** Evaluate the following:

a. $125^{\frac{1}{3}}$

b. $32^{\frac{1}{5}}$

c. $9^{\frac{1}{2}}$

d. $(-9)^{\frac{1}{2}}$

e. $-9^{\frac{1}{2}}$

f. $\left(\frac{27}{125}\right)^{\frac{1}{3}}$



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Define $x^{\frac{m}{n}}$ and show how to use this rule to evaluate $25^{\frac{3}{2}}$

**<u>Examples</u>** Evaluate the following:

a. $27^{\frac{2}{3}}$

b. $4^{\frac{3}{2}}$

c. $16^{\frac{5}{4}}$

d. $-8^{\frac{2}{3}}$

e. $(-8)^{\frac{2}{3}}$

f. $(-16)^{\frac{3}{4}}$

g. $-81^{\frac{3}{4}}$

h. $\left(\frac{125}{27}\right)^{\frac{2}{3}}$



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Define $x^{-\frac{m}{n}}$ and show how to use this rule to evaluate $25^{-\frac{3}{2}}$

**<u>Examples</u>** Evaluate the following:

a. $25^{-\frac{1}{2}}$

b. $27^{-\frac{1}{3}}$

c. $16^{-\frac{3}{4}}$

d. $4^{-\frac{5}{2}}$

e. $\left(\frac{25}{4}\right)^{-\frac{3}{2}}$

f. $(-25)^{-\frac{3}{2}}$

g. $(-27)^{-\frac{2}{3}}$

h. $-16^{-\frac{5}{4}}$



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**Lesson: Using Exponent Rules with Rational Exponents**

First, let's review the main exponent rules:

Next, let's do a quick review on operations with fractions. Complete the following operations:

a. $\frac{2}{3} \times \frac{5}{6}$

b. $\frac{2}{5} \times 10$

c. $\frac{2}{3} + \frac{2}{5}$



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Now we will combine all of these ideas into the following examples!

**<u>Examples</u>** Simplify the following. Your final answer should not include any negative exponents.

a. $x^{\frac{2}{3}} * x^{\frac{1}{5}}$

b. $\left(4x^{\frac{2}{3}}y^{6}\right)^{\frac{3}{2}}$

c. $\frac{x^{\frac{2}{3}}y^{-\frac{1}{5}}}{x^{\frac{1}{6}}y^{\frac{2}{5}}}$



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d. $\left(\frac{125x^4y^{-\frac{1}{3}}}{x^{-\frac{1}{2}}y^{\frac{2}{5}}}\right)^{-\frac{2}{3}}$

e. $\left(\frac{4x^6y^{-\frac{1}{3}}}{x^{-3}y^{\frac{1}{2}}}\right)^{-\frac{3}{2}}$



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**Lesson: Simplifying Radicals with Rational Exponents**

Convert the following radicals to rational exponents, simplify, and then rewrite in radical form.

a. $\sqrt{7^2}$

b. $\sqrt{9^2}$

c. $\sqrt[3]{x^{15}}$

d. $\sqrt[4]{x^2}$



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**Rational Exponents**
**Video 3: Convert a radical expression to exponential form**

**<u>Examples</u>** Convert the following expressions to exponential form, and then simplify if needed.

a. $\sqrt{7}$

b. $\sqrt[3]{8}$

c. $\sqrt{5^2}$

d. $\sqrt[3]{7^6}$

e. $\sqrt[4]{x^{16}}$

f. $\sqrt[6]{x^2}$



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**Simplifying Radicals**
**The Basics**

We will look at how to simplify radicals like this:

$$\sqrt{50} = 5\sqrt{2} \quad \text{or} \quad \sqrt[3]{250} = 5\sqrt[3]{3}$$

You might want to have the list of squares / cubes / quartics ready for this exercise. If you don't, pause the video and write them all down on a separate sheet of paper so that you can have it handy for this entire lesson.

First we look at **the multiplication property of radicals:**

**<u>Examples</u>** Multiply the following:

a. $\sqrt[3]{2} * \sqrt[3]{3}$

b. $\sqrt{7} * \sqrt{5}$

c. $\sqrt[3]{3} * \sqrt[3]{9}$

d. $\sqrt[3]{6} * \sqrt[3]{9}$

How are examples c and d related?



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Being able to multiply radicals together is powerful, because it also allows us to break apart radicals and simplify them. Using the example $\sqrt{12}$, write down how to simplify:

**<u>Examples</u>** Simplify the following:

a. $\sqrt{50}$

b. $\sqrt{20}$

c. $\sqrt[3]{54}$

d. $\sqrt[4]{32}$

e. $\sqrt[3]{24}$



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At this point, we should probably define when a radical is actually simplified.

**A simplified radical will ……**

**<u>Examples</u>** Explain why the following radicals are not simplified:

a. $\sqrt{24}$

b. $\frac{5}{\sqrt{7}}$

c. $\sqrt[4]{\frac{3}{5}}$

d. $\sqrt[3]{8x^8y^2}$



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**Examples** Simplify the following:

a. $\sqrt{24}$

b. $\sqrt[3]{32}$

c. $\sqrt[4]{64}$

d. $\sqrt[5]{64}$

e. $\sqrt{250}$

f. $\sqrt[3]{250}$



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**Lesson: Simplifying Radicals with Variables**

We are going to rely on a skill we've already learned to help with simplifying radicals.

**<u>Examples</u>** Show you can simplify the following utilizing rational exponents.

a. $\sqrt{x^6}$

b. $\sqrt[3]{x^{15}}$

c. $\sqrt{x^6y^8}$

d. $\sqrt[4]{x^{12}y^{20}}$

How can we simplify $\sqrt{x^5}$? Watch the 2 explanations in the video and summarize the explanation that makes more sense to you.



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**Example** Simplify the following.

a. $\sqrt[3]{x^{13}}$

b. $\sqrt[4]{x^{22}}$

c. $\sqrt{12x^6y^7}$

d. $\sqrt[3]{250x^8y^{16}}$

e. $\sqrt[4]{\frac{32x^7}{y^8}}$



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**Lesson: Multiplying Radicals**

Write a one-sentence explanation of how to multiply $\sqrt[3]{3} * \sqrt[3]{4}$

**<u>Examples</u>** Multiply the following. Simplify if possible.

a. $\sqrt[3]{6} * \sqrt[3]{4}$

b. $\sqrt[4]{4x^3} * \sqrt[4]{12x^2}$

c. $\sqrt{5x} * \sqrt{10xy}$



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**Simplifying Radicals**
**Multiplying and Dividing Radicals with Different Indices**

We will look at how to perform operations like:

$$\sqrt{x} * \sqrt[3]{x} \qquad \text{or} \qquad \frac{\sqrt[4]{x}}{\sqrt[3]{x}}$$

Before we begin, you should be comfortable with doing problems like this:

$8^{\frac{1}{3}}$

$25^{\frac{3}{2}}$

Write down the approach for performing this operation: $\sqrt{x} * \sqrt[3]{x}$

**<u>Examples</u>** Perform the following operations:

a. $\sqrt[3]{x} * \sqrt[4]{x}$

b. $\frac{\sqrt[5]{x^2}}{\sqrt[3]{x}}$



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**<u>Examples</u>** Perform the following operations:

a. $\sqrt[4]{x^3} * \sqrt[5]{x^3}$

b. $\frac{\sqrt{x}}{\sqrt[3]{x^2}}$

c. $\sqrt[3]{x^2} * \sqrt{x}$



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**Adding and Subtracting Radicals**

In this video we will look at how to perform operations like:

$$\sqrt{3} + \sqrt[3]{5} - 5\sqrt{3} + 7\sqrt[3]{3} \qquad \text{or} \qquad \sqrt{24x^4y^3} - 3x^2y\sqrt{6y}$$

Before you begin, you should already be able to simplify radicals like this:

a. $\sqrt{24x^6y^9}$

b. $\sqrt[4]{16x^7y^{11}}$

What are like terms?

Add the like terms in this problem: $3x + 7y - 9x + 7y^2 - 2x - 2y$

What are like radicals?

Add the like radicals in this problem: $5\sqrt{3} - 2\sqrt[3]{3} + 8\sqrt[3]{3} - 2\sqrt{5} + 8\sqrt{3}$



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**Examples** Perform the following operations:

a. $4\sqrt{7} - 5\sqrt[3]{3} + 2\sqrt{3} - 3\sqrt[3]{7} - 9\sqrt{3} + \sqrt[3]{3}$

b. $2x\sqrt{7} - 3\sqrt{7} + 8x\sqrt{7} - 2\sqrt{7}$

c. $3x\sqrt[3]{5} + 2x\sqrt[3]{4} - 3\sqrt[3]{4} + 5x\sqrt[3]{4} - 7x\sqrt[3]{5} + 5\sqrt[3]{4}$

It is possible that you might have to simplify a radical before you can add it. Note how this works with the example:

$$\sqrt{24} + 2\sqrt{54}$$

**Examples** Perform the following operations:

a. $5\sqrt{8} - \sqrt{18}$

b. $4\sqrt[3]{54} - 7\sqrt[3]{16}$



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**Examples** Perform the following operations:

a. $\sqrt{8x^2y^4} - 9xy^2\sqrt{2}$

b. $4x\sqrt[3]{16x^6y^7} + 9x^3y\sqrt[3]{250y^4}$

c. $5x^2y^3\sqrt[4]{32x^7z^{10}} - 2x^3z^2\sqrt[4]{2x^3y^{12}z^2}$

d. $3x\sqrt[3]{24x^5y^7} + 3xy\sqrt[3]{3x^2y} + y^2\sqrt[3]{375x^8y}$



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**Lesson: Multiplying Radicals Expressions**

First, let's review how to multiply radicals in general. Show how to multiply:

a. $\sqrt[3]{4x^2y} * \sqrt[3]{6x^2y}$

b. $\sqrt{6xy} * \sqrt{10y}$

Now we will apply that logic to larger rational expressions.

Write a one-sentence explanation of how to multiply: $\sqrt{2}\left(\sqrt{3} + \sqrt{10}\right)$, then multiply.

**<u>Example</u>** Multiply $2\sqrt[3]{3}\left(\sqrt[3]{9} + 5\sqrt[3]{6}\right)$



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**Example** Multiply $3x\sqrt{5}\left(\sqrt{10x} - 5x\sqrt{2}\right)$

Write out an explanation of how to multiply: $\left(\sqrt{3} - \sqrt{2}\right)\left(\sqrt{6} + \sqrt{3}\right)$

**Example** Multiply: $\left(3\sqrt{5} - \sqrt{2}\right)\left(\sqrt{10} + 4\sqrt{2}\right)$



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**<u>Example</u>** Multiply: $\left(3\sqrt[3]{4}-\sqrt[3]{6}\right)\left(5\sqrt[3]{2}+\sqrt[3]{6}\right)$

How do you multiply $\left(\sqrt{2}+\sqrt{3}\right)^2$

**<u>Examples</u>** Multiply: $\left(\sqrt[3]{9}+\sqrt[3]{6}\right)^2$



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**Lesson: Rationalizing a Denominator with One Square Root**

What does it mean to rationalize the denominator?

How would you rationalize $\frac{2}{\sqrt{3}}$ ? Explain *why* we can do this.

Can every fraction with a radical in the denominator be rationalized like this? Circle: Yes No

What types of problems can be rationalized this way?



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**Examples** Rationalize the following.

a. $\frac{20}{\sqrt{8}}$

b. $\sqrt{\frac{40}{60}}$

c. $\sqrt{\frac{81x^7}{2y}}$

d. $\frac{\sqrt{22}}{\sqrt{x^9}}$



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**Dividing Radicals/Rationalizing the Denominator**
**Part 2: Higher Roots**

In this video we will look at how to simplify expressions like:

$$\frac{5}{\sqrt[3]{3}} \quad \text{or} \quad \frac{\sqrt[4]{2}}{\sqrt[4]{9}}$$

Before you begin, you should already be able to simplify problems like this:

a. $\frac{5}{\sqrt{3}}$

b. $\frac{\sqrt{6}}{\sqrt{18}}$

First, let's observe why higher roots are a different situation. Write down what happens if we use the same methods as above to rationalize the following: $\frac{5}{\sqrt[3]{3}}$

We have to pivot our thinking to "count powers". Note how this works with this examples: $\sqrt[3]{3} * ___ = 3$



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**Examples** Determine what to fill in the blank to create a true statement:

a. $\sqrt[3]{25} * ? = 5$

b. $\sqrt[4]{2} * ? = 2$

c. $\sqrt[4]{27} * ? = 3$

d. $\sqrt[5]{4} * ? = 2$

e. $\sqrt[3]{7} * ? = 7$

f. $\sqrt[4]{25} * ? = 5$

g. $\sqrt[5]{3} * ? = 3$

h. $\sqrt[3]{36} * ? = 6$

Now you should have a sense of what number you want this to equal. This is a very strange exercise, but using the pattern above, determine what to multiply by to get to the desired root:

a. $\sqrt[3]{4} * ? =$

b. $\sqrt[4]{5} * ? =$

c. $\sqrt[5]{9} * ? =$

d. $\sqrt[4]{36} * ? =$



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Now we should have a decent sense of how to rationalize and what the answer should equal. Explain how to rationalize the following:

a. $\frac{5}{\sqrt[3]{2}}$

b. $\frac{7}{\sqrt[4]{5}}$

**<u>Examples</u>** Rationalize the following denominators:

a. $\frac{6}{\sqrt[3]{5}}$

b. $\frac{2}{\sqrt[4]{9}}$

c. $\frac{2}{\sqrt[5]{125}}$

d. $\frac{7}{\sqrt[4]{8}}$



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**Examples** Rationalize the following denominators:

a. $\frac{\sqrt[3]{5}}{\sqrt[3]{4}}$

b. $\frac{2\sqrt[3]{3}}{\sqrt[3]{4}}$

c. $\frac{5\sqrt[4]{2}}{\sqrt[4]{27}}$

d. $\frac{6\sqrt[3]{5}}{\sqrt[3]{3}}$

e. $\frac{3}{\sqrt[5]{x^3}}$

f. $\frac{\sqrt[5]{25}}{\sqrt[5]{16}}$



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**Lesson: Rationalizing a Denominator with Two Square Roots**

How do you find the conjugate of $4 + \sqrt{2}$? Write one sentence to explain what you need to do.

**<u>Examples</u>** What is the conjugate of the following?

a. $3 - \sqrt{5}$

b. $-4 - \sqrt{2}$

Show how to rationalize: $\frac{5}{\sqrt{2}-\sqrt{3}}$



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**Examples** Rationalize the following.

a. $\frac{\sqrt{2}}{\sqrt{3}-\sqrt{6}}$

b. $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{3}+\sqrt{5}}$

c. $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$



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**Solving Equations with Radicals**
**Part 1: One Square Root in the Equation**

In this video, we will look at how to solving equations like this:

$$\sqrt{3x+1} = 2 \qquad \text{or} \qquad \sqrt{4x-2} = 5 - x$$

Use the example $\sqrt{3x+1} = 4$ to show the general procedure for solving these types of equations:

Why must we check the solutions?

**<u>Example</u>** Solve $\sqrt{3x+1} = 8$



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**Examples** Solve

a. $\sqrt{5x-4}+2=8$

b. $\sqrt{1+3x}+5=2$

Explain what is different about solving $\sqrt{5x+4}=x+2$



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**Examples** Solve the following.

a. $\sqrt{3x-6} = x - 2$

b. $\sqrt{2x+4} = x - 2$

c. $\sqrt{5x+1} + 5 = 3x$



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**Solving Equations with Radicals**
**Part 2: Two Square Roots in the Equation**

In this video, we will look at how to solving equations like this:

$$\sqrt{4x-2} = 3\sqrt{2x+6} \qquad \text{or} \qquad \sqrt{3x-1} + 2 = \sqrt{4x-2}$$

Let's begin by explaining how to solve: $\sqrt{5x+2} = \sqrt{3x+6}$

**Example**s Solve:

a. $\sqrt{3-2x} = \sqrt{2x+7}$

b. $\sqrt{5+3x} = \sqrt{3+2x}$

c. $4\sqrt{3-2x} = 2\sqrt{1+3x}$



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**Example** Solve: $6\sqrt{x} = 4\sqrt{2x+1}$

Now write out the detailed explanation of how to solve $\sqrt{4x+1} - 2 = \sqrt{2x+1}$



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**Examples** Solve:

a. $\sqrt{3x+6} = 2 + \sqrt{2x-4}$

b. $\sqrt{3x+1} = 2 + \sqrt{2x-2}$



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**Examples** Solve:

a. $\sqrt{2x+4}+\sqrt{1-2x}=3$

b. $\sqrt{2x+4}=2+\sqrt{2x+3}$



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**Solving Equations with Radicals**
**Part 3: Two Cube Roots of Higher in the Equation**

In this video, we will look at how to solving equations like this:

$$\sqrt[3]{3x+1} = 2 \qquad \text{or} \qquad \sqrt[4]{2x-1} = 5$$

Explain how to solve: $\sqrt[3]{3x+1} = 2$

Explain how to solve: $\sqrt[4]{3x+4} = 2$

When is it more likely a solution will not work?



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**Examples** Solve the following:

a. $\sqrt[3]{4x+7} = 3$

b. $\sqrt[3]{3-5x} = 2$

c. $\sqrt[4]{3-2x} = 3$

d. $\sqrt[4]{4x-1} = -7$



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**Lesson: Review of Function Operations**

Given that $f$ and $g$ are functions, how do we define the function operations?

$(f+g)(x) =$

$(f-g)(x) =$

$(fg)(x) =$

$\left(\frac{f}{g}\right)(x) =$

**<u>Example</u>** Let $f(x) = 3x+1, g(x) = x^2-4, h(x) = 3x^2+7x+2$, and $k(x) = x^2+6x+9$. Find each of the following:

a. $(f+g)(x) =$

b. $(f+g)(3) =$

c. $(g-k)(1) =$



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**<u>Example</u>** Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = 3x^2 + 7x + 2$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $\left(\frac{h}{f}\right)(x) =$

b. $(gh)(x) =$

c. $\left(\frac{k}{h}\right)(1) =$



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**Lesson: Review of Composition of Functions**

For two functions $f$ and $g$, what is the composition $(f \circ g)(x)$?

**<u>Example</u>** Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = \sqrt{x}$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $(f \circ g)(x) =$

b. $(g \circ f)(x) =$



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**<u>Example</u>** Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = \sqrt{x}$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $(k \circ f)(1) =$

b. $(g \circ h)(4) =$

c. $(h \circ k)(x) =$



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**Lesson: The Difference Quotient**

Explain what the difference quotient is and draw some sketches to explain how it works. In other words, watch the video explanation and then explain it in your own words on this page.



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State the formula for the difference quotient here:

**<u>Example</u>** Calculate the different quotient for the following.

a. $f(x) = 4x + 5$

b. $f(x) = x^2 - 1$



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**Lesson: The Difference Quotient**

Find the difference quotient.

a. $f(x) = \frac{1}{x+3}$

b. $f(x) = x^2 - 4x + 1$

c. $f(x) = -x^2 + 3$



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**Lesson: Determining if Functions are One-to-One**

Before we begin, you should already be familiar with the concept of functions and the vertical line test. First let's do some review.

**<u>Review</u>** Determine whether the following relations are functions, then determine the domain and range.

a. $\{(2,3),(5,7),(2,4)\}$

b.

Now we will discuss a specific type of function: a one-to-one function. Define this below:

How can we visualize this?



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**Example** Determine if any of the following are i) functions and ii) one-to-one functions

a. $\{(2,4),(-1,7),(3,7)\}$

b. $\{(3,6),(-1,2),(-3,-4)\}$

Similar to how we have a test to determine if a graph represents a function, we have another test to determine if a function is one-to-one.

Describe the horizontal line test below:

**Examples** Determine whether the following graphs represent i) functions and ii) one-to-one functions (draw your own):

a. b. c.



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**Lesson: Finding the Inverse of Functions**

One-to-one functions have inverses. Describe what the inverse of a function is:

**<u>Example</u>** Find the inverse of the following function: $\{(1,4),(7,8),(3,-2)\}$

Describe the steps to find the inverse of a function:



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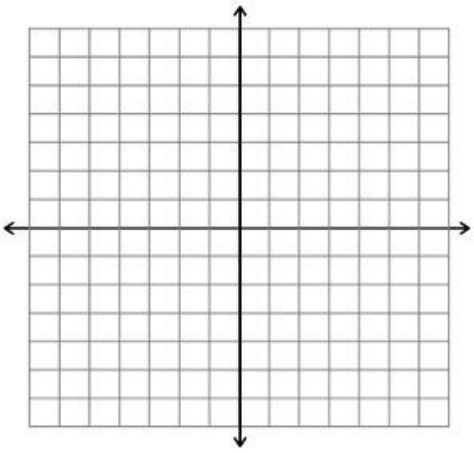
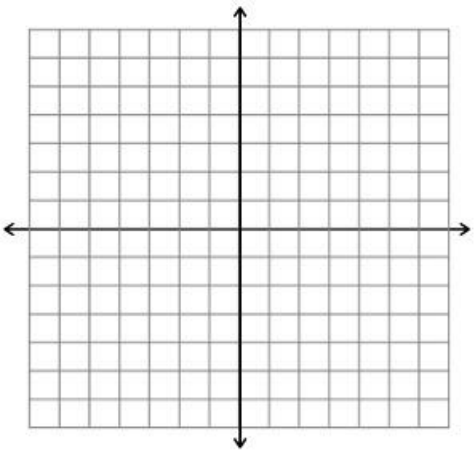
**<u>Examples</u>** Find the inverse of the following one-to-one functions. Then plot the original function on the graph with its inverse.

a. $f(x) = 3x + 1$



b. $g(x) = -\frac{1}{2}x + 4$



What is the relationship of a graph with its inverse?



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**<u>Examples</u>** Find the inverse of the following one-to-one functions.

a. $h(x) = \sqrt[3]{x}$

b. $f(x) = x^2, x \geq 0$



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**Lesson: A Review of Inverse Functions**

What is a one-to-one function?

**<u>Examples</u>** Are the following one-to-one functions? If no, why not?

a. $f(x) = x^2$

b. $g(x) = 2x + 1$

What is the horizontal line test?



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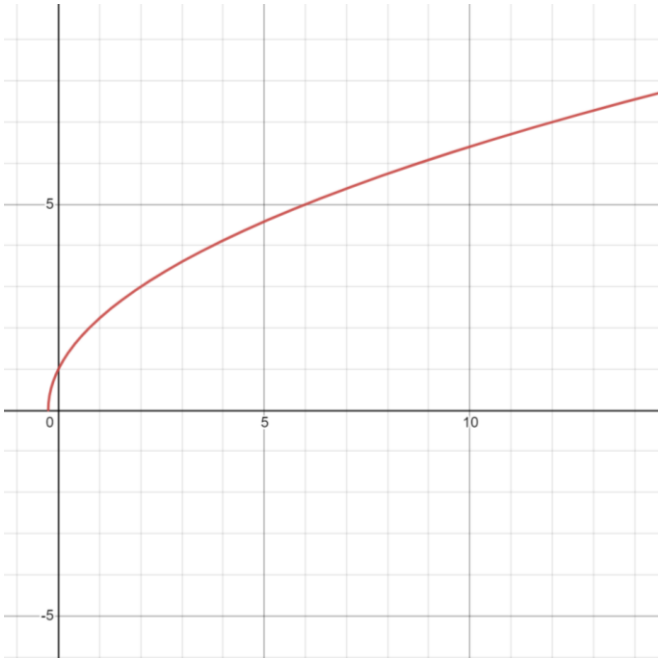
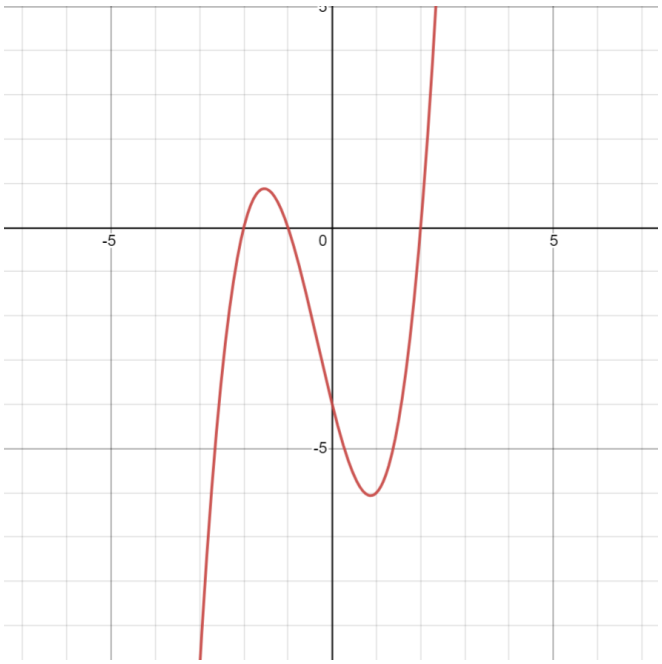
**Example** Use the horizontal line test to determine if the functions are one-to-one:

a.




b.




What is an inverse function?

What does an inverse function have to do with one-to-one functions?



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Describe how to use composition to determine if 2 functions are inverses of one another.

**Example** Use composition to determine if the functions $f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x - 2$ are inverses.

**Example** Use composition to determine if the functions $f(x) = \frac{2}{x+6}$ and $g(x) = \frac{6+x}{2}$ are inverses.



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Using $f(x) = 4x + 2$, outline the steps for how to find the inverse of a function.



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**Example** Find the inverse of the following functions.

a. $f(x) = \sqrt[3]{3x+1}$

b. $f(x) = \frac{x+2}{x-1}$

c. $f(x) = \sqrt{x-5}$



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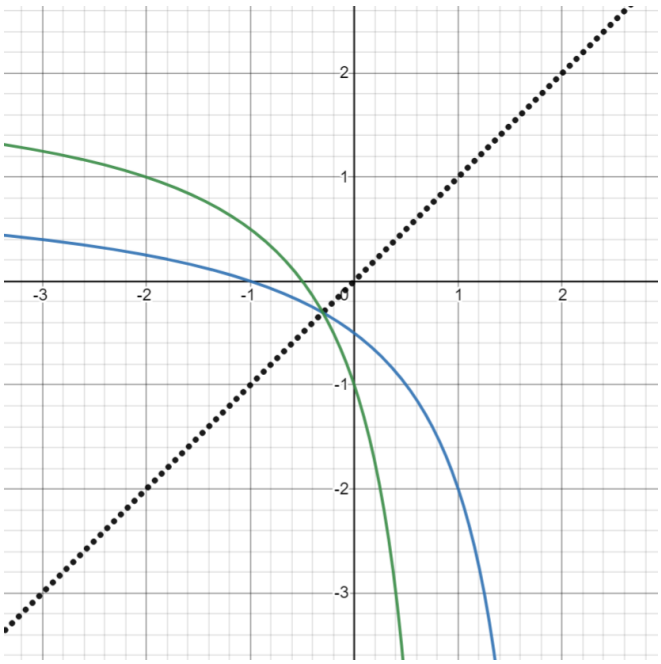
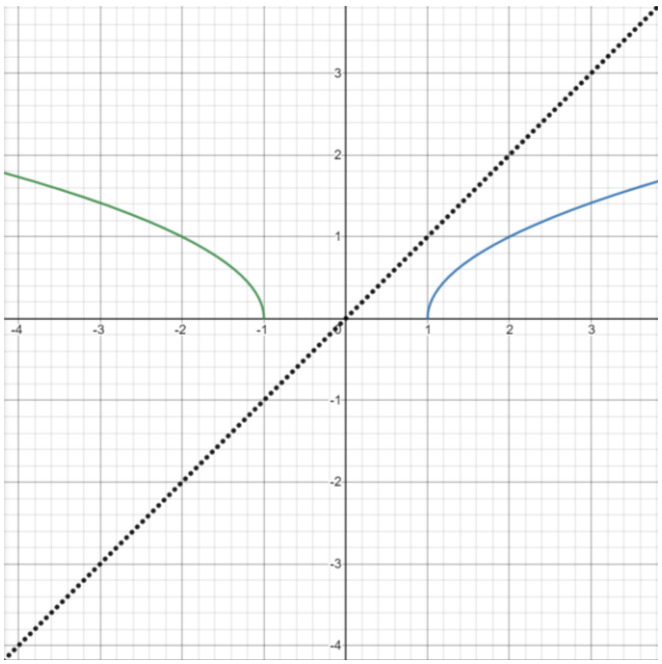
How can we tell on a graph if 2 functions are inverses of one another?

**<u>Examples</u>** Are the following graphs inverses of one another?

a.




b.



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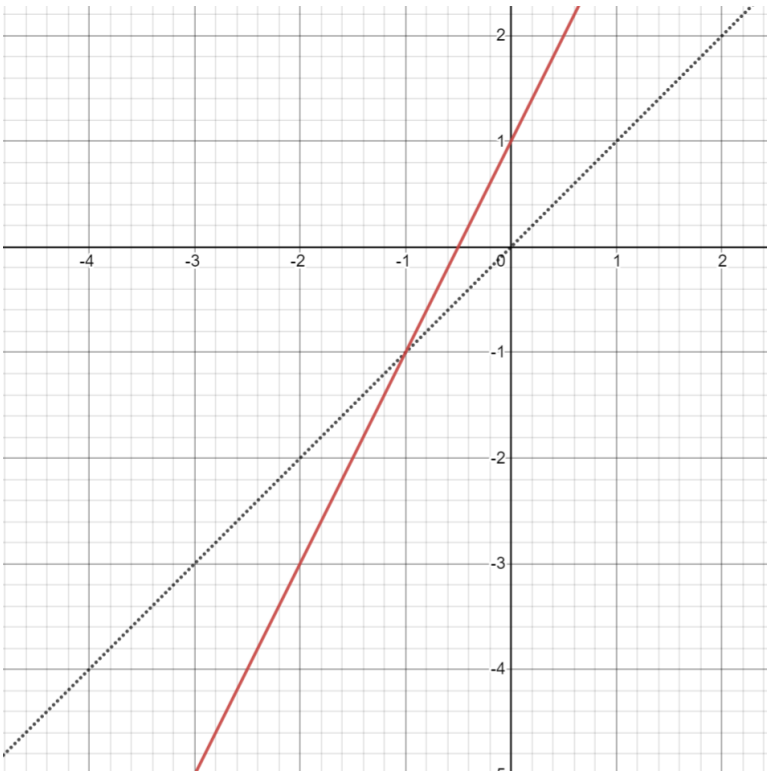
**Example** Draw the inverse of the following graph.



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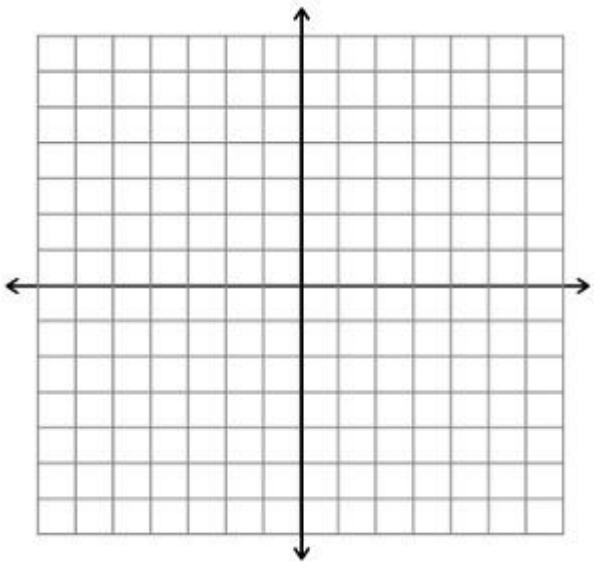
**Lesson: Exponential Functions**

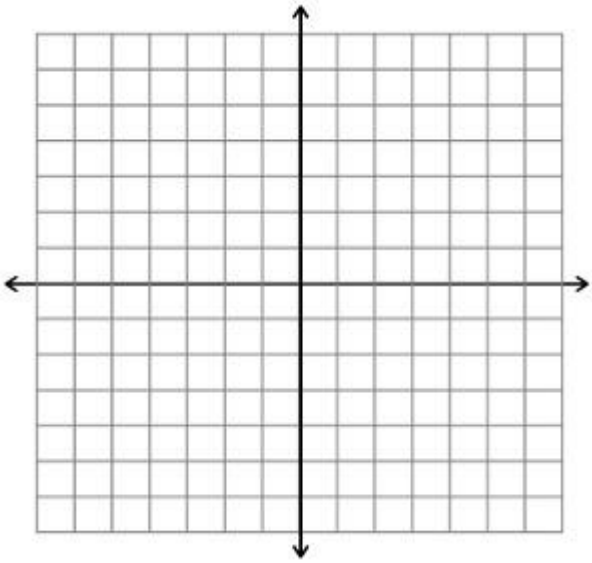
An exponential function is a function of the form _____ where $a > 0, a \neq 1$.

**<u>Examples</u>** Graph the following functions, then determine their domain and range.

a. $f(x) = 2^x$



b. $g(x) = \left(\frac{1}{2}\right)^x$



Using the graphs above, summarize some of the characteristics of these graphs:

The number e is an irrational number that is frequently used in math. What is an approximation of e?



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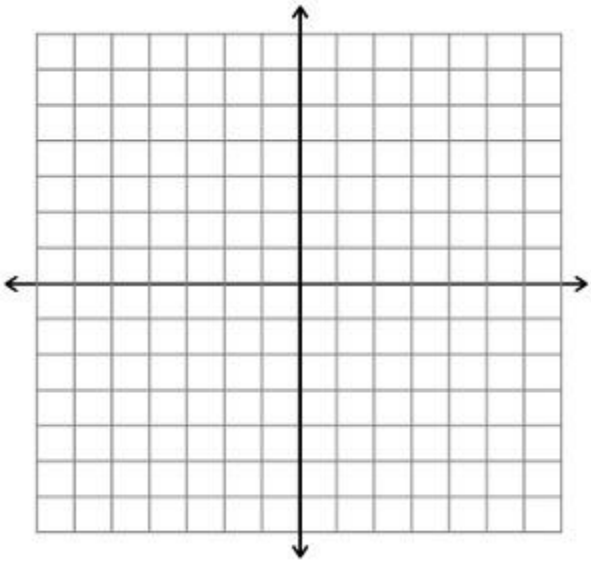
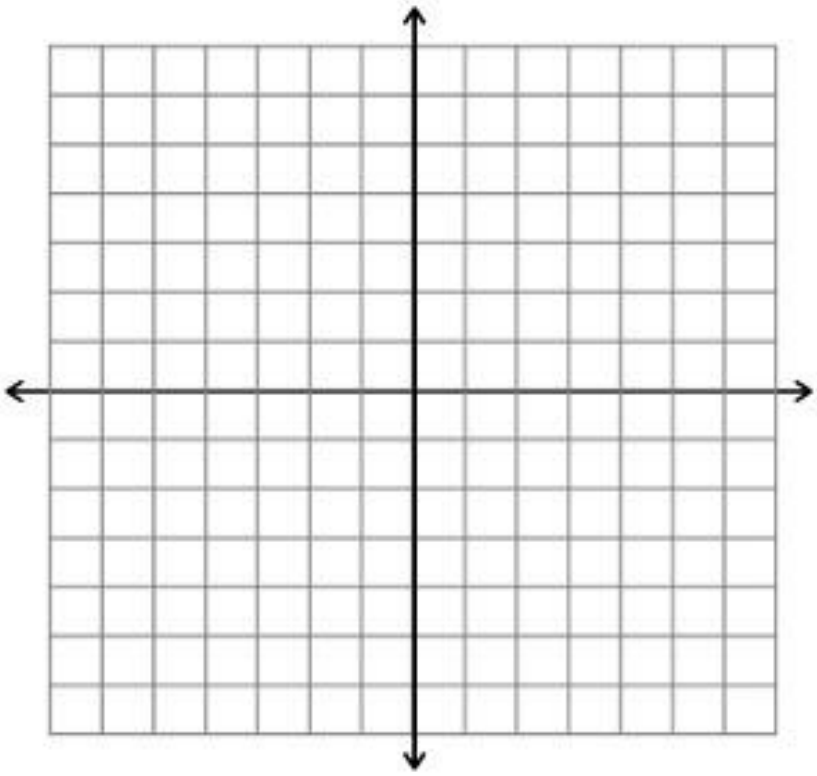
**<u>Example</u>** Graph $f(x) = 2^x, g(x) = e^x, h(x) = 3^x$ all on the same graph below for comparison:



Translations work similarly to other graphs we have discussed. Using the graph of $f(x) = 2^x$ as a "base graph", draw the following graphs together on the same graph: $f(x) = 2^x, g(x) = 2^x + 3, h(x) = 2^{x-1}$





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**Lesson: A Review of Exponential Functions and their Graphs**

What is the form of a general exponential equation?

**<u>Example</u>** Let $f(x) = 3^x$ and $g(x) = \left(\frac{1}{2}\right)^x$. Find the following.

a. $f(-2)$

b. $f\left(\frac{3}{2}\right)$

c. $g(-3)$

d. $g(3.2)$



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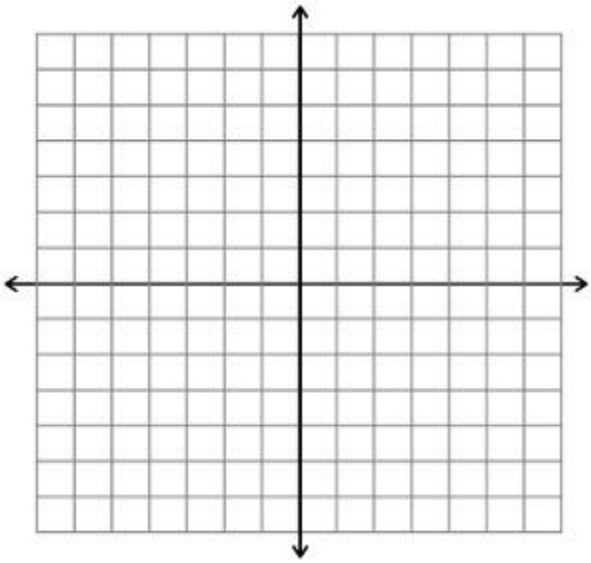
Sketch the graph of $f(x) = 2^x$ and create a table to go with it.



When will an exponential function $f(x) = a^x$ take this shape?

What is the x-axis for this graph?

What are 3 standard points to note here?

What is the domain and range?



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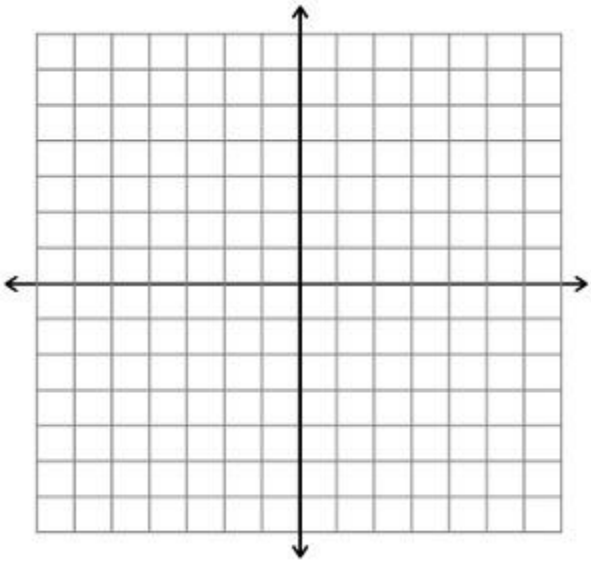
Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^x$ and create a table to go with it.



When will an exponential function $f(x) = a^x$ take this shape?

What is the x-axis for this graph?

What are 3 standard points to note here?

What is the domain and range?



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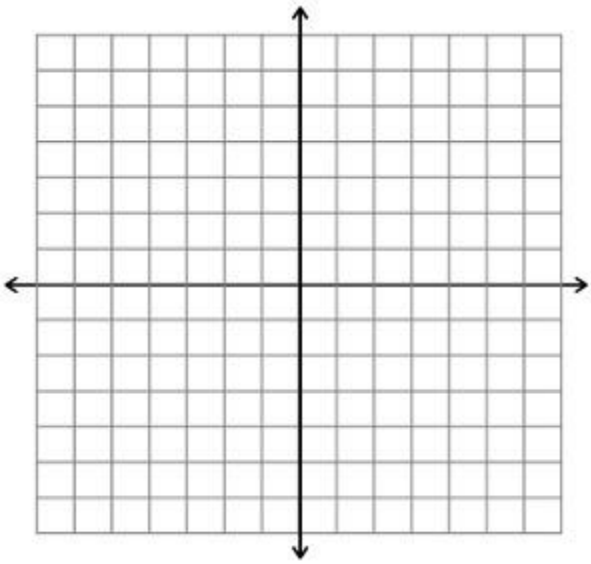
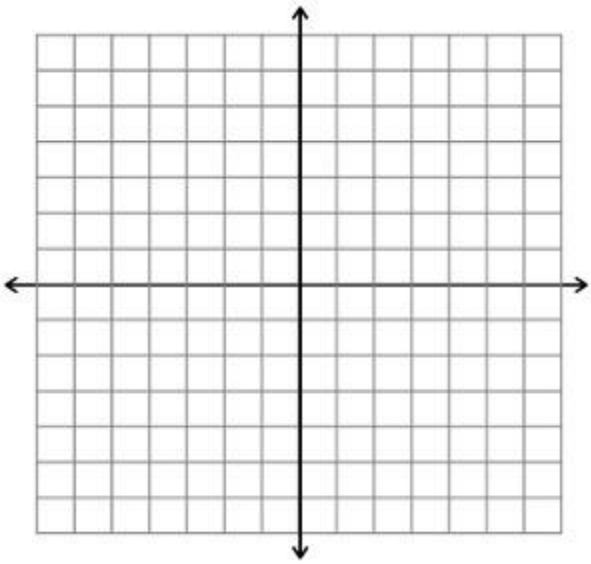
**Examples** Describe the translations and sketch the following graphs.

a. $f(x) = -3^x + 1$



b. $f(x) = \left(\frac{1}{2}\right)^{x+1} - 2$





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**Lesson: Solving Basic Exponential Equations**

Outline the general steps to solve $3^{x+1} = 81$.

Outline the general steps to solve $4^{x-2} = 8^{x+1}$.



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**<u>Examples</u>** Solve the following.

a. $27^{2x} = 9^{x-1}$

b. $e^{3-x} = (e^2)^{-x}$

c. $\left(\sqrt{3}\right)^{x+3} = 9^x$



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**Examples** Solve the following.

a. $x^{\frac{3}{2}} = 8$

b. $x^{\frac{5}{2}} = 4$

c. $x^{-4} = \frac{1}{16}$



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**Lesson: Compound Interest**

What is the formula for compound interest and what does it mean?

**<u>Example</u>** Suppose $2000 is invested into an account paying 2.5% interest per year compounded monthly. How much is in the account after 10 years, and how much interest is earned in that time?

What else do we call the A from the compound interest formula?

What else do we call the P from the compound interest formula?



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**Example** In 5 years, you want to have $8000 in cash to buy a car (including taxes). Let's use savings to help us reach this goal.

a. You find an account with a 3.25% interest rate compounded biannually (2x a year). What amount would you have to deposit today to ensure you have $8000 in 5 years?

b. Let's pretend that as of today, you have $5000. What interested rate would you need to ensure that this will grow to $8000 in 5 years?



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What is the number e?

What is the formula for continuous compounding?

**<u>Example</u>** Suppose $2000 is invested into an account paying 2.5% interest per year compounded continuously. How much is in the account after 10 years, and how much interest is earned in that time?



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**Lesson: Solving Exponential Equations**

In this lesson, we will solve equations that look like this:

$$3^x = 27 \qquad \text{or} \qquad 4^{x+3} = 8^{x-2}$$

To do this, we will write directions around the following example: $2^{3x} = 8^{x+1}$

**<u>Examples</u>** Solve:

a. $49^{x-3} = 7^{2x}$

b. $9^{4x} = 27^{x+4}$



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**<u>Examples</u>** Solve:

a. $5^{x-2} = \frac{1}{25}$

b. $\left(\frac{3}{5}\right)^{2x} = \left(\frac{27}{125}\right)^{x+2}$

c. $27^{3x+1} = 9^{4x+2}$



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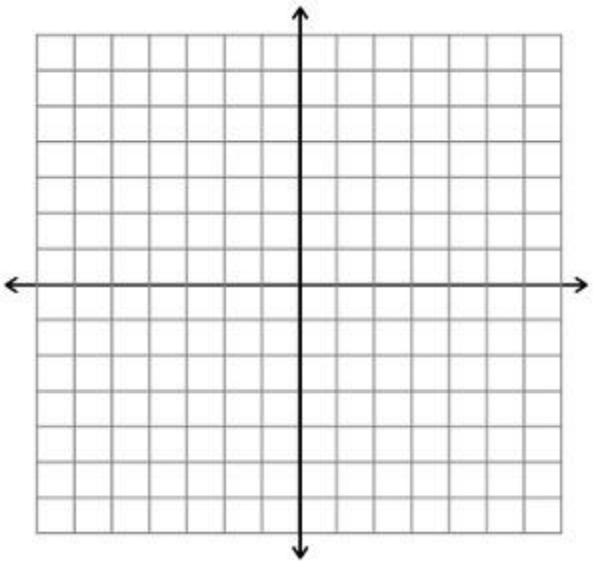
**Lesson: Intro to Logarithmic Functions**

First, we want to review how to function inverses of functions. Find the inverse of: $f(x) = 2x + 8$

Graph the exponential function $f(x) = 3^x$ below:



Recall the exponential functions are one-to-one, which means that they have inverses. Attempt to find the inverse of $f(x) = 3^x$ below:



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As we can see, there is an issue with finding the inverse – we have no way to solve for y! As such, we need new mathematics to help solve this.

Write down the definition of a logarithm:

**Examples** Write the following in exponential form:

a. $\log_3 81 = 4$

b. $\log_2 \frac{1}{8} = -3$

c. $\log_5 125 = 3$

d. $\log_9 9 = 1$

**Examples** Write the following in logarithmic form:

a. $2^4 = 16$

b. $3^0 = 1$

c. $6^{-2} = \frac{1}{36}$

d. $8^{\frac{1}{3}} = 2$



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Now we can finally determine what is the inverse of $f(x) = 3^x$:

There are a few common logarithms:

- $\log x$ means:

- $\ln x$ means:

There are also 2 key properties of logarithms:

1. $\log_a a = 1$: explain why this makes sense

2. $\log_a 1 = 0$: explain why this makes sense:



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**Lesson: Basics of Solving Logarithmic Equations and Evaluating Logarithms**

We want to solve equations that look like:

$$\log_3 x = 4 \qquad \text{or} \qquad \log_x 36 = 2$$

Before you begin, you should already be able to do the following:

a. Put $3^5$ in logarithmic form:

b. Put $\log_5 5 = 1$ in exponential form:

Now we can explain how to solve logarithmic equations using the example $\log_3(2x + 1) = 3$

**<u>Example</u>** Solve: $\log_x 16 = 4$



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**Example** Solve:

a. $\log_9 x = 2$

b. $\log_6(2x + 4) = 2$

Now we can also discuss how to evaluate logarithms. Explain how to evaluate: $\log_3 27$:

**Examples** Evaluate:

a. $\log_5 \frac{1}{25}$

b. $\log 100$



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**Examples** Evaluate:

a. $\ln e$

b. $\log_3 27$

c. $\log_{25} 5$

d. $\log \frac{1}{10}$



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**Lesson: How to Graph a Logarithm**

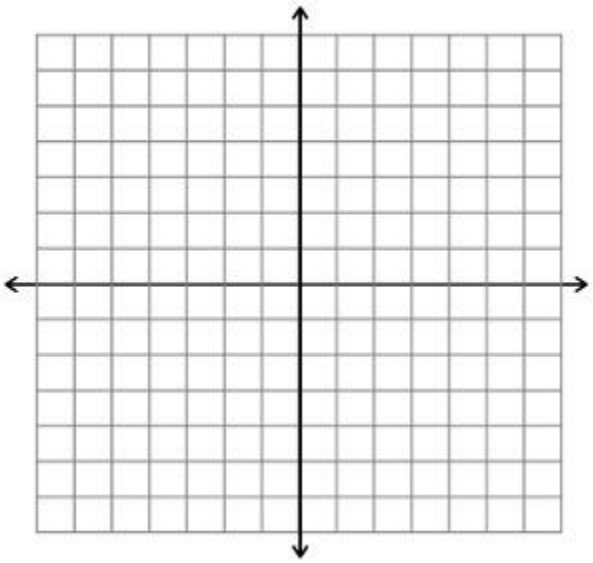
Before we begin, you should be able to do the following:

a. Convert to logarithmic form: $25 = 5^2$

b. Convert to exponential form: $\log_2 8 = 3$

c. Find the inverse of $y = 2^{\mathrm{x}}$

Now we can explain how to graph $y = \log_3 x$



For comparison, also graph $f(x) = 3^x$ on the same graph above.



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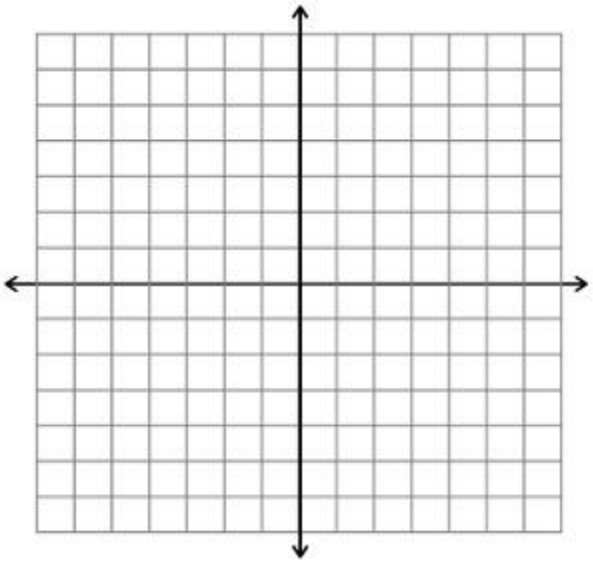
**<u>Example</u>** Graph $y = \log_{\frac{1}{2}} x$ below:





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**Lesson: Review of the Basics of Logarithmic Functions**

Explain how to find the inverse of $f(x) = 3^x$, and explain how this relates to logarithms.

Define a logarithm:

**<u>Example</u>** Write the following in logarithmic form.

a. $3^2 = 9$ b. $5^0 = 1$ c. $4^{-2} = \frac{1}{16}$

**<u>Example</u>** Write the following in exponential form.

a. $\log_3 1 = 0$ b. $\log_5 125 = 3$ c. $\log_3 81 = 4$



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**<u>Example</u>** Show how to solve the following basic logarithmic equations.

a. $\log_x 8 = 3$

b. $\log_3 x = -2$

c. $\log_{49} 7 = x$



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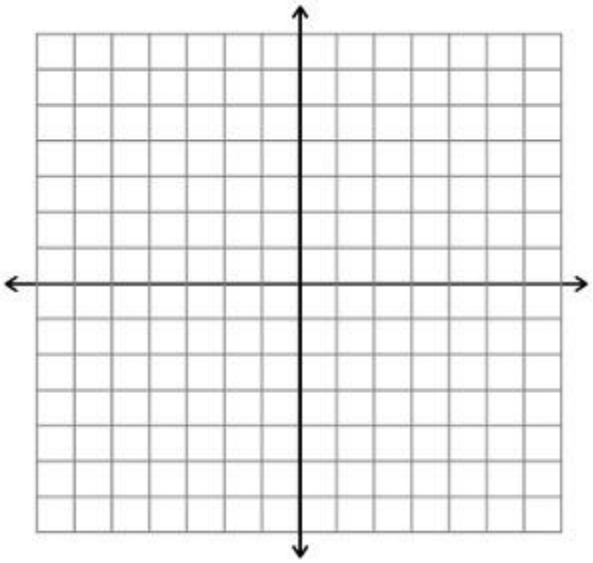
Create a table and show how to graph $f(x) = \log_2 x$



What 3 points can you easily find on the graph?

When does the graph take a shape like this for $f(x) = \log_a x$?

What is the y-axis for this graph?

What are the domain and range?



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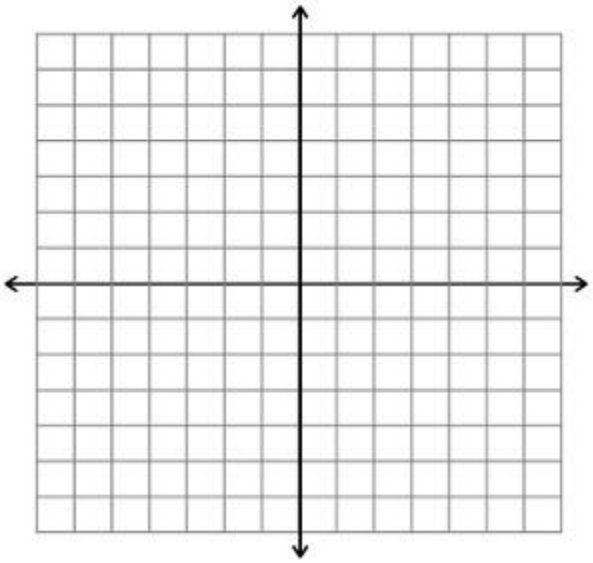
Create a table and show how to graph $f(x) = \log_{\frac{1}{2}} x$



What 3 points can you easily find on the graph?

When does the graph take a shape like this for $f(x) = \log_a x$?

What is the y-axis for this graph?

What are the domain and range?



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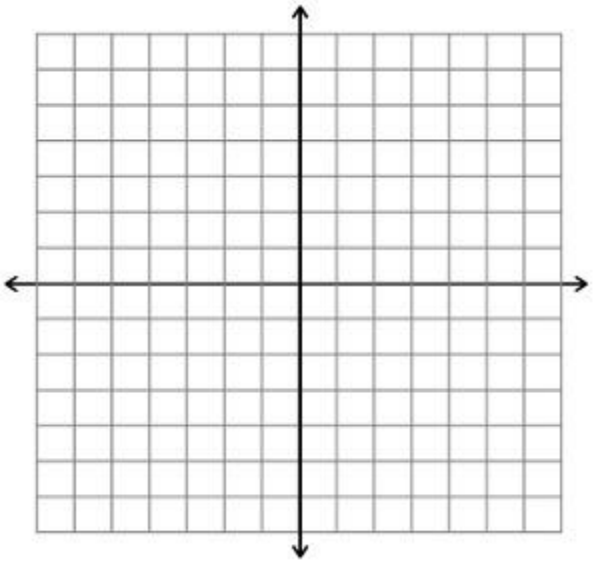
**<u>Examples</u>** Graph the following: $g(x) = \log_2(x-1) + 1$





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**Lesson: Review of the Basics of Properties of Logarithsm**

List the major properties of logarithms.

Explain what the common logarithms mean:

$\log x =$

$\ln x =$



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**Examples** Expand the following as much as possible.

a. $\log_3\left(\frac{3x^3y^7}{\sqrt{z}}\right)$

b. $\ln\left(\sqrt[3]{x^2y}\right)$

c. $\log_3\sqrt{6}$

d. $\log_5\frac{25x^3y^2}{p^7r^3}$



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**Example** Write as a single logarithm.

a. $\log(x+2)+\log(x-2)-\log(x+3)$

b. $3\log_2 3+2\log_2 y-5\log_2 x$

c. $\frac{1}{2}\log_5 16+3\log_5 y^2-\log_5 x^7-\log_5 y$

d. $4\ln 2+\ln x^6-2\ln y^3-3\ln x^2$



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**Lesson: Change of Base Theorem**

State the change of base theorem:

**<u>Example</u>** Show how to evaluate the following:

a. $\log_5 17$

b. $\log_2 3$

c. $\log_{\frac{1}{2}} 5$



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**Lesson: Solving Exponential Equations**

Outline the steps to solve: $2^{x+3} = 7^{3x}$

**<u>Examples</u>** Solve the following.

a. $e^{x^2} = 100$

b. $e^{2x-3} * e^{3x} = 5e$



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**Examples** Solve.

a. $3(2)^{x+2} + 6 = 105$

b. $e^{2x} + 8e^{x} + 15 = 0$

c. $3^{2x} + 2(3^{x}) = 24$



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**Lesson: Solving Logarithmic Equations**

Outline the steps to solve: $\log(3 - x) = 0.75$

Compare these steps to: $\log(x + 20) = \log(x + 10) + \log(6)$. How are these problems different?



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**Examples** Solve the following.

a. $\log_4(x^3 + 37) = 3$

b. $\log x + \log x^3 = 4$



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**<u>Examples</u>** Solve the following.

a. $\log(x^2 - 4) - \log(x - 2) = \log(8)$

b. $\log(5 + 4x) - \log(3 + x) = \log 3$



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**Lesson: Solving Exponential and Logarithmic Formulas**

**<u>Examples</u>** Solve for the desired variable.

a. for b: $p = a + \frac{c}{\ln b}$

b. for n: $A = \frac{Pr}{1-(1+r)^{-n}}$



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c. for A: $\log A = \log B - C \log D$

d. for x: $D = 100 + 5 \log x$



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**Lesson: Models of Exponential Growth and Decay**

Explain the equation for the general model:

Some examples of growth can come from populations or investments. Some examples of decay can come from half-life/radioactive decay, or carbon dating.

Define the following:

Doubling time:

Half-life:

What is the formula for continuous compounding?



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**Example** A sample of 400g of radioactive lead decays to polonium according to the function: $A(t) = 400e^{-0.03t}$.

a. Find the amount of lead remaining after 5 years.

b. Find the half-life.

**Example** The amount of carbon-14 present in a paint after t years if given by $y = y_0 e^{-0.0002t}$. The paint contains 37% of its carbon-14. How old are the paintings?



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**Example** Find the doubling time of an investment earning 5% interest if interest is compounded continuously.

**Example** During an epidemic, the number of people who have never had a particular virus and are not immune decreases exponential, where this decline is modeled by the function: $f(x) = 20{,}000e^{-0.025t}$, where t is time in days.

a. How many people were susceptible at the beginning of the epidemic?

b. How many people are susceptible after 2 weeks?



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**Variation**
**Video Notes**

The term "variation" refers to a particular way of describing how certain things relate to one another.

There are three types of variation we will look at. Write down the definitions for each, you will remember it better!

1. Direct Variation:

2. Inverse Variation

3. Joint Variation

It's also possible to combine different types of variation.



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The first thing we have to do is learn how to interpret and write variation equations.

**Examples** Write the variation equation:

a. P varies directly with Q

b. J varies inversely with L

c. H varies jointly with A and B

d. Z varies inversely with the square of R

e. M varies directly as the square root of N

f. Y varies jointly with the cube of M and the square root of L

g. M varies directly as L and inversely as N



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Now we can take a look at how to solve general variation equations.

**Example** Write down the detailed steps / directions for this example.

J varies directly with L. If J = 12 when L = 4, find J when L = 7.

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:



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**Examples** Using the steps outlined from the previous example, try the following:

a. M varies inversely as the square of L. If M = 12 when L = 2, find M when L = 4.

b. A varies jointly with B and C. If A = 30 when B = 2 and C = 5, find A when B = 3 and C = 4.

c. L varies directly as M and inversely as N. If L = 15 when M = 5 and N = 2, find L when M = 7 and N = 3.



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**Example** Jada's weekly income varies directly with the amount of hours that she works. Last week, she earned $570 for working 38 hours. This week she worked 36 hours, how much will she earn?



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**Lesson: The Basic Graphs of Rational Functions**

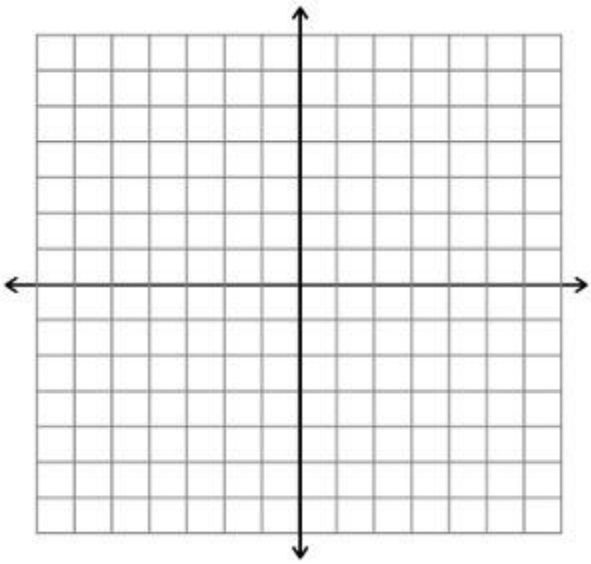
A rational function is a function of the form:

$\frac{f(x)}{g(x)}, g(x) \neq 0$

Where $f(x)$ and $g(x)$ are polynomials.

There are 2 graphs that we view as the most basic version of rational functions.

Make a table and graph $f(x) = \frac{1}{x}$



What is the domain and range?

Where does the graph not exist (ie where is it discontinuous)

What are the y-axis and x-axis for this graph?

What kind of symmetry does this graph have?



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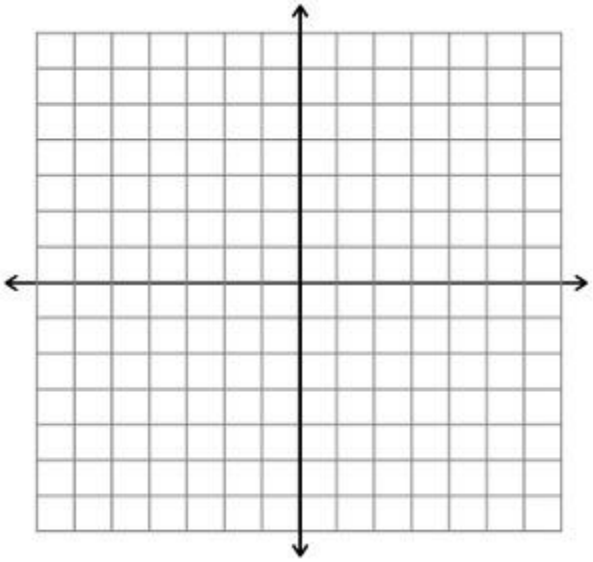
Make a table and graph $f(x) = \frac{1}{x^2}$



What is the domain and range?

Where does the graph not exist (ie where is it discontinuous)

What are the y-axis and x-axis for this graph?

What kind of symmetry does this graph have?



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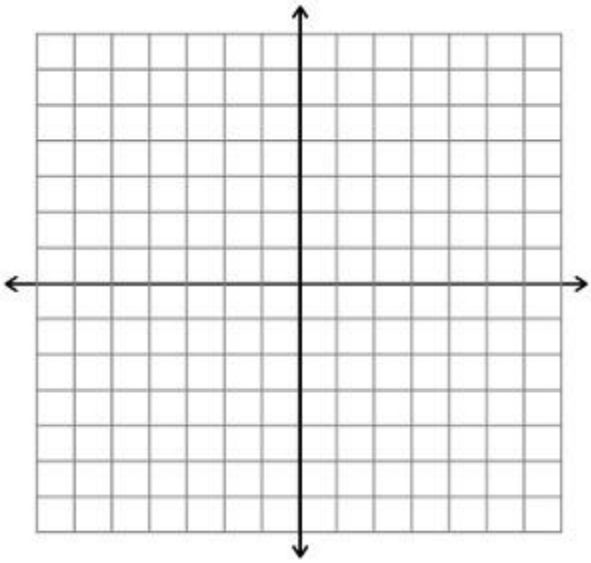
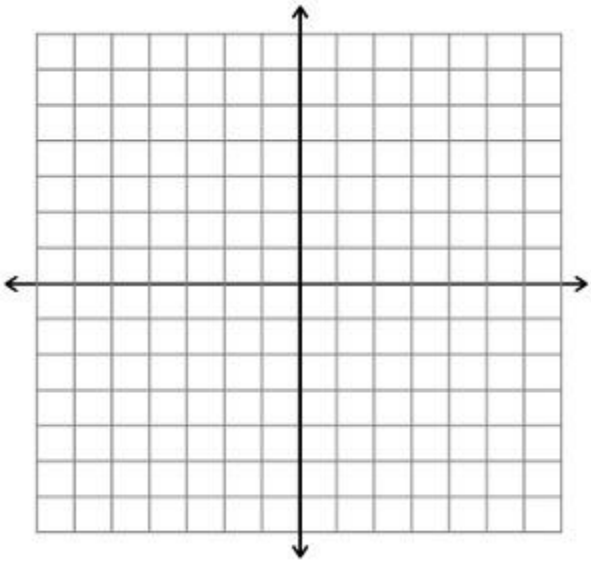
**Example** Describe the translations for the following graphs, and then draw an approximate sketch of the graph.

a. $f(x) = \frac{1}{(x+1)^2} - 2$



b. $g(x) = -\frac{2}{x} + 1$





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Sketch what happens for odd powers of n for: $f(x) = \frac{1}{x^n}$, ie just draw a sketch of $y = \frac{1}{x^3}, y = \frac{1}{x^5}, y = \frac{1}{x^7}$

Sketch what happens for higher even powers of n for: $f(x) = \frac{1}{x^n}$, ie just draw a sketch of $y = \frac{1}{x^4}, y = \frac{1}{x^6}, y = \frac{1}{x^8}$



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**Lesson: Asymptotes of Rational Functions**

What is an informal definition of asymptotes?

Is it possible that a graph would graph an asymptote?

Here are the formal definitions of asymptotes:

A graph $f(x) = \frac{p(x)}{q(x)}$ $(q(x) \neq 0)$ has a

- **Vertical asymptote** if $|f(x)| \to \infty$ as $x \to a$. The asymptote is x=a.
- **Horizontal asymptote** if $f(x) \to b$ as $|x| \to \infty$. The asymptote is y=b.

What is super important about asymptote?

Where do you typically find vertical asymptotes?



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How can you determine horizontal asymptotes? Describe the two ways, and also explain how oblique asymptotes. (In other words, explain what we talked about in the video).



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**<u>Examples</u>** Find any vertical, horizontal, or oblique asymptotes.

a. $f(x) = \frac{x+2}{x^2+7x+12}$



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b. $f(x) = \frac{3x+1}{4x-2}$



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c. $f(x) = \frac{x^2+3x+2}{x^2-9}$



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d. $f(x) = \frac{x^2+x}{x+3}$



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**Examples: Finding Oblique Asymptotes**

Find all the asymptotes for the following functions.

a. $f(x) = \frac{x^2+2x}{2x+1}$

b. $f(x) = \frac{x^2+2}{x+1}$



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**Lesson: Graphing Rational Functions**

To learn how to graph rational functions, we will outline the steps and then graph. **Clearly write directions for each step as we go through this process!!!**

Graph $f(x) = \frac{x+3}{x^2+7x+10}$. Outline all the relevant steps discussed in the video.



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Graph $f(x) = \frac{x+1}{x-3}$. Outline all the relevant steps discussed in the video.



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Graph $f(x) = \frac{x^2+1}{x-3}$. Outline all the relevant steps discussed in the video.



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**Examples: Graphing Rational Functions**

Graph $f(x) = \frac{(x+2)^2}{(x-1)(x+3)}$. Outline all the relevant steps discussed in the video.



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Graph $f(x) = \frac{2x^2-3x+1}{x-2}$. Outline all the relevant steps discussed in the video.



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**Examples: Equations of Rational Functions**

Copy the graphs from the video and then describe how to find the equations of those graphs.



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**Rational Expressions**
**Video 1: The Basics**

What are rational expressions?

What do rational expressions look like? Write down some examples:

**<u>Simplifying Rational Expressions</u>**
Take down some notes on this:



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**Examples** together:

a. $\frac{25x^7y^9}{15x^3y^2z^7}$

**Examples** on your own:

b. $\frac{21x^7y^9}{14y^8z^3}$

c. $\frac{16x^4y^7z^3}{15x^3y^4z^8}$



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**Examples** together:

a. $\frac{x^2+5x+6}{x^2-9}$

**Examples** on your own:

b. $\frac{x^2+7x+10}{x^2+10x+25}$

c. $\frac{x^2-x-6}{x^2-9x+14}$

d. $\frac{x^2+10x+16}{x^3+2x^2+x+2}$

Play the video and check your work.



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**Examples** on your own:

e. $\frac{x^3-8}{x^2-9x+14}$

f. $\frac{10x^3+15x^7}{10x^3+40x^2+40x}$



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**Rational Expressions**
**Video 2: Other Things to Know**

When it comes to fractions, where are you allowed to have a zero? In the numerator (top) or denominator (bottom)?

Therefore, what do we have to worry about with rational expressions?

**Examples** on your own:

Evaluate the following rational expressions for $x = 2$.

a. $\frac{x^2+6x+8}{x^2+8x+15}$

b. $\frac{x^2+9x+20}{x^2+3x-10}$



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**Example** on your own: (evaluate for x=2)

c. $\frac{x^2-4}{x^2+9x+14}$

**To summarize…**

When a rational expression has a zero in the numerator, what does this mean?

When a rational expression has a zero in the denominator, what does this mean?

Define the term **domain:**

What are we concerned with for the domain of rational expressions?



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**A review of interval notation:**

**Examples**:

Write the following in interval notation:

a.

b.

c.



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**Example** together:

Determine i) where the expression equals zero ii) where the expression is undefined iii) the domain of the expression

a. $\frac{x+3}{x-5}$

b. $\frac{2x+1}{x^2+7x+10}$

**Examples** on your own:

Determine i) where the expression equals zero ii) where the expression is undefined iii) the domain of the expression

c. $\frac{x-3}{x-1}$

d. $\frac{x^2-81}{x^2+5x+6}$



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**Rational Expressions**
**Video 3: The Negative One Trick with Simplifying**

In its current form, can you simplify: $\frac{x-3}{3-x}$? Why or why not?

Show how you can manipulate this expression to simplify it.

There is no formal name for this trick. Some might refer to it as a *multiplicative property of -1,* but I refer to as "that -1 trick." I know, it's not exactly formal sounding!

Let's do one more example of this. Simplify $\frac{5-x}{3x-15}$



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The first thing to get used to it how to rewrite expressions using this trick.

**Examples**

Rewrite the order of each expression by factoring out -1 first.

a. $4 - x$

b. $3x - 1$

c. $x - 6$

Now let's use this to simplify rational expressions.

**Examples** Simplify the following.

a. $\frac{6x-12}{2-x}$



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**Examples** Simplify the following.

b. $\frac{4x-20}{30x-6x^2}$

c. $\frac{x^2-4}{2-x}$

d. $\frac{x-6}{36-x^2}$

e. $\frac{5-x}{x^2-7x+10}$



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**Examples** Simplify the following.

f. $\frac{x^2-10x+25}{25-x^2}$

g. $\frac{27-x^3}{x^2-5x+6}$

h. $\frac{x^3-3x^2+5x-15}{6-3x}$



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**Examples** Simplify the following.

i. $\frac{54-9x}{x^2-36}$

j. $\frac{8-x^3}{x^2-9x+14}$

k. $\frac{y^3-x^3}{x^2-y^2}$



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**Example/Question: Troubleshooting the -1 Trick**

Explain the actual DETAIL behind WHY $\frac{4}{5-x} = -\frac{4}{x-5}$

**<u>Examples</u>** Write an equivalent fraction for the following:

a. $\frac{5}{x-8}$

b. $\frac{x-3}{x+2}$



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**Lesson: Multiplying Rational Expressions**

How do you multiply the fractions $\frac{a}{b} \times \frac{c}{d}$? Include a one-sentence explanation.

**<u>Examples</u>** Multiply the following.

a. $\frac{4}{5} \times \frac{7}{9}$

b. $\frac{2x^3}{7y^2} \times \frac{5x^5}{9y^7}$



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Show how to make cancellations before multiplying $\frac{12}{25} \times \frac{15}{16}$

We can apply this same logic to rational expressions.

**<u>Examples</u>** Multiply the following and simplify when possible.

a. $\frac{21x^7}{15y^3} \times \frac{5y^8}{42x^4}$

b. $\frac{2x^2+5x+3}{x^2-25} \times \frac{x^3+125}{4x^2+6x}$



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**Lesson: Dividing Rational Expressions**

How do you divide $\frac{a}{b} \div \frac{c}{d}$? Write a one sentence explanation of how to do this too.

**<u>Example</u>** Divide the following: $\frac{5x}{9} \div \frac{2}{7x^4}$

When can you make cancellations in the process?



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**<u>Examples</u>** Divide.

a. $\frac{12x^7}{45} \div \frac{42x^3}{15}$

b. $\frac{x^2+4x+4}{x+5} \div (x+2)^5$

c. $\frac{x^3+1}{6x^3-6x^2+6x} \div \frac{x^2-1}{8}$



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Explain why $\frac{\frac{5x+5y}{24}}{\frac{15x^2+15xy}{12}}$ is both a fraction and a division problem.

**<u>Example</u>** Divide: $\frac{\frac{16x^2-25}{x^5}}{\frac{36x-45}{9x^7}}$



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**Lesson: Finding the Least Common Denominator for Addition and Subtraction – A Quick Review for Fractions (with only numbers)**

We are going to work on adding and subtracting rational expressions with different denominators. Before we do that, we are going to remind ourselves how to find common denominators for fractions that only have numbers.

The simplest case: what is the least common denominator between the fractions $\frac{1}{2}$ and $\frac{1}{3}$?

Why is that the common denominator?

Now let's look at something slightly more complicated. What is the least common denominator between $\frac{1}{6}$ and $\frac{1}{10}$?

First, take a guess. It's totally fine if this guess is wrong, but just make a guess.

*Why* is this your guess? What did you do?



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There is a specific procedure that you want to use when finding least common denominators in this class. Write down instructions that were given in the video, but write them our in a way that makes sense to you.

Repeat this procedure to find the least common denominator $\frac{1}{15}$ and $\frac{1}{10}$.



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Repeat this procedure to find the least common denominator $\frac{1}{12}$ and $\frac{1}{8}$.

Now we will focus on how to write fractions with this desired denominator.

The simplest case: using the least common denominator you found for $\frac{1}{2}$ and $\frac{1}{3}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.



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Using the least common denominator you found for $\frac{1}{6}$ and $\frac{1}{10}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.

Using the least common denominator you found for $\frac{1}{12}$ and $\frac{1}{8}$, show how to write 2 equivalent fractions with the desired denominator. Write out instructions on how you do this. Summarize what was discussed in the explanation in the video.



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**Lesson: Finding the Least Common Denominator for Rational Expressions**

The simplest case: what is the least common denominator between the fractions $\frac{1}{2x}$ and $\frac{1}{3y^2}$?

**<u>Examples</u>** Find the least common denominator between the following fractions.

a. $\frac{1}{2x^2}, \frac{1}{3xy^2}$

b. $\frac{1}{2x^2y^5}, \frac{1}{3x^7y^2}$

c. $\frac{1}{2x^3y^5z^2}, \frac{1}{3x^5y^7z^8}$



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**Examples** Find the least common denominator for the following.

a. $\frac{1}{14x^3y^2}, \frac{1}{21x^2y^7}$

b. $\frac{1}{24x^3y}, \frac{1}{12x^2y^5}$

What is the least common denominator between $\frac{1}{x+2}$ and $\frac{1}{x}$?



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What is the least common denominator between $\frac{1}{x^2-4}$ and $\frac{1}{x^2+5x+6}$?

**<u>Examples</u>** Find the least common denominator between the following fractions.

a. $\frac{3}{5x-10}, \frac{1}{7x-14}$

b. $\frac{1}{x^2-12x+35}, \frac{1}{x^2+x-20}$



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**Examples** Find the least common denominator between the following fractions.

a. $\frac{1}{x-3}, \frac{1}{3-x}$

b. $\frac{1}{x^2-4}, \frac{1}{2-x}$

c. $\frac{1}{x^2+9x+20}, \frac{1}{x^2-25}, \frac{1}{x^2-x-20}$



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**Lesson: Writing Equivalent Rational Expressions with the Same Denominator**

**<u>Examples</u>** Find the least common denominator between the following rational expressions, and then write each rational expression with it's equivalent and the LCD.

a. $\frac{2}{15x^2y^5}, \frac{4}{9x^3y^2}$

b. $\frac{5}{x+3}, \frac{4}{x}$



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**<u>Examples</u>** Find the least common denominator between the following rational expressions, and then write each rational expression with it's equivalent and the LCD.

a. $\frac{x}{x^2+9x+18}, \frac{2}{x^2+5x+6}$

b. $\frac{x}{5x^2-30x}, \frac{x-2}{2x^2-17x+30}, \frac{2}{2x^2-5x}$



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**Lesson: Adding and Subtracting Rational Expressions**

Show how to add or subtract the following:

a. $\frac{5}{x+2} - \frac{3}{x+2}$

b. $\frac{5}{x-4} + \frac{7}{x}$

c. $\frac{5}{12xy^2} - \frac{7}{8x^3y}$



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d. $\frac{x}{x+3} - \frac{2}{x^2-9}$

e. $\frac{x}{x^2-25} - \frac{5}{x^2-3x-40}$



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**Example/Question: When Are You Allowed to Cancel with Rational Expressions?**

What is the wrong way to cancel $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$ and explain why you can't cancel like that? How can you tell it's wrong?

What is the difference between $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$ and $\frac{3(x+1)(x-3)}{2(x-3)(x+1)}$?

How do you simplify $\frac{(x+1)-3(x-3)}{2(x-3)+4(x+1)}$?



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**Lesson: Complex Fractions**

First let's review how to simplify a complex fraction we've already discussed. Show how to simplify:

$$\frac{\frac{2x}{x+5}}{\frac{2}{x^2-25}}$$

Explain what makes the problem below different from the one we just reviewed above.

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{5}{6}+\frac{1}{3}}$$



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Outline the steps required to simplify. Write out the instructions as they were explained in the video.

$$\frac{\frac{1}{2}-\frac{1}{3}}{\frac{5}{6}+\frac{1}{3}}$$

**<u>Example</u>** Simplify the following

$$\frac{\frac{7}{9}-\frac{1}{3}}{1+\frac{5}{9}}$$



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**Examples** Simplify the following.

a. $\dfrac{\frac{5}{x}+\frac{2}{y}}{\frac{1}{y}-\frac{3}{x}}$

b. $\dfrac{\frac{6}{x+3}-\frac{4}{x-1}}{\frac{2}{x-1}+\frac{5}{x+3}}$



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**<u>Example</u>** Simplify the following.

$$\frac{1-\dfrac{4}{t+5}}{\dfrac{4}{t^2-25}+\dfrac{t}{t-5}}$$



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**Complex Fractions**
**Video 2: More Complicated Examples**

Complex fractions look like this:

$$\frac{\frac{4x}{15y^2}}{\frac{6x^4}{25y^8}} \quad \text{or} \quad \frac{\frac{1}{3}-\frac{1}{6}}{\frac{3}{2}+\frac{5}{6}} \quad \text{or} \quad \frac{\frac{4}{x^2}-\frac{4}{y^2}}{\frac{1}{xy}+\frac{5}{y}}$$

First we will discuss the basics.

**<u>Examples</u>**

a. $\frac{\frac{1}{x^2}-\frac{1}{y}}{\frac{3}{x}-\frac{4}{y^2}}$

b. $\frac{\frac{3}{x}-x}{\frac{2}{x}+x}$



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**Examples**

a. $\dfrac{\frac{3}{x-1}-\frac{2}{x+3}}{\frac{5}{x+3}-\frac{2}{x-1}}$

b. $\dfrac{\frac{5}{x-2}-\frac{3}{x+5}}{\frac{1}{x+5}+\frac{2}{x-2}}$



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**Examples**

a. $\dfrac{1-\frac{1}{x+5}}{\frac{x}{x-5}+\frac{4}{x^2-25}}$

b. $\dfrac{1+\frac{12}{x^2-16}}{\frac{x}{x+4}-\frac{6}{x-4}}$

c. $\dfrac{\frac{1}{x+2}+\frac{4}{x^2-4}}{\frac{3}{x-2}+1}$



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**Lesson: Solving Rational Equations**

How do you know that you need to solve a problem versus simplify it?

Explain how to solve the rational equation. Include some instructions in your notes.

$$\frac{2}{x-1}+\frac{1}{x+4}=\frac{4}{x+4}$$



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What do we have to watch out for when solving rational equations?

**<u>Examples</u>** For the following, which solutions would NOT work for these rational equations?

a. $\frac{4}{x+1} - \frac{2}{x+3} = \frac{5}{x+7}$

b. $\frac{4}{x^2-9} + \frac{3}{x-3} = \frac{2}{x+3}$



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**<u>Examples</u>** Solve.

a. $1 - \frac{8}{x} = \frac{10}{x}$

b. $\frac{5}{x-3} - \frac{2}{x^2-9} = \frac{5x+13}{x^2-9}$



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c. $\frac{x^2}{2} = \frac{x^2-6x}{3}$

d. $\frac{4}{x+1} = \frac{10}{x^2-1} - \frac{5}{x-1}$



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**Solving Equations with Rational Expression**

We will learn how to solve equations that look like this:

$$z = \frac{x+3y}{4w} \quad \text{or} \quad \frac{1}{a} - \frac{2}{b} = \frac{3}{c}$$

**<u>Examples</u>**

a. Solve for y: $z = \frac{3xy}{5w}$

b. Solve for x: $w = \frac{7+2y}{xz}$

c. Solve for z: $y = \frac{4xw}{z-w}$



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**Examples**

a. Solve for x: $\frac{3}{x} + \frac{1}{y} = \frac{2}{z}$

b. Solve for b: $\frac{1}{a} - \frac{1}{b} = \frac{3}{c}$

c. Solve for z: $\frac{5}{x} - \frac{3}{y} = \frac{7}{z}$



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**Lesson: Three Ways to Use the LCD with Rational Expressions**

We have spent quite a bit of learning how to find the LCD between rational expressions and then using it in very different ways. We are going to compare the 3 techniques for how we used the LCD in this video.

First, simplify completely:

$$\frac{9}{x+5} - \frac{4}{x+1}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?



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Simplify completely:

$$\frac{\frac{9}{x+5}-\frac{4}{x+1}}{\frac{10}{x^2+6x+5}-\frac{3}{x+1}}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?



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Solve:

$$\frac{9}{x+5} - \frac{4}{x+1} = \frac{10}{x^2+6x+5}$$

Before we begin, should x have a solution? Why or why not?

Now show your work below.

What type of problem is this? How did we leverage the LCD in this case?



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**Example/Question: When Can You Cross Multiply**

Explain what a ratio is:

When can you cross-multiply?

Solve: $\frac{8}{x+2} - \frac{12}{x-4} = \frac{2}{x+2}$. Show that you CANNOT cross-multiply in this situation.

Now show why cross-multiplying works with $-\frac{x}{5} = \frac{3}{x+8}$



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**Rational Expression Applications**
**Video 1: Proportion Type Problems**

What is a proportion?

A bakery can make 300 cookies in 1.5 hours. How long would it take to make 525 cookies?

The ratio of patrons at a café who purchase coffee versus those who purchase tea is 5 to 3. On Monday, the café sold 45 teas. How many coffees were sold?



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A painter makes a particular shade of orange by using 3 parts red and 4 parts yellow. The painter has four more gallons of yellow paint than he has of red paint. How many gallons of red paint should he use?



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**Rational Expression Applications**
**Video 3: Work Problems**

What is the setup for work problems?

It takes Maria 3 hours to paint a room, while it takes her younger sister Lupe 5 hours to paint the same room. How long would it take if the girls painted the room together?

It takes Max 30 minutes to shovel the driveway, while it takes his brother Darrell 45 minutes to do the same job. How long would it take for the guys to do it together?



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**Rational Expression Applications**
**Video 2: Distance, Rate, and Time Problems**

In these types of problems, we need to use the equation:

$$d = rt$$

Where d=distance, r=rate/speed of a vehicle, and t=time

We can also see that we can rearrange this formula in a few ways:

A boat travels 30 miles downstream in the same time that it takes to travel 20 miles upstream. The current of the river is 5mph. How fast would the boat travel in still water?



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A boat travels 16 miles downstream in the same time that it travels 12 miles upstream. In still water, the boat can travel 6mph. What is the speed of the current?

A plane travels 550 miles with the wind. In the same amount of time, it travels 375 miles against the wind. If the wind is blowing at a speed of 25 mph, what is the speed of the plane when there is no wind?



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**Lesson: Sequences**

Define a sequence.

**<u>Examples</u>** Write the first five terms of each sequence.

a. $a_n = \frac{1}{n}$

b. $a_n = \frac{n\mp 1}{n+3}$

c. $a_n = (-1)^{n+1} * (2n + 1)$



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What does it mean for a sequence to converge?

What does it mean for a sequence to diverge?

What is a recursively defined sequence?



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**Examples** Find the first four terms in the sequence.

a. $a_1 = 3; a_n = 2a_{n-1} - 4$

b. $a_1 = 2; a_n = a_{n-1}^2 + n - 2$



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**Lesson: Series**

What is the summation sign and what does it mean? What do we called the “i” in this formula?

Define a finite series.

Define an infinite series.



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**<u>Examples</u>** Evaluate the following sums.

a. $\Sigma_{i=1}^{4}\left(3^i + 1\right)$

b. $\Sigma_{j=3}^{6} a_j$

c. $\Sigma_{k=1}^{4}(4x_k - 1)$ if $x_1 = 2, x_2 = 3, x_3 = 5, x_4 = 8$

d. $\Sigma_{i=1}^{3} f(x_i)\Delta x$, if $f(x) = 3x, x_1 = 1, x_2 = 4, x_3 = 7, \Delta x = 3$



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What are the summation properties?

What are the summation rules for summing up terms?



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**Examples** Use the summation properties and rules to evaluate each series.

a. $\Sigma_{i=1}^{50} 3$

b. $\Sigma_{k=1}^{25} 3k$

c. $\Sigma_{j=1}^{33} (4j^2 + 1)$



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**Lesson: Arithmetic Sequences**

What is an arithmetic sequence?

Give some examples of an arithmetic sequence.

**<u>Example</u>** Determine the common difference, d, for the arithmetic sequence:

$$-7, -5, -3, -1, \ldots$$



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**Example** Find the first five terms of each arithmetic sequence.

a. The first term is 3, and the common difference is -2.

b. $a_1 = -5, d = 7$

How do we define the general nth term of an arithmetic sequence?



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**Example** Determine $a_{11}$ and $a_n$ for the arithmetic sequence $-2, 2, 6, 10, \ldots$

**Example** Determine $a_n$ and $a_{20}$ for the arithmetic sequence having $a_3 = 9$, and $a_4 = 16$.

**Example** An arithmetic sequence has $a_7 = -15$ and $a_{14} = -35$. Determine $a_1$.



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**Lesson: Arithmetic Series**

When we sum up the terms of an arithmetic sequence, how can we define that sum? (sum of an arithmetic sequence)

**<u>Example</u>** Consider the arithmetic sequence $-7, -3, 1, 5, \ldots$

a. Evaluate $S_{10}$

b. Evaluate the sum of the first 50 positive integers.



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**Example** The sum of the first 15 terms of an arithmetic sequence is 345. If $a_{15} = 65$, find $a_1$ and d.

**Examples** Evaluate each sum.

a. $\Sigma_{i=1}^{19}(-3 - 5i)$

b. $\Sigma_{k=4}^{15}(2 + 4k)$



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**Lesson: Geometric Sequences and Series**

Define a geometric sequence:

**<u>Example</u>** Determine $a_6$ and $a_n$ for the geometric sequence 4, 12, 36, 108, ….

**<u>Example</u>** Determine r and $a_1$ for the geometric sequence with $a_2 = -18$ and $a_5 = 486$



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How do we define the sum of the first n terms of a geometric sequence?

**<u>Example</u>** Evaluate: $\Sigma_{i=1}^{8} 3 * 2^i$

If we start with 2 and use a common ratio of $\frac{1}{2}$, what eventually happens as the series continues infinitely? Summarize the explanation below.



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**Examples** Evaluate the following infinite sums.

a. The first term is 1 and the common ratio is $\frac{1}{3}$

b. $\Sigma_{i=1}^{\infty}\left(-\frac{1}{2}\right)\left(-\frac{3}{4}\right)^{i-1}$

c. $\Sigma_{i=1}^{\infty}\left(\frac{3}{7}\right)^{i}$



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**Lesson: Angle Relationships**

Draw and explain what vertical angles are:

Draw and explain what makes a transversal line. Draw a big picture, you will use it for the next part of your notes:

Label the diagram and note the relationships between the different angles below:



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**Example** Draw and label the lines from the video. Find the measures of angles 1, 2, 3, and 4.

**Example** Draw and label the lines from the video. Find the measures of angles 1, 2, 3, and 4.



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**Lesson: Review of Triangles and Similar Triangles**

The sum of angle measures of any triangle is:

Draw the different types of triangles:

**<u>Examples</u>** Draw the triangles in the example, then classify the triangles as acute, right, or obtuse. Also classify as equilateral, isosceles, or scalene.



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What makes 2 triangles similar?

What makes 2 triangles congruent?

What is the key relationship between similar and congruent triangles?

**<u>What are the 2 conditions for similar triangles?</u>**

In other words, triangle ABC is similar to triangle DEF if:



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**Example** Draw the similar triangles in the video, and then find any missing angle measures.

**Example** Draw the similar triangles in the video, and then find any missing side lengths.

**Example** Draw the similar triangles in the video, and then find any missing measures.



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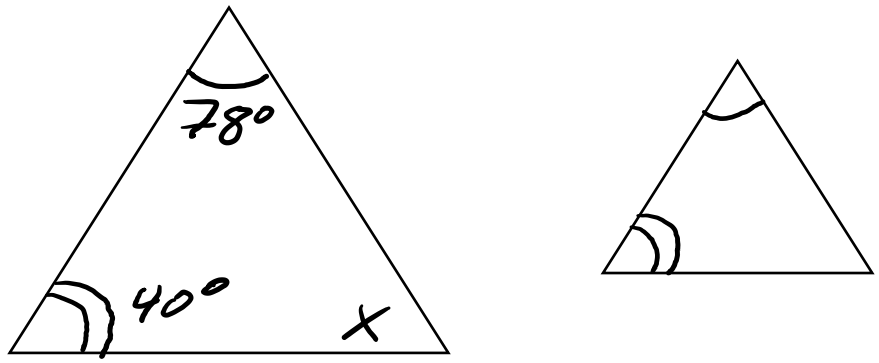
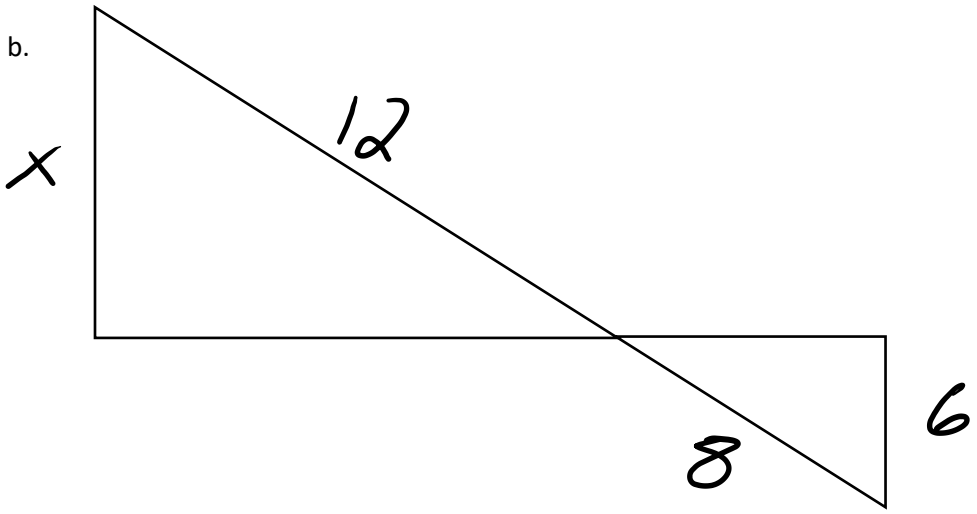
**Examples: Similar Triangles**

For each example, assume the triangles are similar. Triangles are not drawn to scale. Find any missing angle or side measures.

a.




b.



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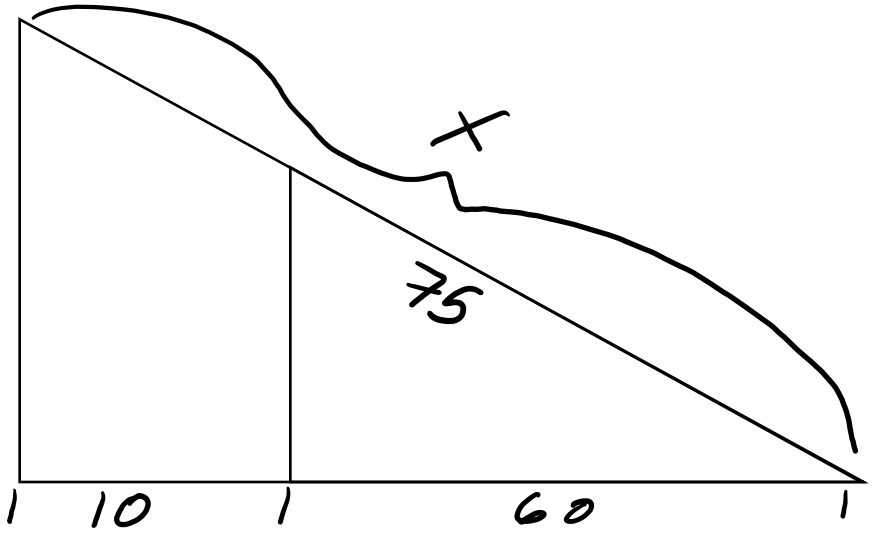
c.



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**Lesson: Right-Triangle Based Definitions of Trigonometric Functions and Cofunctions**

Instead of using standard position to describe trig functions, we can alternatively use right triangles.

Draw a right triangle to orient yourself.

What does A represent?

Show how to define the 6 trig functions with this right triangle.

1.

2.

3.

4.

5.

6.



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What mnemonic device can help you remember the trig functions?

**Example** Draw the triangle in the video, and then show how to find the values of sine, cosine, and tangent.

**Example** Using the same example above, show how to find the values of cosecant, secant, and cotangent.



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Draw a new right triangle, and show an alternative angle measure we can find from this picture:

These relationships are called the **cofunction identities,** summarize these below:



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**Examples** Write each function in terms of its cofunction

a. $\sin 10°$

b. $\cot 65°$

c. $\csc 34°$

**Example** Find one solution for each equation. Assume all angles involved are acute angles.

a. $\cos\theta = \sin(2\theta - 30°)$

b. $\sec(\theta + 15°) = \csc(2\theta + 25°)$



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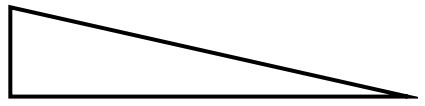
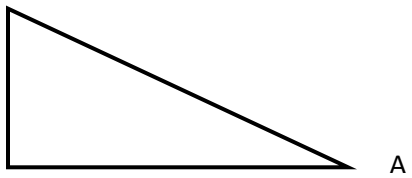
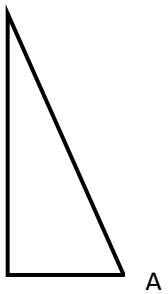
**Lesson: More on Trig Function Values as Angles Change and Special Angles**




Explain how angle A changes in these pictures:

**<u>Examples</u>** Determine whether each statement is true or false.

a. $\tan 25° < \tan 22°$

b. $\cos 25° < \sin 25°$

There are a few angles that occur frequently in trig and have some *nice* properties that we like to call them out. Draw out a triangle where each angle is $60°$ and describe any other properties:



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If you bisect this triangle, what happens? Draw a picture to illustrate, and then describe the trig functions associated with it:



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Finally, draw an isosceles right triangle and describe any special characteristics with it:



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We have now examined some *extremely* common trig measures that will come up all the time in trig. They are so important that we will create a separate reference sheet for them.

**Read this!** I highly recommend that you remove the last page from this packet, and instead put it at the very front of your notes so that you can easily reference it. If you are moving on to calculus, KEEP THIS PAGE IN GOOD CONDITION FOR THE NEXT FEW YEARS. You will thank yourself later that you have this reference as these angles will come up over and over again, and students frequently forget them. I will occasionally create these handouts and make these suggestions to you as we go along in trig – you will thank yourself (and me) later if you keep these. It is extremely common for students to forget some of KEY parts of trig and then struggle in calculus – I don't want this to happen to you!

# Some Function Values of Special (Common) Angles

| $\theta$ | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ | $\cot\theta$ | $\sec\theta$ | $\csc\theta$ |
|---|---|---|---|---|---|---|
| $0°$ | | | | | | |
| $30°$ | | | | | | |
| $45°$ | | | | | | |
| $60°$ | | | | | | |
| $90°$ | | | | | | |



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**Lesson: A Review of Angles and Degrees**

Draw, label, and define how to make an angle:

Show how to create an angle of positive versus negative measure:

**Degree** is generally how we measure angles.

Draw the complete rotation of a ray. How many degrees is this?



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Define:

- Acute angle

- Right angle

- Obtuse angle

- Straight angle

Define:

- Complementary angles

- Supplementary angles



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**Example** Find the complement and supplement of a $55°$ angle.

**Example** Find the measures of the marked angles shown in the video.



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**Examples: Converting Between Minutes and Seconds**

**<u>Examples</u>** Convert the following to degrees.

a. $23°15'32''$

b. $38°27'15''$

**<u>Examples</u>** Convert the following to degrees, minutes, and seconds.

a. $32.713°$

b. $72.17°$



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**Examples: Adding and Subtracting Degrees, Minutes, and Seconds**

Add or subtract.

a. $45°15' + 23°57'$

b. $23°18'29'' + 14°19'47''$

c. $94°16' - 27°29'$

d. $72°18'15'' - 15°20'30''$



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**Examples: Applications of Minutes and Seconds**

**Example** A turntable makes 45 revolutions in a minute. How many revolutions is that per second?

**Example** A windmill makes 80 revolutions per minute. How many revolutions is that per second?

**Example** Wind power is caused by a turbine spinning 10 to 20 revolutions per minute. How many revolutions does it make per second?



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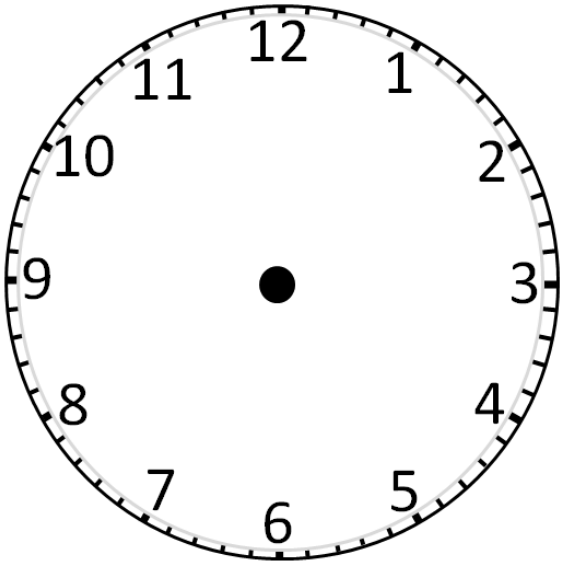
**Lesson: Angle Measure with a Clock**

Explain the basics of angle measure, ie where is $90°, 180°$?

Each number represents how many degrees? SHOW how to actually determine this?

**<u>Example</u>** Find in the clock below, and show how to find the angle measure for 5:00. Draw pictures to help illustrate how this works, pictures are key to remembering!!!



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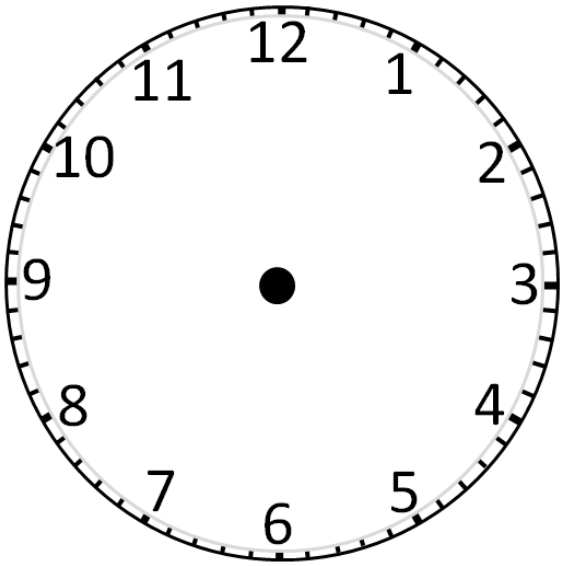
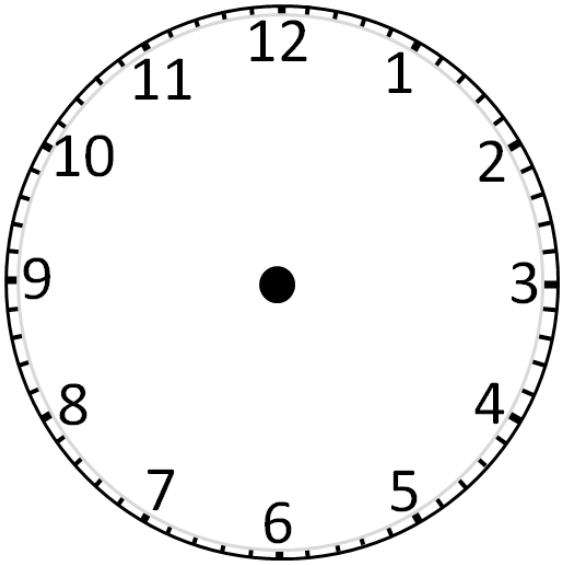
**Example** Find in the clock below, and show how to find the angle measure for 8:20. Draw pictures to help illustrate how this works, pictures are key to remembering!!!




**Example** Find in the clock below, and show how to find the angle measure for 10:05. Draw pictures to help illustrate how this works, pictures are key to remembering!!!



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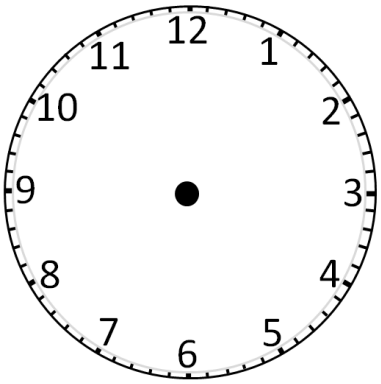
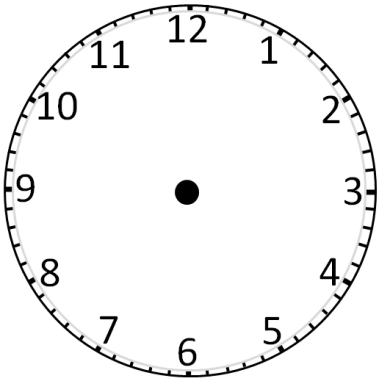
**Examples: Angle Measure from a Clock**

Find the angles associated with each time.

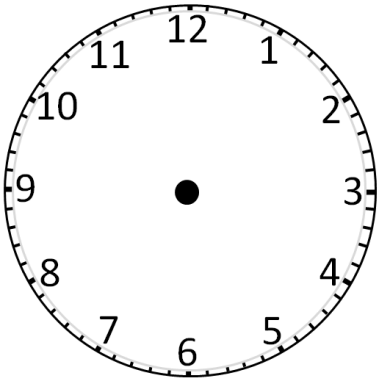
a. 6:25




b. 3:15




c. 2:38



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**Lesson: Standard Position and Coterminal Angles**

When is an angle in standard position? Draw an example.

Show some examples of how we can determine angle measure by quadrant.

What are quadrantal angles? Draw a picture to illustrate.



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What makes two angles coterminal? Draw a picture to explain.

**Example** Find the angle of least positive measure that is coterminal with each angle:

a. $715°$

b. $-80°$

**Example** How can you find all angles coterminal with $45°$? Prove it by filling in the following table.

| Value of n | Angle coterminal with $45°$ |
|---|---|
| 2 | |
| 0 | |
| -1 | |



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**Examples** Fill in the table below for which angles are coterminal with the quadrantal angles

| Quadrantal Angle | Coterminal Angles |
|---|---|
| $0°$ | |
| $90°$ | |
| $180°$ | |
| $270°$ | |



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**Lesson: Radian Measure**

Radian measure is another way to describe angle measure, as opposed to degrees.

What does radian measure allow us to do?

What is the definition of a radian?

Are radians a unit of measure like feet or meters? Circle: Yes No

What is the formula for the distance around a circle?

What does this formula show?

Write a way to relate radians to degrees:



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So what is $\pi$ radians?

Show a conversion rate between radians and degrees:

**<u>Examples</u>** Convert each degree to radians:

a. $35°$

b. $-175°$

c. $27.8°$

Convert each radian measure to degrees:

a. $\frac{7}{4}\pi$

b. $-\frac{2\pi}{3}$

c. $7$



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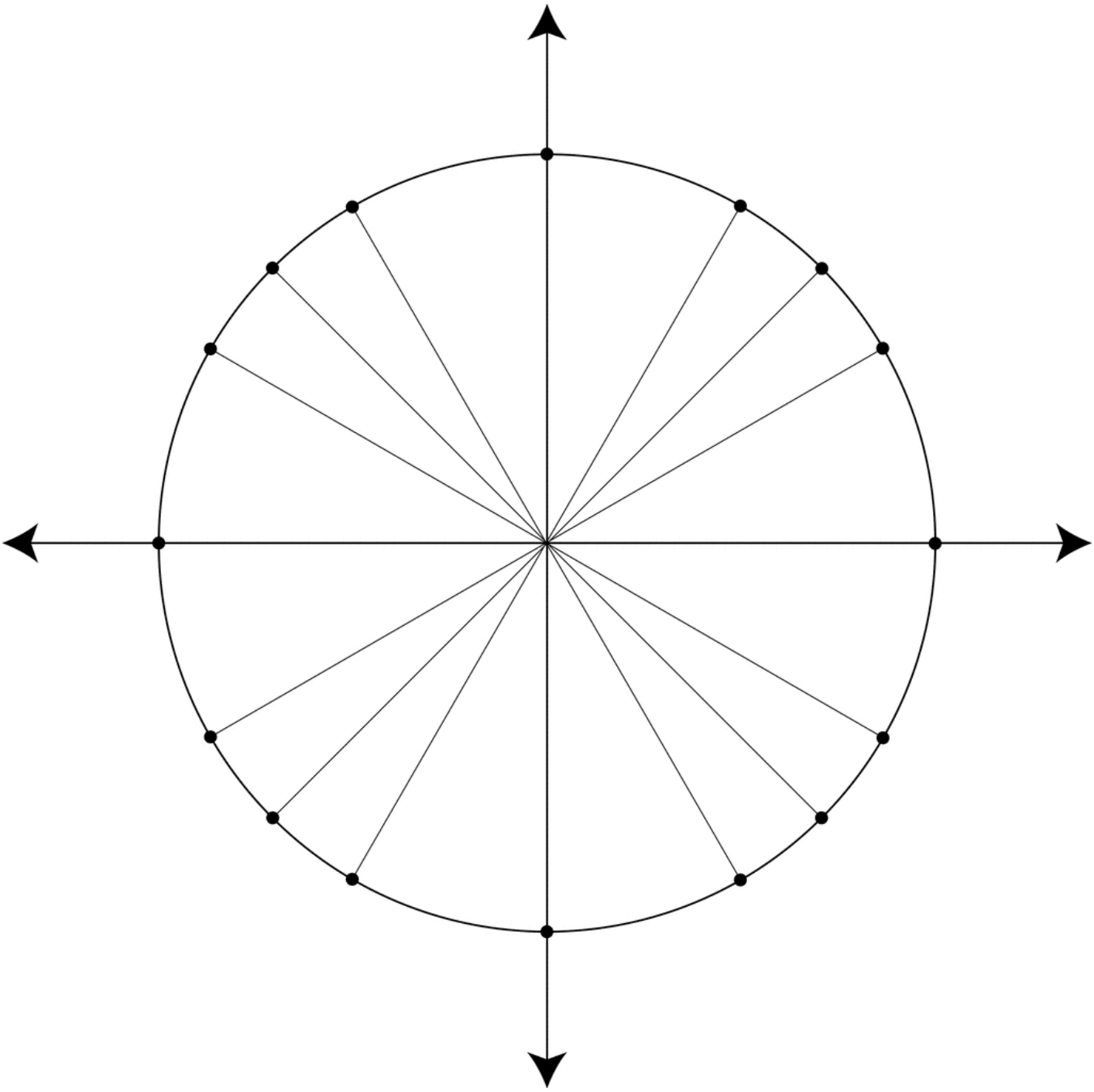
If no unit of angle measure is specified, what should you assume?

Below we will construct an illustration of all the common degree measures we have discussed, and then write their radian equivalent. Take your time to copy this perfectly!





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You will need to know all of those common degrees AND radian measures for the exam!

You may want to reference cheat sheets to help you with the next few problems.

**<u>Examples</u>** Find the value of each function.

a. $\sin\frac{5\pi}{6}$

b. $\tan\frac{3\pi}{4}$

c. $\csc\frac{5\pi}{3}$

d. $\cos\left(-\frac{2\pi}{3}\right)$



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**Lesson: Arc Length on a Circle and Some Applications**

We can determine the formula for the arc length of a circle from the definition of radians.

Regarding radians, what does $\theta$ equal? Show how this relates to arc length.

What is key about the arc length formula?

**<u>Examples</u>** A circle has radius 22.30 cm. Find the length of the arc intercepted by a central angle having each of the following measures.

a. $\frac{5\pi}{6}$ radians

b. $66°$



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**Examples** A circle has radius 15.22 cm. Find the length of the arc intercepted by a central angle having each of the following measures.

a. $\frac{3\pi}{8}$

b. $125°$

What is the definition of latitude?

**Example** The latitude of Panama City, Panama is $9°$ N and the latitude of Pittsburgh, PA is $40°$ N. Find the north-south distance of these 2 cities. The radius of the Earth is 6400 km.



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**Example** A rope is being wound around a drum with a radius of 0.7525 ft. How much rope will be wound around the drum if the drum is rotated through an angle of $42.25°$?

**Example** Two gears are adjusted so that the smaller gear drives the larger one. If the smaller gear rotates through an angle of $150°$, through how many degrees will the larger gear rotate? Refer to the photo in the video.



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**Examples: Applications using Arc Length**

Charleston, SC and Toronto, Canada, are 1100 km apart and lie on the same north-south line. If the latitude of Charleston is $33°$ N, what is the latitude of Toronto?

Find the radius of the larger wheel in the figure if the small wheel rotates $75.0°$ when the large wheel rotates $45°$.



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How many inches will a small weight rise if a pulley is rotated through an angle of $65°15'$? Through what angle, to the nearest minute, must the pulley be rotated to raise the weight 4 inches?



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**Lesson: Area of a Sector of a Circle and Some Applications**

What do we define as a sector of a circle?

What is the formula for the area of a section of a circle with radius r?

**<u>Examples</u>** Find the area of a sector of a circle having the given radius and central angle $\theta$. Answers should be to the nearest tenth.

a. $r = 11.5$ ft, $\theta = \frac{5\pi}{6}$ radians

b. $r = 13.5$ m, $\theta = 125°$



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**Example** Find the measure (in radians) of a central angle of a sector of a circle of area $25\ in^2$ in a circle of radius 2.5 in.

**Example** A center-pivot irrigation system provides water to a sector-shaped field as shown in the figure in the video. Find the area of the field if $\theta = 35.0°$ and $r = 155$ yd.

**Example** A circular sector has an area of $60\ in^2$. The radius of the circle is 6 inches. What is the arc length of the sector?



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**Lesson: Linear Speed and Angular Speed and Applications**

What is the formula for linear speed?

When are some situations where you might use linear speed?

What is the formula for angular speed?

When are some situations where you might use angular speed?



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**Examples** Use the formula $\omega = \frac{\theta}{t}$ to find the value of the missing variable.

a. $\omega = \frac{\pi}{3}$ radians per second, $t = 6$ seconds

b. $\theta = 3.721$ radians, $t = 18.34$ seconds

**Examples** Use the formula $v = r\omega$ to find the value of the missing variable.

a. $r = 10\ m$, $\omega = \frac{9\pi}{5}$ radians per second

b. $v = 10$ m per second, $\omega = \frac{3\pi}{2}$ radians per second



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**<u>Example</u>** Suppose a point P is on a circle with a radius of 5 cm, and the ray OP Is rotating with an angular speed of $\frac{\pi}{10}$ radian per second.

a. Find the angle generated by P in 3 seconds.

b. Find the distance traveled by P along the circle in 3 seconds.

c. Find the linear speed of P in cm per second.



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The formula $\omega = \frac{\theta}{t}$ can be rewritten as $\theta = \omega t$. We can then use the formula $s = r\theta$ to create a different formula. Show the new formula we can create:

**<u>Examples</u>** Use the formula above to find the value of the missing variable.

a. $r = 5\ cm, \omega = \frac{2\pi}{3}$ radians per second, $t = 8$ seconds

b. $s = \frac{3\pi}{2}$ km, $r = 4$ km, $t = 3$ seconds



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**Example** The orbit of the earth around the sun is almost circular. Suppose that the orbit has a radius of 93 million miles.

a. We know that a year is 365 days. Find the angle formed by the Earth's movement in one day.

b. Give the angular speed in radians per hour.

C. Find the linear speed of the Earth in miles per hour.



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**Examples: Applications of Linear Speed and Angular Speed**

**Example** Find the angular speed of the minute hand of a clock.

**Example** Find the linear speed of the tip of the second hand of a clock, if the hand is 25 mm long.

**Example** A bicycle has tires with a 12-inch radius. The tires are turning at the rate of 200 revolutions per minute. How fast if the bike traveling in miles per hour?



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**Example** A pulley has a radius of 13.23 cm. It took 15 seconds for 45 cm of belt to go around the pulley.

a. Find the linear speed of the belt in cm per sec.

b. Find the angular speed of the pulley in radians per second.

**Example** A thread is being pulled off a spool at a rate of 55.4 cm per second. Find the radius of the spool if it makes 155 revolutions per minute.



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**Lesson: The Pythagorean Theorem, The Distance Formula, and Intro to Trig Functions**

State the Pythagorean theorem:

State the distance formula:

How does the Pythagorean theorem relate to the distance formula?

Now we will introduce **trigonometric functions!** Explain what makes trig functions unique:



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How do trig functions relate to the distance formula? Draw the picture associated with this.

Explain the 6 trig functions and how they relate to what we discussed above.

Sometimes you will see expressions such as $\sin^2 90°$. What does this mean?



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**Example** The terminal side of an angle $\theta$ in standard position passed through the point (-12, -5). Find the values of the 6 trig functions of angle $\theta$.

**Example** Find the 6 trig function values of an angle $\theta$ in standard position if the terminal side of $\theta$ is defined by

$3x + 5y = 0, x \geq 0$



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**Example** Suppose the point $(x, y)$ is in the indicated quadrant. Determine whether given ratio is positive or negative.

$$III, \frac{y}{r}$$



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**Examples: Finding Values of Trig Functions**

For each given point, sketch the angle of $\theta$ in standard position such that $\theta$ has the least positive measure and the given point is on the terminal side. The find the values of the 6 trig functions.

a. $(-3, -4)$

b. $(7, 4)$

c. $\left(-\sqrt{2}, \sqrt{2}\right)$



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**Examples: Finding the 6 Trig Functions Given an Equation of the Terminal Side**

An equation of the terminal side of an angle $\theta$ (in standard position) is given with a restriction on x. Sketch the situation with least positive angle $\theta$, and find the values of the 6 trig functions.

a. $-6x - y = 0, x \geq 0.$

b. $x + y = 0, x \geq 0$

c. $x = 0, y \geq 0.$



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**Lesson: Quadrantal Angles and Trig Functions**

What were the quadrantal angles?

If one of the quadrantal angles is involved, how does this impact how we think about the trig functions? Draw a sketch to help illustrate this point. Explain how these are different from non-quadrantal angles, and how we choose points in these cases.



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**Example** Find the indicated value of: $\sin 90°$.

How should we think about something like $\sin^2 90°$?

**Example** Use trigonometric function values of quadrantal angles to evaluate: $\tan 0° - 6 \sin 90°$.

**Example** Use trigonometric function values of quadrantal angles to evaluate: $\sin^2 180° + \cos^2 180°$.



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**Examples: Quadrantal Angles and Trig Functions**

Find the function values:

a. $\cot 90°$

b. $\sec 180°$

c. $\sin(-270°)$

d. $3\sec 180° - 2\tan 360°$

e. $\sin^2 90° - 3\cos^2 90°$



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**Lesson: Reference Angles and Non-Acute Angles**

Review: what is a quadrantal angle?

What is associated with every non-quadrantal angle?

Show how we can find reference angles by drawing the reference graphs in the video:

**<u>Example</u>** Find the reference angle for each angle.

a. $274°$

b. $822°$



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Draw how to find the reference angle based on the quadrant (follow along with the video and draw the diagram on your own):

Now let's talk about using $30°$, $45°$, and $60°$ as reference angles.

**<u>Example</u>** Find the values of the 6 trig functions for $240°$. Draw a reference!!! (This class is all about pictures!!)

Don't forget the helpful device: All Students Take Calculus 😊



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Let's outline the steps for finding trig function values for any non-quadrantal angle:

1.

2.

3.

4.

How can you avoid making an error with finding a trig function value?

**<u>Examples</u>** Find the exact values:

a. $\sin(-150°)$

b. $\cos(-510°)$



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**Example** Evaluate: $\sin^2 120° + \cos^2 120°$

**Example** Find all values of $\theta$, if $\theta$ is in the interval $[0°, 360°)$ and has the given function value: $\cos\theta = \frac{\sqrt{3}}{2}$

**Example** Is the following true or false? $\cos 60° = 2\cos 30°$



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**Examples: Reference Angles and Finding Trig Value Functions**

Find the exact values of all 6 trig functions of each angle. Rationalize when applicable.

a. $315°$

b. $-855°$

Evaluate

a. $\cot^2 120° + \tan^2 60° - \cos^2 330°$

b. $\cot^2 135° - \tan^4 45° - \sin^2 150°$



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**Examples: Finding the Value of Theta**

Find all values of $\theta$ if $\theta$ is in the interval $[0°, 360°)$ and has the given value.

a. $\sec\theta = -\sqrt{2}$

b. $\cot\theta = -\frac{\sqrt{3}}{3}$

c. $\cos\theta = \frac{1}{2}$



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**Lesson: The Unit Circle and Circular Functions**

Explain how we create the unit circle:

What is the equation for the unit circle?

Recall the formula for arc length is $s = r\theta$. When the radius of the circle is 1, what does this formula become?

Why does this matter / what is cool about this?



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Since r=1 on the unit circle, what can we say about our previously studied trig functions? Just summarize what we discussed in the video as best as you can.

Write out the **circular functions**

1.

2.

3.

4.

5.

6.



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Explain the symmetry of the unit circle:

If a point $(a, b)$ in on the unit circle, what other points are guaranteed to be on the unit circle?

How can we use symmetry with drawing the unit circle? How is it handy?



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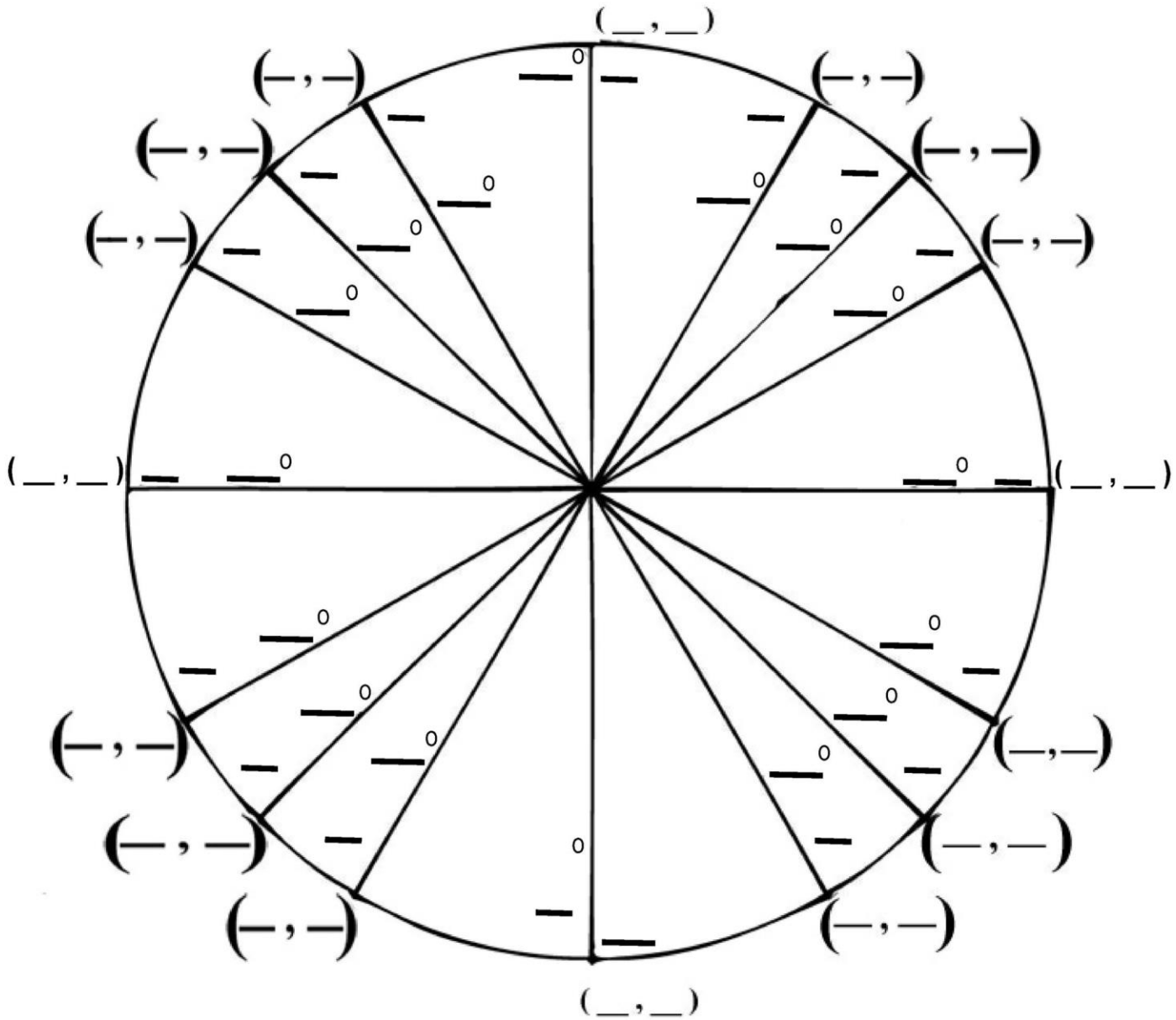
Label the unit circle. Note: you will have to have this memorized! These are the standard angle measures that you'd be expected to do without a calculator!




Now, going back to the circular functions we discussed, how do you view the x-coordinates?

How do you view the y-coordinates?



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**Examples** Use the unit circle to find the values of the following:

a. $\sin\frac{\pi}{3}$

b. $\cos\frac{5\pi}{4}$

c. $\sin\frac{11\pi}{6}$



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**Lesson: Arc Length on a Circle and Some Applications**

Using the unit circle, we can also determine the domains of our circular functions.

What is the domain of sine and cosine, and why?

What is the domain of tangent and secant, and why?

What is the domain of cotangent and cosecant, and why?



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**Lesson: Values of Circular Functions**

We have previously discussed the unit circle (you will want to have it handy). One the most important things about the unit circle is that it will quickly tell you the value of the sine and cosine functions.

On the unit circle, what does the y-coordinate represent?

On the unit circle, what does the x-coordinate represent?

When we think about the values of these functions, what do we assume?

What does this mean for your calculator?

**<u>Examples</u>** Use the unit circle to find the values of:

a. $\sin\frac{\pi}{2}$ b. $\cos\frac{\pi}{2}$ c. $\tan\frac{\pi}{2}$



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**Examples** Use the unit circle to find the values of:

a. $\sin\frac{5\pi}{4}$ b. $\cos\frac{5\pi}{4}$ c. $\tan\frac{5\pi}{4}$

**Example** Compare methods. Use reference angles and degrees to find the exact value of $\sin\frac{2\pi}{3}$, then confirm your answer with the unit circle.

For approximate values, we still must use a calculator. Remember, when you are not specifically told you are using degrees, then you assume you are using radians.

What must you ensure with your calculator?

**Examples** Use a calculator to approximate the function values.

a. $\cos(1.72)$

b. $\sin(-3.54)$

c. $\cot(1.438)$



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**Lesson: Values of Inverse Circular Functions**

What do the functions $\sin^{-1} x$ , $\cos^{-1} x$, $\cot^{-1} x$, etc. represent?

Your calculator makes a few key assumptions if it uses these functions (the reasons why are explained later):

1. For all x in $[-1, 1]$, using $\sin^{-1} x$ with a calculator will return a value between:

2. For all x in $[-1, 1]$, using $\cos^{-1} x$ with a calculator will return a value between:

3. For all real numbers x, using $\tan^{-1} x$ with a calculator will return a value between:



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Sometimes you will be asked to find answers in different quadrants than what your calculator will return!

**<u>Example</u>** Solve for s, given that s is in the interval $\left[0, \frac{\pi}{2}\right]$: $\sin s = 0.3311$

**<u>Example</u>** Solve for s, given that s is in the interval $\left[0, \frac{\pi}{2}\right]$: $\tan s = 0.2155$



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**Example** Solve for s, given that s is in the interval $\left[\pi, \frac{3\pi}{2}\right]$: $\tan s = \sqrt{3}$

**Example** Solve for s, given that s is in the interval $[0, 2\pi)$: $\cos^2 s = \frac{1}{2}$



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**Lesson: The Unit Circle and Circular Functions as Line Segments**

The unit circle is really just an extension of how we have previously thought about trig functions as line segments / right triangles.

Draw the illustration in the video, then show how these sides relate to our trig functions.

From your illustration, how do we define the following?

$\cos\theta =$ $\tan\theta =$ $\csc\theta =$

$\sin\theta =$ $\cot\theta =$ $\sec\theta =$



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**Example** Using your illustration from the previous page, suppose the measurement of TVU is $45°$. Find the lengths of the following line segments:

OQ

OU

OV

VR

US

PQ



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**Lesson: Sine and Cosine as Periodic Functions**

What is a periodic function?

What are some examples?

What are 2 known periodic functions?

What can we say about the behavior of these functions as we move along the unit circle?

| AS $s$ CHANGES FROM | THE VALUE OF $\sin s$ | THE VALUE OF $\cos s$ |
|---|---|---|
| $0$ TO $\frac{\pi}{2}$ | | |
| $\frac{\pi}{2}$ TO $\pi$ | | |
| $\pi$ TO $\frac{3\pi}{2}$ | | |
| $\frac{3\pi}{2}$ TO $2\pi$ | | |



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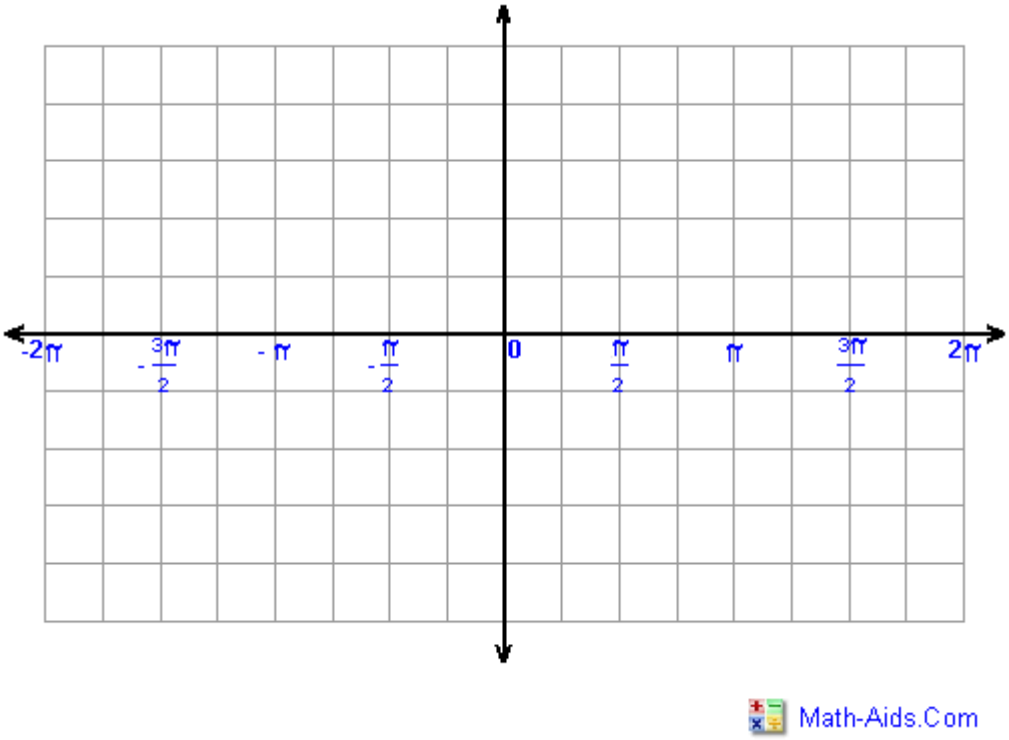
But let's change s to x going forward so we don't get confused.

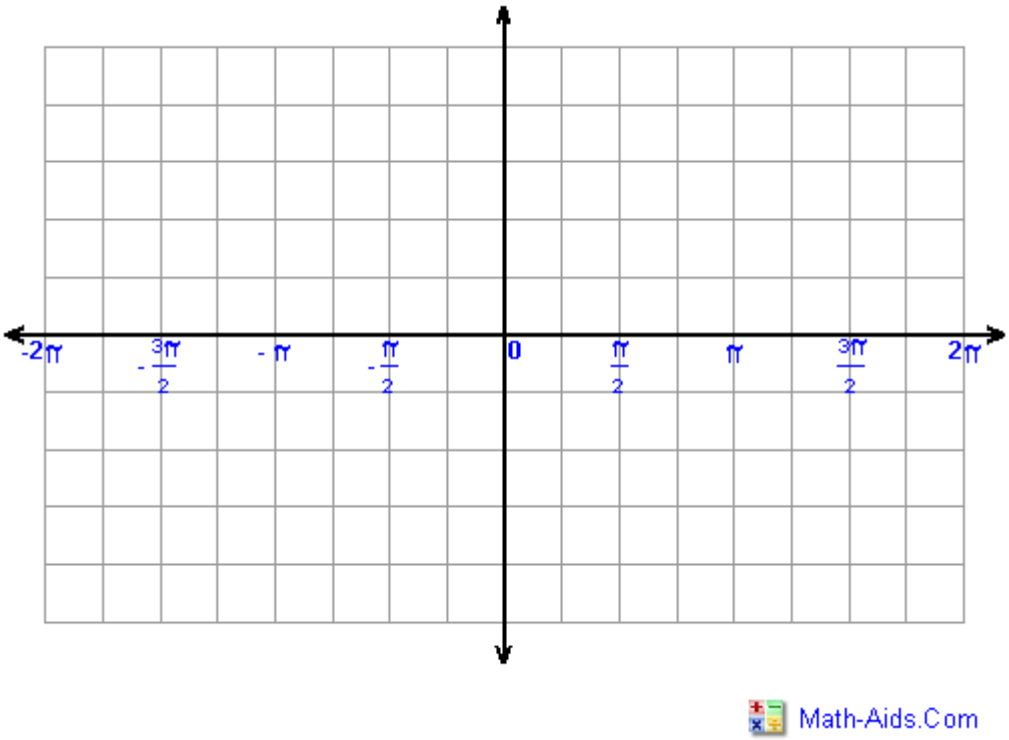
**Draw the graph of sine, filling in the table and then using the graph paper.**

| $x$ | $y = \sin x$ |
|---|---|
| $0$ | |
| $\frac{\pi}{4}$ | |
| $\frac{\pi}{2}$ | |
| $\pi$ | |
| $\frac{3\pi}{2}$ | |
| $2\pi$ | |




**Draw the graph of cosine, filling in the table and then using the graph paper.**

| $x$ | $y = \sin x$ |
|---|---|
| $0$ | |
| $\frac{\pi}{4}$ | |
| $\frac{\pi}{2}$ | |
| $\pi$ | |
| $\frac{3\pi}{2}$ | |
| $2\pi$ | |



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What term do we use to describe the graph of sine?

How do we describe the graph of cosine?

What is the period of both the sine and cosine functions?

Where are the graphs continuous?

What is the domain and range of the graphs?

How can we describe the x-intercepts of sine?

How can we describe the x-intercepts of cosine?



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What is the test for even or odd functions?

What does it mean to be an even function?

What does it mean to be an odd function?

How do you know that $y = \sin x$ is odd? How would you show it?

How do you know that $y = \cos x$ is even? How would you show it?



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**Lesson: Using Amplitude and Period to Graph Sine and Cosine**

What is amplitude?

Using the equation $y = a \sin x$, when will this graph stretch or shrink? Draw 2 comparison sketches to help you remember (just a quick sketch so you remember how these visually compare)

Using the equation $y = a \cos x$, when will this graph stretch or shrink? Draw 2 comparison sketches to help you remember (just a quick sketch so you remember how these visually compare)



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What happens when $a$ is negative? Demonstrate this with some brief sketches so that you can remember.

What is period?

Using the equation $y = \sin bx$, watch the video and write an explanation with sketches to demonstrate period.



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Using the equation $y = \cos bx$, watch the video and write an explanation with sketches to demonstrate period.

What formula can you use to determine the period of a function?

**<u>Examples</u>** Determine the periods of the following functions:

a. $y = \sin 3x$

b. $y = \cos \frac{1}{2} x$

c. $y = \sin 2\pi x$



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Summarize amplitude and period when the function is of the form $y = a \sin bx$ and $y = a \cos bx$

**<u>Examples</u>** Graph the following over 2 period intervals, completing the table with strategic points to help you graph. Give the period and amplitude in each. You will have to sketch your own graph and create your own axes.

$y = \cos \frac{1}{3} x$

**Period:** **Amplitude:**

| $\boldsymbol{x}$ | $\boldsymbol{y = cos \frac{1}{3} x}$ |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |



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$y = 2\sin x$

**Period:** **Amplitude:**

| $\boldsymbol{x}$ | $\boldsymbol{y = 2\,sin\,x}$ |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |



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$y = 3\cos 2\pi x$

**Period:** **Amplitude:**

| $x$ | $y$ |
|---|---|
| | |
| | |
| | |
| | |
| | |
| | |



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**Examples: Using Amplitude and Period to Graph Sine and Cosine**

Graph the following over two-period intervals.

a. $y = -\frac{1}{2}\sin x$



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b. $y = 3\cos 2x$

d. $y = -\sin \pi x$



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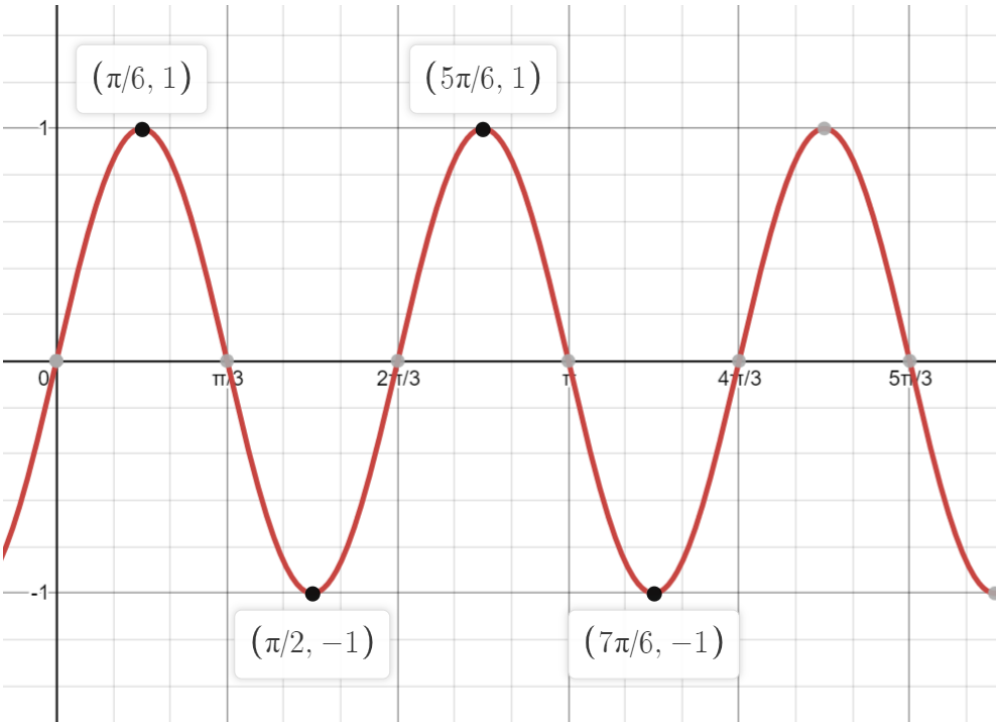
**Lesson: Determining the Equations of Sine and Cosine from Graphs**

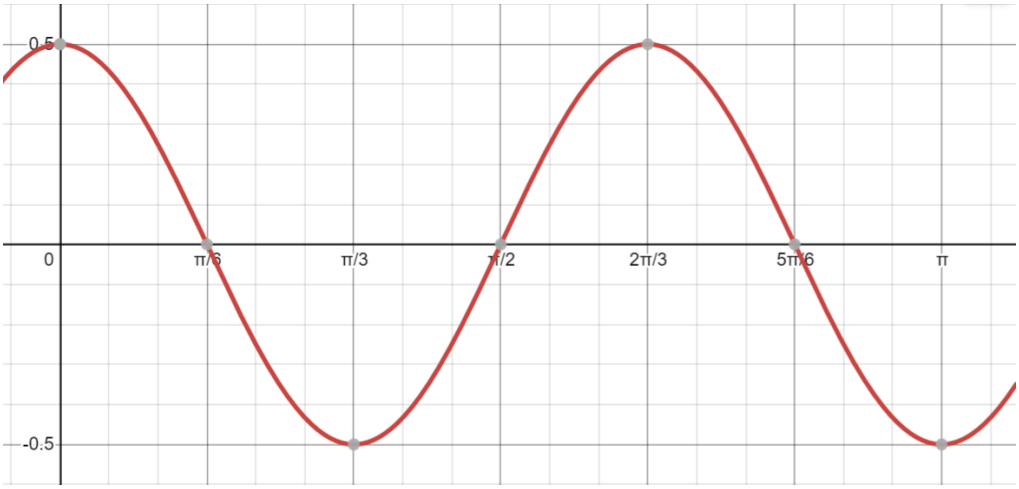
In each graph, determine the equation of the form $y = a\cos bx$ or $y = a\sin bx$, with $b > 0$, for each graph.

Desmos.com was used to create these graphs.

a.




b.



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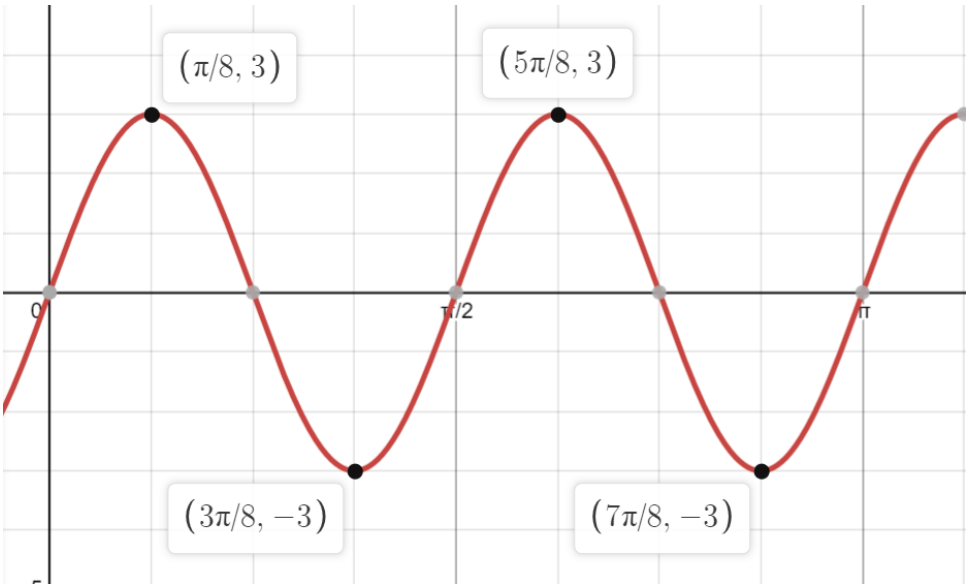
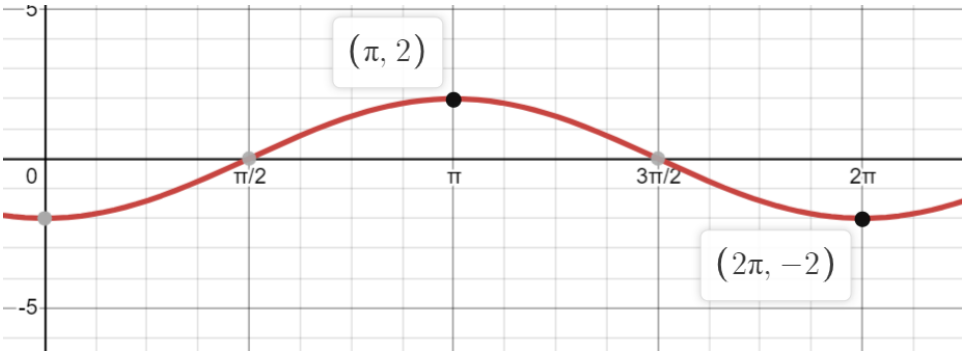
c.




d.



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**Lesson: Translations of the Sine and Cosine Graphs**

How do you describe a horizontal translation? In your own words / explanation, summarize with sketches how the video show these translations work.

What is phase shift?

What is an argument?

How do you describe a vertical translation? In your own words / explanation, summarize with sketches how the video show these translations work.

What happens when a function is of the form: $f(x) = a\cos(bx + bd)$ with the phase shift?



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**Examples** Identify the amplitude, period, phase shift, argument, and vertical translation.

a. $y = \sin\left(x + \frac{\pi}{4}\right)$

b. $y = 2 - \sin x$

c. $y = 3 + \sin(x + 2)$

d. $y = -4 + \frac{1}{2}\cos x$

e. $y = 2 + \sin(2x - 2\pi)$

f. $y = 1 - 2\cos\left(3x + \frac{\pi}{2}\right)$

g. $y = 2 - 4\cos(3x + 1)$

h. $y = 3\pi - 2\sin(\pi x - 3)$



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**Examples** Show how to find the start and end points of an interval for us to graph.

a. $y = \sin\left(x + \frac{\pi}{2}\right)$

b. $y = 1 - 2\sin\left(3x - \frac{\pi}{2}\right)$

c. $y = 3\pi - 2\sin(\pi x - 3)$



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**<u>Example</u>** Using the graph $y = \sin\left(x + \frac{\pi}{4}\right)$, and *clearly write out the steps* required to graph any sine or cosine graph. Identify the amplitude, period, phase shift, and vertical translation.



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**Example** Graph $y = 1 + \cos x$. Identify the amplitude, period, phase shift, and vertical translation.



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**<u>Example</u>** Graph $y = -1 + \frac{1}{2}(\cos 2x - \pi)$. Identify the amplitude, period, phase shift, and vertical translation.



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**Examples: Translations with Sine and Cosine**

Graph the following: $y = \frac{5}{2}\sin\left(4\left(x + \frac{\pi}{4}\right)\right)$



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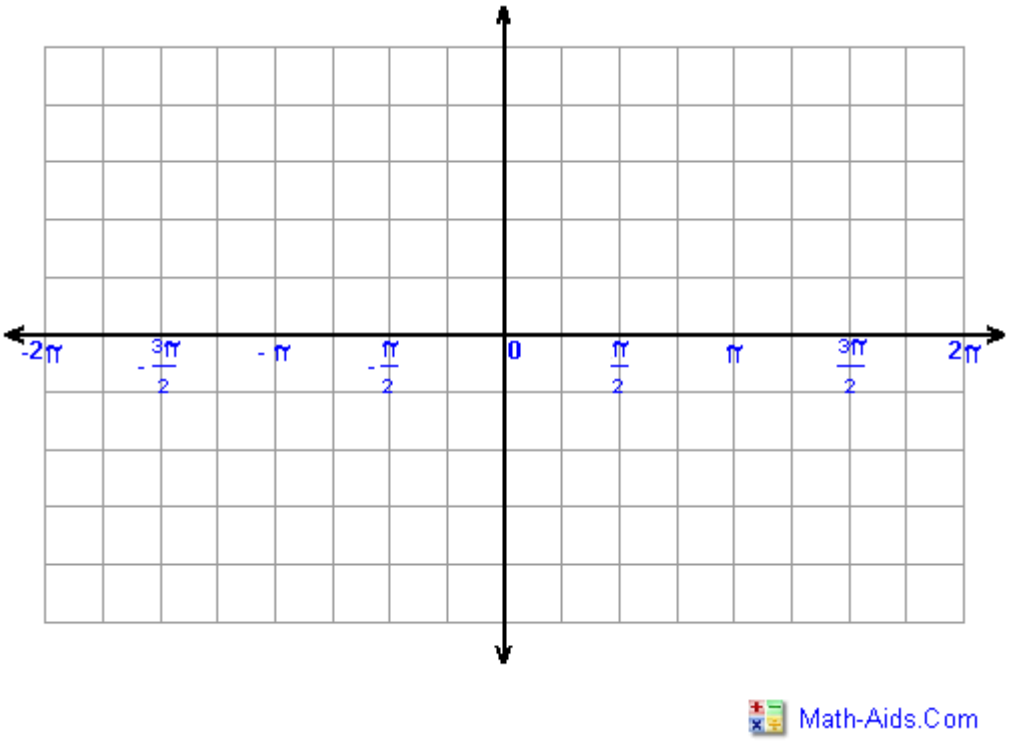
**Lesson: The Graphs of Tangent and Cotangent**

What is an asymptote? Draw a sketch to help explain.

**Draw the graph of tangent, filling in the table and then using the graph paper.**

| $x$ | $y = \tan x$ |
|---|---|
| $-\frac{\pi}{2}$ | |
| $-\frac{\pi}{4}$ | |
| $0$ | |
| $\frac{\pi}{4}$ | |
| $\frac{\pi}{2}$ | |
| $\frac{3\pi}{2}$ | |



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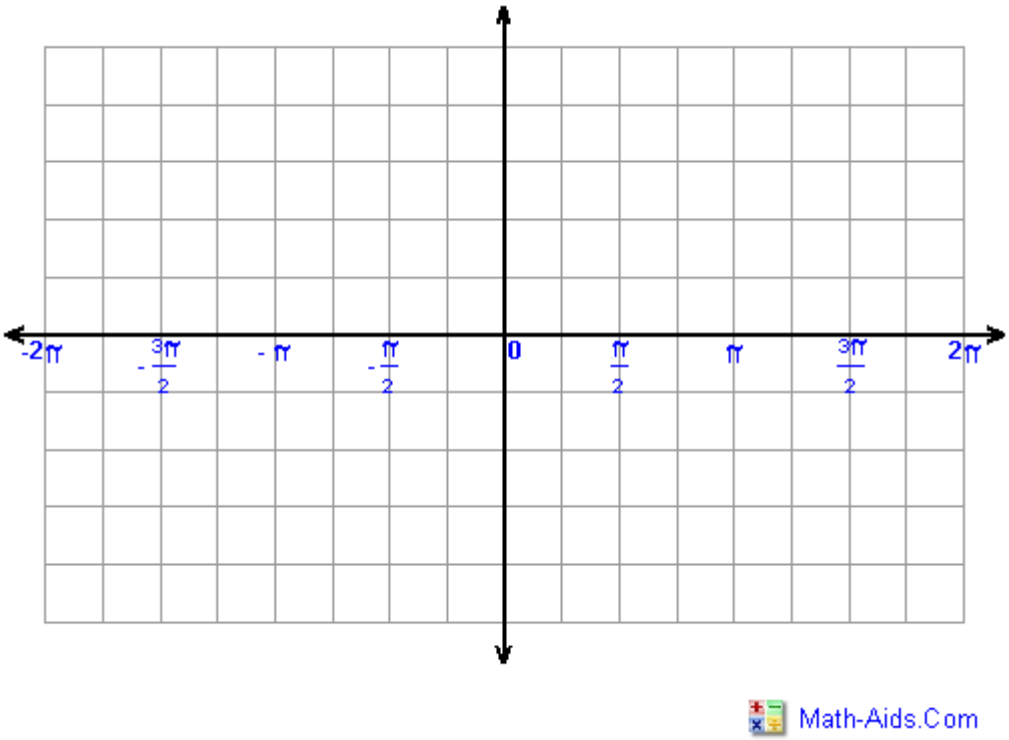
**Draw the graph of cotangent, filling in the table and then using the graph paper.**

| $x$ | $y = \cot x$ |
|---|---|
| $0$ | |
| $\frac{\pi}{4}$ | |
| $\frac{\pi}{2}$ | |
| $\frac{3\pi}{2}$ | |
| $\pi$ | |
| $\frac{5\pi}{4}$ | |




How would we describe the period for these functions?

Where are the graphs continuous (and discontinuous)?



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What is the domain and range of the graphs?

How can we describe the x-intercepts of tangent?

How can we describe the x-intercepts of cotangent?

Are either function even or odd?

How do you interpret amplitude with these functions and why?

What is the key for graphing? (Be specific with the equations you need to leverage)



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**<u>Example</u>** Graph over a one-period interval: $y = \tan 2x$



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**<u>Example</u>** Graph over a two-period interval: $y = -\frac{1}{2}\cot 4x$



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**Lesson: Translations with The Graphs of Tangent and Cotangent**

Tangent and cotangent can experience the same types of translations that sine and cosine did (which we discussed in a previous lesson).

**<u>Examples</u>** Describe the period, phase shift, and vertical translations for each function.

a. $y = 3\tan 4x$

b. $y = -\cot(x + \pi)$

c. $y = 2\tan\left(3x - \frac{\pi}{2}\right)$

d. $y = 3 + \cot(x - 4)$

e. $y = -2 + \tan\left(2\pi x - \frac{\pi}{4}\right)$

f. $y = \frac{1}{2} - \cot(5x + 2)$

**<u>Examples</u>** Show how to find 2 adjacent asymptotes for graphing each of the following:

a. $y = 3\tan 4x$

b. $y = 2\tan\left(3x - \frac{\pi}{2}\right)$

e. $y = -2 + \tan\left(2\pi x - \frac{\pi}{4}\right)$

f. $y = \frac{1}{2} - \cot(5x + 2)$



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**<u>Example</u>** Graph $y = 3 + \cot\left(x + \frac{\pi}{4}\right)$



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**<u>Example</u>** Graph $y = -1 + 2\tan\,(4x + \pi)$



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**Examples: Translations with The Graphs of Tangent and Cotangent**

Graph over a two-period interval.

a. $y = \tan\left(3x + \frac{\pi}{4}\right)$



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b. $y = -2 + 3\cot(4x - \pi)$



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c. $y = 1 - \tan(2\pi x - \pi)$



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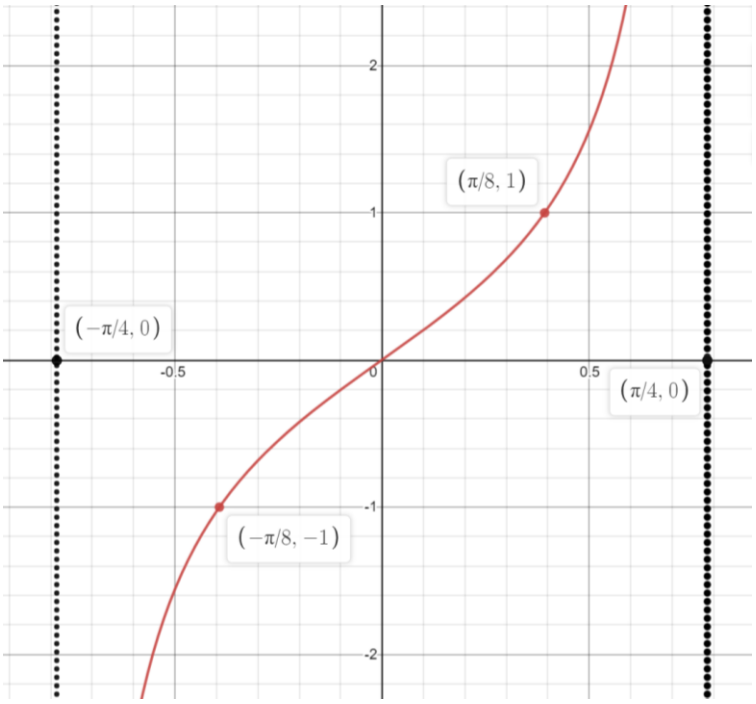
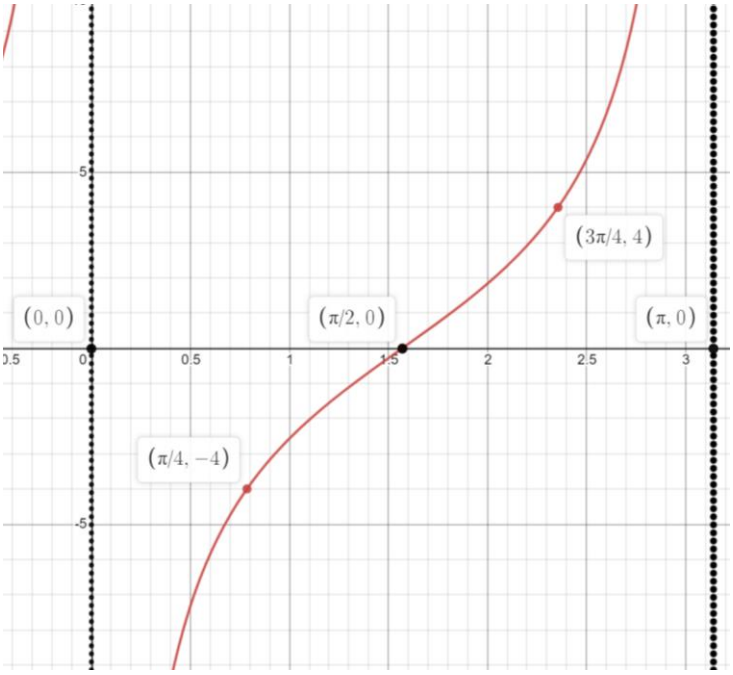
**Lesson: Determining the Equations of Tangent and Cotangent from Graphs**

Graphs from desmos.com of course!

a.




b.



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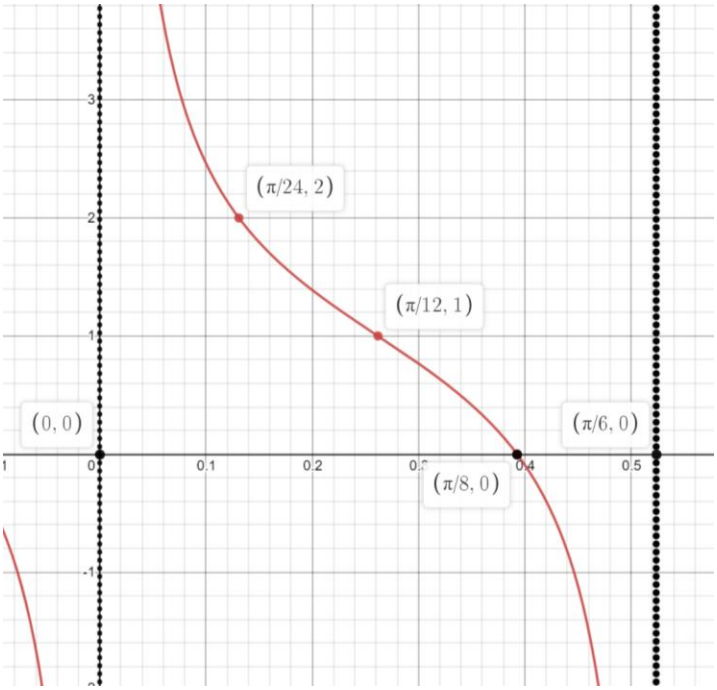
c.



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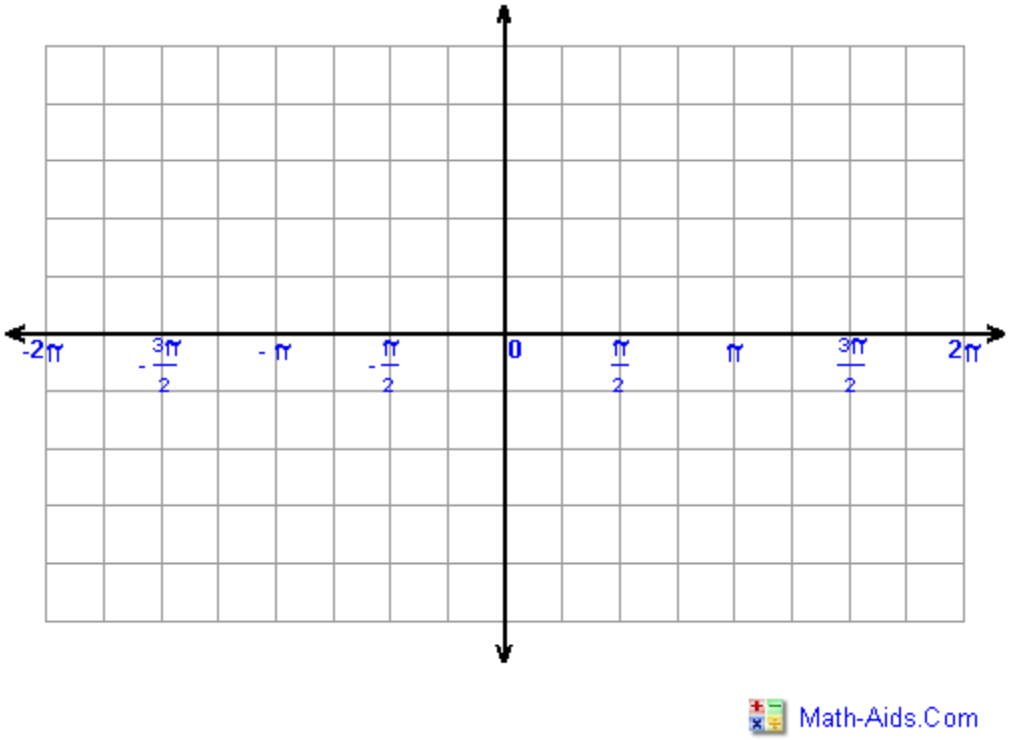
**Lesson: Translations of The Graphs of Secant and Cosecant**

**Draw the graph of secant, filling in the table and then using the graph paper.**

| $x$ | $y = \sec x$ |
|---|---|
| $-\frac{\pi}{2}$ | |
| $-\frac{\pi}{4}$ | |
| $0$ | |
| $\frac{\pi}{4}$ | |
| $\frac{\pi}{2}$ | |
| $\frac{3\pi}{4}$ | |
| $\pi$ | |
| $\frac{3\pi}{2}$ | |



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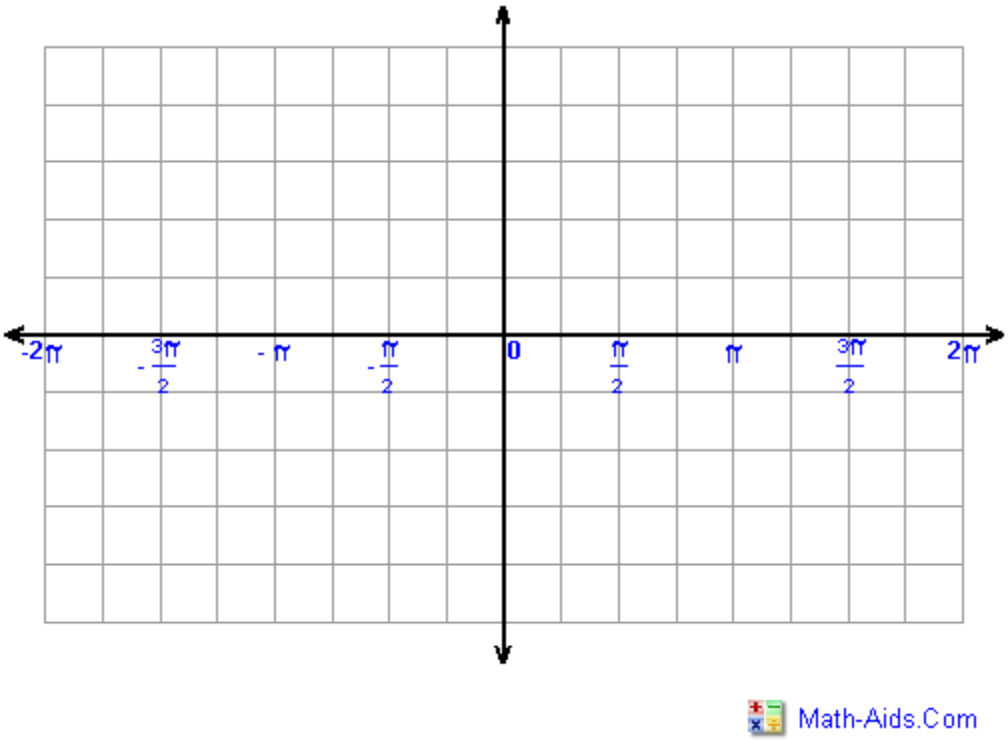
**Draw the graph of cosecant, filling in the table and then using the graph paper.**

| $x$ | $y = \sec x$ |
|---|---|
| $0$ | |
| $\frac{\pi}{6}$ | |
| $\frac{\pi}{3}$ | |
| $\frac{\pi}{2}$ | |
| $\frac{2\pi}{3}$ | |
| $\pi$ | |
| $\frac{3\pi}{2}$ | |
| $2\pi$ | |



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How would we describe the period for these functions?

Where are the graphs continuous (and discontinuous)?

What is the domain and range of the graphs?

How can we describe the x-intercepts of both graphs?



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Are either function even or odd?

How do you interpret amplitude with these functions?

What is the key for graphing? (Be specific with the equations you need to leverage)



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**<u>Examples</u>** Graph $y = 2\csc\frac{1}{2}x$



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**<u>Examples</u>** Graph $y = 1 + \frac{1}{2}\sec\left(x - \frac{\pi}{2}\right)$



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**Examples: Translations of The Graphs of Secant and Cosecant**

Graph the following over a one-period interval.

a. $y = -2\sec\left(x + \frac{3\pi}{4}\right)$



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b. $y = \frac{1}{2}\csc(2x - \pi)$



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c. $y = 2 + \csc\left(\frac{1}{2}x - \pi\right)$



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d. $y = 2 - 3\sec\left(2\pi x - \frac{\pi}{2}\right)$



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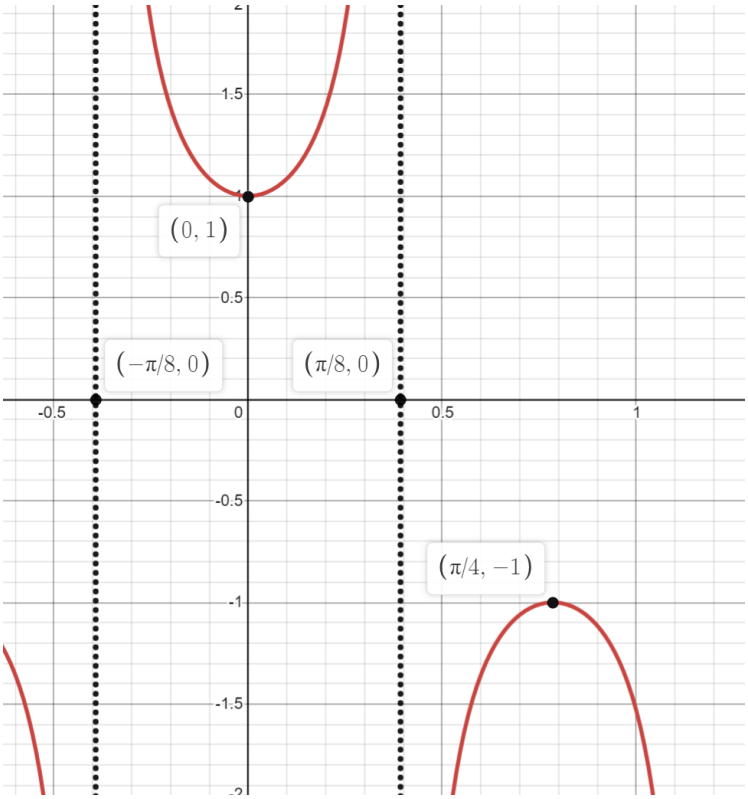
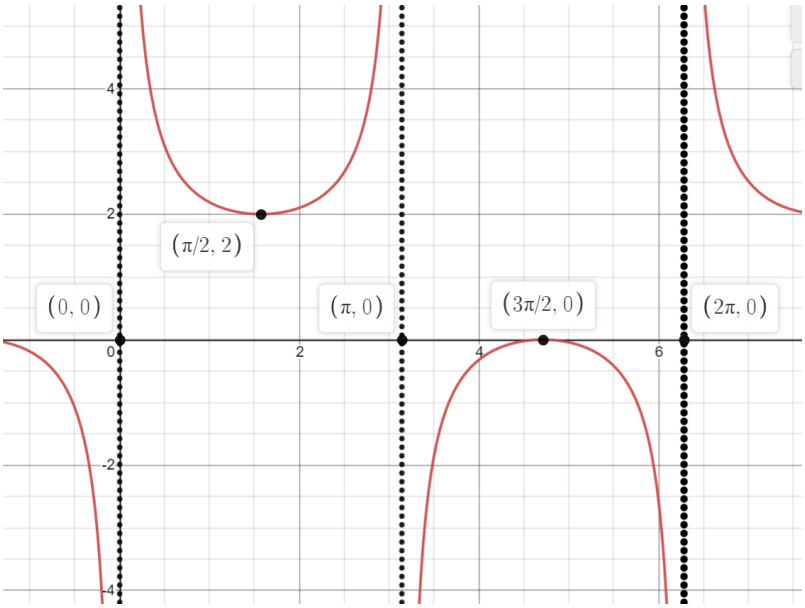
**Lesson: Equations of Secant and Cosecant from Graphs**

Determine the equation of secant or cosecant from the graphs.

a.




b.



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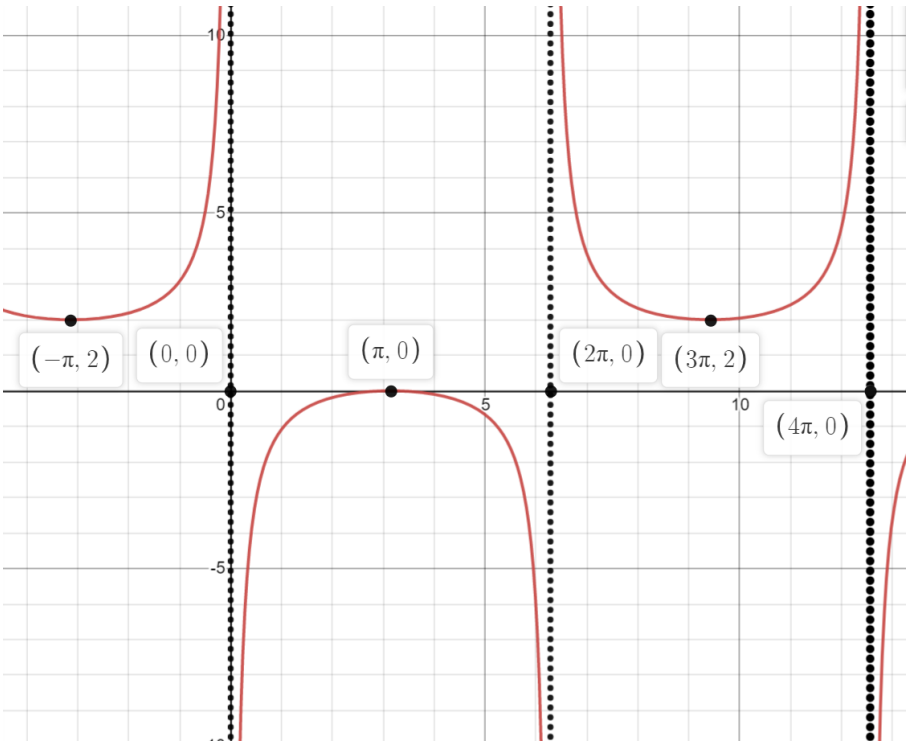
c.



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**Lesson: How to Approximate Trigonometric Function Values**

We will discuss using a calculator to do approximations of trig functions.

If you are going to use a calculator, what is key?

**<u>Examples</u>** Use a calculator to evaluate the following expressions.

a. $\cos 32°15'$

b. $\sec 55.321°$

c. $\sin(-105°)$

d. $\frac{1}{\csc(233.45°)}$

To find the value of $\theta$ (certain angle measures), we can use inverse trig functions. What are the three inverse trig functions that a calculator will typically have?

What do these functions means?



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**<u>Example</u>** Show how to use completely proper notation to find the angle $\theta$ in the interval $[0°, 90°)$.

a. $\cos\theta = 0.931145$

b. $\cot\theta = 1.7538932$



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**Examples: Solving for Theta with a Calculator**

Find all the values of $\theta$ in $[0, 360°)$ where $\tan\theta = 0.9845331$



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**Lesson: Significant Digits and Solving Triangles**

What is an exact number?

We've seen that many values for trig functions are not exact. Consider this: a room is 12ft x 18ft. We are doing a project where we need to know the length of the diagonal. What is it?

What's the issue with reporting this number?

What do we call the number of digits obtained by actual measurement?

Show some examples of significant digits:



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**Significant Digits for Angles**

| Angle Measure to Nearest | Number of Significant Digits Used | Write an Example |
|---|---|---|
| | | |
| | | |
| | | |
| | | |

How do you perform calculations in this case?

What should you keep in mind about the answer in this case?



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We can practice significant digits by solving triangles.

**Example** Solve the right triangle presented in the video.

**Example** Solve the right triangle presented in the video.



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**Examples: Significant Digits and Rounding**

How many significant digits are there in each?

| | |
|---|---|
| a. 421 | e. 400.01 |
| b. 420 | f. 400.00 |
| c. 400 | g. $72°31'$ |
| d. 400. | h. $72°30'$ |

Your answer needs to be 2 significant digits. You calculate the answer 421. What should your final answer be for significant digits?

Your answer needs to be 4 significant digits. You calculate the answer 421. What should your final answer be for significant digits?

Your answer needs to be 3 significant digits. You calculate the answer $42°18'$. What should your final answer be for significant digits?



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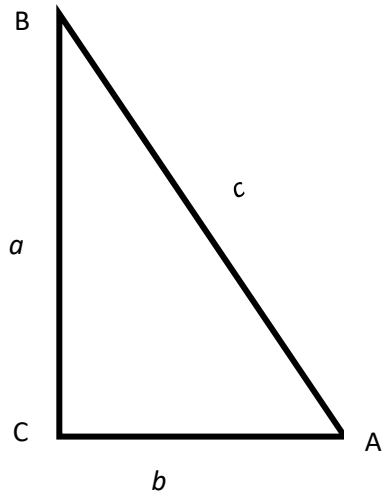
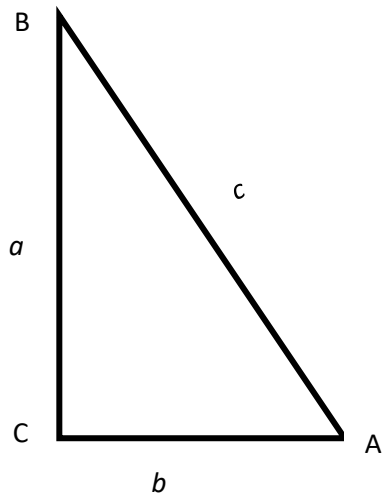
**Examples: Solving Triangles**

Find all missing lengths and angle measures for the given triangles. Use significant figures.

a. $A = 28.3°, c = 18.5$




b. $b = 85m, c = 112m$



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**Lesson: Angles of Elevation or Depression**

What is an angle of elevation and what is its context?

What is an angle of depression?

What should you be careful when interpreting either?



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**Example** Outline the general steps to solving the application problem: the angle of elevation from a point on the ground 122ft from the base of a flagpole is $25°20'$. Find the height of the flagpole.

Step 1 (write out the general approach and then show how to do it for each step)

Step 2:

Step 3:



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**Example** A 14.00m fire truck ladder is leaning against a building. What is the distance that the ladder goes up the wall if the ladder makes an angle of $42°25'$ with the horizontal?

**Example** A company safety committee installs a floodlight to illuminate the exit of a building, with the dimensions shown in the video. Draw the situation, and then find the angle of depression to the nearest minute.



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**Examples: Angles of Elevation or Depression**

A pyramid has a square base with side 600 ft long and height 300 ft. Find the angle of elevation made from the sides of a corner to the center of the pyramid. (Hint: The base of the triangle in the figure is half the diagonal of the square base of the pyramid.



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**Lesson: Applications of Right Triangles with Bearing**

What is bearing?

What are some examples of bearing?

Explain method 1 to express bearing:

What is key with these types of problems?



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**Example** Two radar stations are on an east-west line, 9.2 km apart from one another. The first station detects a plane on a bearing of $63°$. The second station detects the same plane on a bearing of $333°$. Find the distance from the second station to the plane.



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Explain method 2 to express bearing:

Show some examples:

**Example** A ship leaves port and sails on a bearing of N 35° E for 3 hours. Then it turns and sails on a bearing of S 55° E for 2.5 hours. If the ships rate is 22 knots (nautical miles per hour), find the distance that the ship is from the port.



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**Examples: Applications of Right Triangles with Bearing**

Two lighthouses are located on a north-south line. From lighthouse A, the bearing of a ship 3520 meters away is 139°43^′. From lighthouse B, the bearing of a ship is 49°43^′. Find the distance between the lighthouses.

The bearing from A to C is S 52°$E$. The bearing from A to B is N 84° $E$. The bearing from B to C is S 38°$W$. A plane flying at 375 mph takes 1.5 hours to go from A to B. Find the distance from B to C.



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**Lesson: Applications of Right Triangles with Distance and Height**

What is the subtense bar method?

**<u>Example</u>** Draw a situation between points P and Q, then solve the following:

a. Find d when $\theta = 2°3'15''$ and $b = 3.5000$cm.

b. How much change would there be in the value of d if $\theta$ measured $1''$ larger?



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We will now examine a few other problems involving distances and heights.

**Example** Copy the illustration shown in the video of the angle of elevation from a point on the ground to the top of a pyramid. The angle of elevation is $32°15'$. The angle of elevation from a point 120ft farther back is $20°25'$. Find the height of the pyramid.

**Example** In a certain area, the lowest angle of elevation of the sun in winter is $22°15'$. Find the minimum distance x that a plant needing full sun can be placed from a fence $4.75$ ft high.



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**Example: Modeling Distance with Right Triangles**

A highway curve connects 2 straight sections road and may be circular. Use the diagram in the video to illustrate the situation.

a. Find the distance between A and C if r=925ft and $\theta = 34°$.

b. Find an expression in terms of r and $\theta$ for the distance from P to Q.



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**Lesson: Reciprocal Identities of Trig Functions**

First, what is a reciprocal? Give an example.

What are the 6 reciprocal identities?

1.

2.

3.

4.

5.

6.

You should NOT write out something like _____ or _____as a shortcut for a reciprocal identities. Why not?

How can you shortcut the reciprocal identity then with notation?



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What are some other ways we can relate the reciprocal identities to one another?

**<u>Examples</u>** Find the function values.

a. $\tan\theta$ given that $\cot\theta = 3$

b. $\sec\theta$ given that $\cos\theta = -\frac{5}{\sqrt{50}}$

c. $\sin\theta$ given that $\csc\theta = 1.25$



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**Lesson: Signs and Ranges of Trig Functions**

Review: what does r represent when we are talking about trig functions?

Since r is a distance, we know _____.

What determines if a trig function has a positive or negative value?

Draw a picture to help breakdown and analyze the signs of theta:

Now fill in this table to help you remember the sign of each function:

| $\theta$ IN QUADRANT | $\sin\theta$ | $\cos\theta$ | $\tan\theta$ | $\cot\theta$ | $\sec\theta$ | $\csc\theta$ |
|---|---|---|---|---|---|---|
| I | | | | | | |
| II | | | | | | |
| III | | | | | | |
| IV | | | | | | |

One way to remember the signs is with the phrase All Students Take Calculus. Break down this this is helpful.



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**Examples** Determine the signs of the trig functions of an angle in standard position with the given measure.

a. $23°$

b. $255°$

c. $-30°$

**Examples** Determine which quadrant of an angle $\theta$ that satisfies the given conditions.

a. $\sin\theta > 0, \csc\theta > 0$

b. $\sec\theta > 0, \csc\theta > 0$



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Now we will show how to get ranges of the six trig functions.

Draw a picture to demonstrate how angle $\theta$ increase in measure from $0° - 90°$.

As the measure of an angle _____, _____ _____ but never exceeds _____. Therefore:

Let's repeat this idea but in quadrant IV. Follow along with the video below and write down the observations again.



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What can we conclude from these sketches and observations?

What can we conclude about the values of $\tan\theta$ and $\cot\theta$? And why?



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What can we conclude about the values of $\sec\theta$ and $\csc\theta$? Why?

We have now discovered the ranges of all the trig functions. Let's summarize this in the table.

| TRIG FUNCTION | RANGE (WRITE IN BOTH SET-BUILDER NOTATION AND INTERVAL NOTATION) |
|---|---|
| $\sin\theta, \cos\theta$ | |
| $\tan\theta, \cot\theta$ | |
| $\sec\theta, \csc\theta$ | |

**Example** Determine if each statement is possible or impossible.

a. $\cot\theta = -0.98$ b. $\cos\theta = -1.25$ c. $\sec\theta = 0.72$



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**Lesson: Pythagorean and Quotient Identities**

Show how we can build the first Pythagorean identity. These are actually pretty darn important, so you really do need to understand this argument!!

Now let's build another one:

And now let's build the last one:



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**Summarize the Pythagorean identities:**

What is only key about these identities?

Now let's show how to build the quotient identities:



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By the way, similar reasoning holds for $\cot\theta$.

**Summarize the Quotient Identities below:**

**<u>Example</u>** Find the value of $\cos\theta$ and $\tan\theta$, give that $\sin\theta = -\frac{\sqrt{2}}{2}$ and $\cos\theta > 0$.

**<u>Example</u>** Find $\sin\theta$, given that $\cos\theta = \frac{4}{5}$ and $\theta$ is in Quadrant IV.



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**<u>Example</u>** Give all 6 trig function values given that $\cot\theta = \frac{\sqrt{3}}{8}$ and $\theta$ is in quadrant I.



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**Examples: Using Pythagorean and Quotient Identities to Find Trig Function Values**

Find all 6 trig function values given the information.

a. $\cos\theta = -\frac{3}{5}$, $\theta$ is in quadrant III.

b. $\csc\theta = 2, \theta$ is in quadrant II.

c. $\cos\theta = 1$



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**Lesson: Fundamental Identities of Trig**

Review the test for symmetry (and the results):

We know that the sine function is odd, show what this means:

We know that the cosine function is even, show what this means:

These are called **identities.**



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Using these identities, what other identity can we determine about the tangent function:

These are called the **even-odd identities.**

We have already seen a handful of identities, and we call these the fundamental identities. Summarize them below.

**Reciprocal Identities**

**Quotient Identities**

**Pythagorean Identities**

**Even-Odd Identities**



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**Example** If $\cos\theta = \frac{3}{4}$ and $\theta$ is in quadrant I, find $\sin\theta$.

**Example** If $\tan\theta = -\frac{1}{4}$ and $\theta$ is in quadrant IV, find the values of all the other 5 trig functions.



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We use the identities to work on problems called transformations.

**Example** Write $\tan x$ in terms of $\sec x$.

**Example** Write the following in terms of sine and cosine, and then simplify until no quotients appear and all functions are of $\theta$ only.

a. $\csc\theta \cos\theta \tan\theta$

b. $\cos\ \theta(\cos\theta - \sec\theta)$



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**Examples: Using Fundamental Trig Identities to Find Values of Other Trig Functions**

Find the five remaining trig functions given the following information.

a. $\csc\theta = -\frac{5}{2}$, $\theta$ in quadrant III.

b. $\sec\theta = \frac{7}{2}$, $\tan\theta < 0$



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**Examples: Writing Trig Functions in Terms of Other Trig Functions (Fundamental Identities)**

a. Write $\cot x$ in terms of $\sin x$

b. Write $\csc x$ in terms of $\cos x$



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**Examples: Using Fundamental Identities to Write Expressions in Terms of Sine and Cosine**

Write each expression in terms of sine and cosine, then simplify so that no quotients appear and all functions are in $\theta$ only.

a. $\cos\theta \csc\theta$

b. $\frac{(1+\cot\theta)}{\cot\theta}$

c. $\frac{\sec^2\theta-1}{csc^2\,\theta-1}$

d. $\frac{\csc\theta}{\cot(-\theta)}$



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**Lesson: Verifying Trigonometric Identities**

As you progress through higher mathematics, sometimes you will need to manipulate equations in very creative ways. One way to learn this level of creativity is by verifying trigonometric identities. We will go through a few techniques of how to do this.

**<u>Strategy 1: Rewrite the More Complicated Side / Work from One Side</u>**

a. Verify that $\frac{1-\sin^2 x}{\cos x} = \cos x$. Show *why.*

b. Verify that $\sin^2 x + \tan^2 x + \cos^2 x = \sec^2 x$



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**Strategy 2: Express the Functions in Terms of All Sine and Cosine**

a. $\frac{\tan x}{\sec x} = \sin x$

b. $\sec x\,(\sin x + \cos x) = 1 + \tan x$



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**Strategy 3: Factor or Complete Indicated Algebraic Operations**

**Examples** Factor.

a. $x^2 - 4$

b. $\sin^2 x - 4$

c. $(\tan x + \cot x)^2 - (\tan x - \cot x)^2$

d. $2\sin^2 x + 3\sin x + 1$



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**Examples** Verify the following identities.

a. $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

b. $\frac{1-\cos x}{1+\cos x} = (\cot x - \csc x)^2$



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**Strategy 4: Manipulate Both Sides**

a. $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = \sec^2 x - \tan^2 x$

b. $\tan^2 x \ \ sin^2 x = tan^2 x + cos^2 x - 1$



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**Strategy 5: Use the Conjugate**

What is the conjugate of the following:

a. $x - 4$

b. $\sqrt{2} - \sqrt{3}$

c. $\sin x + 1$

Verify the following identities: $\frac{1-\cos x}{1+\cos x} = \csc^2 x - 2\csc x \cot x + \cot^2 x$



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**Examples: Verifying Trigonometric Identities**

a. $\frac{\cos x}{\cot x \sin x} = 1$

b. $\frac{1}{\sec x - \tan x} = \sec x + \tan x$



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c. $\frac{1+\sin t}{1-\sin t} - \frac{1-\sin t}{1+\sin t} = 4\tan t \sec t$

d. $(\cos\theta - \sin\theta)(\sec\theta + \csc\theta) = \cot\theta - \tan\theta$



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**Lesson: Proofs of the Sum and Difference Identities for Cosine**

Write out the proof for $\cos(A - B) = \cos A \cos B + \sin A \sin B$. Do your best to understand the proof and then explain it in a way that you will understand.



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Write out the identity for $\cos\,(A - B)$ below (yes I know I stated it above, but YOU should write it so you have a better chance of remembering it):

Write out the proof for $\cos(A + B) = \cos A \cos B - \sin A \sin B$

Write out the identity below (yes, write it out so you don't forget!):



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**Lesson: Using Sum and Difference Identities for Cosine**

**<u>Examples</u>** Show how to use the cosine of a sum or difference to find the exact value of the expressions:

a. $\cos 105°$

b. $\cos \frac{\pi}{12}$

c. $\cos 40° \cos 50° - \sin 40° \sin 50°$

Show how the cofunction identities are an extension of the cosine of a difference:



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Write out all the **cofunction identities**

**<u>Examples</u>** Find one value of $\theta$ that satisfies the following:

a. $\sec\theta = \csc 72°$

b. $\tan\theta = \cot(-44°)$

c. $\cos\theta = \sin\frac{5\pi}{6}$



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**Example** Write $\cos(180° - \theta)$ as a trigonometric function of $\theta$ alone.

**Example** Suppose $\sin s = \frac{3}{5}$ and $\sin t = -\frac{12}{13}$, where s is in quadrant I and t is in quadrant III. Find $\cos\,(s + t)$ and $\cos(s - t)$.

**Example** Verify that $\sec(\pi + x) = -\sec x$



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**Lesson: Sum and Difference Identities for Cosine**

Write out the proof for $\sin(A + B)$

Write out the proof for $\sin(A - B)$

Write out the identities for the sum and difference of sine below:

Write out the proof for $\tan\,(A + B)$



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Write all identities for the sum and difference of tangent below:

**<u>Examples</u>** Find the exact value of each expression.

a. $\sin(-15°)$

b. $\tan\left(-\frac{5\pi}{12}\right)$

c. $\sin\left(\frac{5\pi}{9}\right)\cos\left(\frac{\pi}{18}\right) + \cos\left(\frac{5\pi}{9}\right)\sin\left(\frac{\pi}{18}\right)$



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**<u>Examples</u>** Write each function as an expression involving functions of $\theta$ alone.

a. $\cos(30° + \theta)$

b. $\sin\,(45° + \theta)$

c. $\tan\left(\frac{\pi}{4} + x\right)$



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**Examples** Suppose A and B are angles in standard position such that $\sin A = -\frac{3}{5}\ \frac{3\pi}{2} < A < 2\pi$ and

$\cos B = -\frac{7}{25}$, $\pi < B < \frac{3\pi}{2}$. Find the following.

a. $\sin(A + B)$

b. $\tan(A - B)$

c. the quadrant of A-B



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**<u>Examples</u>** Verify the identities.

a. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

b. $\frac{\sin(x-y)}{\sin(x+y)}=\frac{\tan x-\tan y}{\tan x+\tan y}$



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**Examples: Sine of a Sum and Tangent of a Sum**

Let $\cos s = \frac{5}{12}$, $\sin t = \frac{4}{5}$ where s and t are in quadrant I. Find:

$\sin(s + t)$

$\tan\,(s + t)$

The quadrant of $(s + t)$



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**Lesson: Oblique Triangles and the Law of Sines**

First, we should review some facts about congruent triangles. When are 2 triangles congruent?

**<u>Congruence Axioms</u>**

Side-Angle-Side (SAS)

Angle-Side-Angle (ASA)

Side-Side-Side (SSS)



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When does Side-Angle-Angle (SAA) hold?

When a triangle is <u>not</u> a right triangle, what is it it called?

When can a triangle be solved?

What data is required for solving oblique triangles?

Case 1:

Case 2:

Case 3:

Case 4:



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Is there an Angle-Angle-Angle congruency?

When Case 1 or Case 2 is involved, what is required?

When Case 3 or Case 4 is involved, what is required?

This Law of Cosines will be discussed in another video.

Show how we derive the Law of Sines:



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State the Law of Sines:

What is an alternative form of the law of sines?



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**Example** Solve Triangle ABC if $A = 28.5°, C = 102.3°, c = 25.1$ in.

Recall the formula for the area of a triangle:

Using SAS, how else can you find the area of a triangle?



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**Example** Reference the sketch in the video. Find the area of the triangle.

**Example** Find the area of a triangle DEF if $D = 58°10', e = 31.2in, F = 73°30'$.



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**Lesson: Applications of the Law of Sines**

**<u>Example</u>** An engineer wishes to measure the distance across a river. Reference the sketch in the video.
If $C = 117.4°, A = 30.0°, b = 75.8$ ft



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Review the two methods of bearing:

**<u>Example</u>** The bearing of a lighthouse from a ship was found to be $N\ 52°$ W. The ship sailed 12.3 miles south, and the new bearing was $N\ 23°W$. Find the distance, to the nearest mile, between the ship and the lighthouse at each location.



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**Example** A balloonist is directly above a straight road, 2 miles long that joins the 2 villages. The closer town to him is an angle of depression of $45°$, and the farther town is at angle of depression of $41°$. How high above the ground is the balloon?



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**Lesson: The Ambiguous Case of the Law of Sines**

Describe the ambiguous case of the law of sines as describe in the video. There's going to be quite a bit of information presented – watch the entire explanation of why we need the law of sines and do your best to describe the various cases to sum up. Summarize the explanation so that it makes sense to YOU.



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What were some tips for applying the law of sines?

**<u>Example</u>** Solve triangle ABC if $a = 16.2cm, c = 14.1\ cm$, and $C = 72°30'$.



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**<u>Example</u>** Solve triangle DEF if $D = 62°8', a = 34.5in, b = 38.1in.$



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Explain the number of triangles satisfying the ambiguous case (SSA):

**<u>Examples</u>** Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

a. $B = 80°, C = 35°, b = 27$

b. $A = 60°, B = 70°, c = 5$



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**Examples** Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

c. $a = 15, b = 24, B = 33°$

d. $A = 25°, b = 8, a = 11$



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**Examples** Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

e. $A = 61.4°, b = 35.5, b = 39.1$

f. $a = 17.1, c = 13.1, C = 76°$



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**<u>Example</u>** Without using the law of sines, explain why no triangle ABC exists satisfying $B = 95°, b = 40cm, c = 46cm$.



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**Lesson: The Law of Cosines**

What is the property of triangle side length restriction?

Show how to derive the law of cosines:



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State the Law of Cosines:

Where is one place where we've seen this?

**<u>Example</u>** Two boats leave a harbor at the same time, traveling on courses that make an angle of $81°30'$ between them. When the slower boat has traveled $65.2$ miles, the faster boat has traveled $84.3$ miles. At that time, what is the distance between the boats?



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**Example** Solve triangle ABC if $B = 72.4°, a = 31.4ft, c = 44.3ft$.

**Example** Solve triangle DEF if $d = 24.2in, b = 42.6in, c = 59.1in$.



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**Example** The simplest type of roof truss is a triangle. Refer to the sketch in the video. Find angle C to the nearest degree for the truss shown.

Explain the 4 Cases of Solving Oblique Triangles



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**Example: The Law of Cosines with SSS Triangles**

Find the angles of the triangles given the following sides:

a. $a = 3ft, b = 5ft, c = 6ft$

b. $a = 324m, b = 421m, c = 298m$



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**Lesson: Heron's Formula for the Area of a Triangle**

Explain how to use Heron's formula (as explained in the video):

**<u>Example</u>** Find the area of triangle ABC where $a = 12m, b = 16m, c = 25m$



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**Example** Find the area of the Bermuda Triangle if the sides of the triangle have approximate lengths 800mi, 875mi, and 1250mi.



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**Lesson: The Binomial Theorem**

Below we can observe the pattern for binomial expansion:

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Focusing on the VARIABLES x and y only, how can we generalize the exponents for the variables?

Pascal's Triangle can explain how to find the coefficient. Draw out Pascal's triangle below.



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Having to draw out this pattern would be very difficult for larger expansions, so we will explore a few ways to simplify how to expand this pattern.

First, we need to understand factorials.

**n-factorial:** for any positive integer n,

$$n! = n(n-1)(n-2)\dots(3)(2)(1)$$

By definition: $0! = 1$

**<u>Examples</u>** Evaluate:

a. $5!$

b. $8!$

Now that we understand factorials, we can discuss the formula to find binomial coefficients.

For nonnegative integers $n$ and $r$, $r \leq n$, the binomial coefficient is defined as follows:

**<u>Examples</u>** Evaluate each binomial coefficient:

a. $\binom{5}{2}$

b. $\binom{6}{0}$



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We can summarize how to expand $(x+y)^n$ as follows:

1. There are n+1 terms in the expansion

2. The first and last terms are $x^n$ and $y^n$ respectively

3. In each succeeding term, the exponent on x decreases by 1 and the exponent on y increase by 1

4. The sum of the exponents on x and y in any term is n.

5. The coefficient of the term with $x^r y^{n-r}$ or $x^{n-r} y^r$ is $\binom{n}{r}$

These observations are captured in the binomial theorem.

State **The Binomial Theorem:**

**<u>Example</u>** Find the first, last, and seventh term of the binomial expansion: $(3x+2y)^{10}$



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**<u>Example</u>** Use the binomial theorem to expand: $(a-b)^5$



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## Not included (no notes file):

013 (Examples of Simplifying Rational Expressions)

014 (Examples of How to Use the Multiplicative Property of -1 to Simplify Rational Expressions)

015 (Examples of Finding Domains of Rational Expressions)

018 (Examples of Multiplying and Dividing Rational Expressions)

022 (Examples of Finding Common Denominators for Rational Expressions)