

Lesson: Simplifying Fractions

Say we want to reduce the fraction: $\frac{55}{77}$

To do so, we need to understand what numbers must be multiplied to make this fraction. Write the steps below as outlined in the videos:

Example 1

Reduce the following fractions.

a. $\frac{15}{35}$

d. $\frac{42}{54}$

b. $\frac{12}{18}$

e. $\frac{105}{135}$

c. $\frac{18}{42}$

f. $\frac{113}{125}$

How do you convert the mixed number $3\frac{4}{5}$ to an improper fraction?

Example 2

Convert the following to improper fractions

a. $5\frac{1}{2}$

b. $7\frac{2}{3}$

How do you convert the improper fraction $\frac{66}{5}$ to a mixed number?

Example 3

Convert the following to mixed numbers.

a. $\frac{15}{2}$

b. $\frac{25}{3}$

Lesson: Multiplying Fractions

The Rule for Multiplying Fractions

Let a, b, c, and d be real numbers where $b \neq 0$ and $d \neq 0$. Then:

What is actually happening with this rule? Write down a simple non-math explanation of how to multiply fractions:

Is a common denominator required for this? Yes No

Example 1

Multiply the following. There will be no reducing required, just make sure you understand how to multiply these together:

a. $\frac{2}{5} \times \frac{3}{7}$

d. $\frac{1}{2} \times \frac{3}{5} \times \frac{1}{7}$

b. $\frac{7}{9} \times \frac{2}{5}$

e. $5 \times \frac{2}{3}$

c. $\frac{12}{7} \times \frac{10}{7}$

f. $\frac{2}{3} \times \frac{5}{7} \times 2$

Reducing Fractions Before Multiplying

Occasionally you will multiply fractions that will need to be reduced. It can be easier to reduce them before multiplying them out. We now practice this.

Say you have this problem: $\frac{55}{54} \times \frac{45}{77}$

In this case, multiplying first would be messy so it is more advantageous to reduce before we multiply. Write down the steps below:

Example 2

Try this in the next problem. If you aren't sure what factor goes into each number, then break the numbers into their prime factorizations.

$$\frac{21}{28} \times \frac{16}{35}$$

Example 3

Cancel out any common factors before multiplying.

a. $\frac{35}{24} \times \frac{18}{49}$

c. $12 \times \frac{5}{6} \times \frac{1}{10}$

b. $\frac{60}{64} \times \frac{56}{72}$

d. $\frac{30}{75} \times 3\frac{2}{5}$

Lesson: Dividing Fractions

The Rule for Dividing Fractions

Let a , b , c , and d be real numbers where $b \neq 0$, $c \neq 0$, $d \neq 0$. Then:

What is actually happening with this rule? Write down a simple non-math explanation of how to multiply fractions:

Is a common denominator required?

Yes

No

Example 1

Complete the following exercises:

a. $\frac{2}{3} \div \frac{5}{7}$

b. $\frac{2}{5} \div 3$

c. $7 \div \frac{2}{3}$

Similar to multiplication, we can make cancellations *after we multiply by the reciprocal of the divisor*.

Example 2

Divide.

a. $\frac{15}{16} \div \frac{9}{14}$

c. $\frac{6}{7} \div 4\frac{1}{2}$

b. $20 \div \frac{4}{5}$

d. $\frac{24}{65} \div \frac{28}{70}$

Lesson: Intro to Adding and Subtracting Fractions

Adding and Subtracting Fractions When Fractions Have a Common Denominator

When fractions have the same denominator, we do not change the denominator but add the numerators as shown ($c \neq 0$):

Is a common denominator required?

Yes

No

Note this is the first instance where the denominator actually matters in the discussion.

Example 1

Add or subtract.

a. $\frac{3}{5} + \frac{1}{5}$

b. $\frac{3}{10} + \frac{7}{10}$

c. $\frac{5}{3} + \frac{4}{3}$

Adding and Subtracting A Whole Number with a Fraction

Suppose you want to complete the following example: $5 + \frac{1}{2}$

What denominator does the fraction have?

In this case, this is the common denominator we will want.

How to Turn a Whole Number into a Fraction

One property of all whole numbers is that we can rewrite them as fractions, so we know that:

$$5 = \frac{5}{1}$$

Once we convert 5 into this fraction form, we can then convert it to an equivalent fraction with the common denominator of 2. In this case, we say “whatever we do to the bottom, we also do the top.” Show how to do this below:

Example 2

a. $5 - \frac{2}{7}$

c. $2 - \frac{11}{4}$

b. $3 + \frac{2}{3}$

d. $8 - \frac{1}{8}$

Lesson: Adding and Subtracting Fractions when You Need to Find a Common Denominator

This is a much longer process. The ultimate goal is to make two fractions have the same denominator so we can combine them like we did in the first case.

To add two fractions that do not have the same denominator, we must complete the following three steps:

- 1) Find the LCD between the two fractions.
- 2) Convert each fraction as a fraction with the LCD.
- 3) Add or subtract the fractions and reduce if necessary.

Example 1

Find the LCD between: $\frac{1}{6}, \frac{1}{10}$

What is the largest number that divides both 6 and 10?

Now show how you can factor 6 and 10 by using this number that divides each:

Show how to find the LCD:

Example 2

Find the LCD between: $\frac{1}{12}, \frac{1}{18}$

What is the largest number that divides both 12 and 18?

Now show how you can factor 12 and 18 by using this number that divides each:

Show how to find the LCD:

Example 3

Find the LCD of the following fractional pairs. Put a box around your final answer to clearly show what the LCD is.

a. $\frac{3}{28}; \frac{5}{42}$

b. $\frac{5}{24}; \frac{1}{42}$

A Complete Example Showing How to Add Two Fractions with Unlike Denominators

Add the fractions: $\frac{1}{12} + \frac{4}{15}$

First, find the LCD between the denominators first:

What do you have to multiply 12 by to get the LCD?

What do you have to multiply 15 by to get the LCD?

Show how to finish the problem below:

Example 4

Use the guided steps to add $\frac{3}{10} + \frac{2}{15}$

What is the LCD between the fractions?

What must you multiply 10 by to get to the LCD?

What must you multiply 15 by to get to the LCD?

Complete the problem and put a box around the final answer:

Example 5

Add $\frac{1}{24} - \frac{3}{18}$ using the steps in the previous example to guide you.

Examples : Adding and Subtracting Fractions when You Need to Find a Common Denominator

Add or subtract the following:

a. $\frac{5}{16} - \frac{5}{6}$

c. $\frac{9}{10} - 3\frac{1}{2}$

b. $\frac{7}{12} + \frac{5}{18}$

d. $\frac{4}{15} + \frac{6}{25}$

Lesson: Exponents and the Order of Operations

Exponents

Label the parts of this exponent:

$$2^3$$

What does 2^3 literally mean?

Example 1

Evaluate the following exponents

a. 3^3

b. 5^3

c. 2^4

We need to understand exponents so that we can fully understand the order of operations. The order of operations is one of the handiest concepts in arithmetic that is often confused.

Order of Operations

Write down what the order of operations is:

Classic Example of the Order of Operations

Evaluate: $10 \div 5 \times 6$

Example 2

Evaluate the following. This page and the next page are both blank so that you have plenty of space to work out your calculations by hand.

a. $3 + 2^3 - 6 \div 3 \times 7$

b. $2 + 3^2 + 4 \times 2 - 5 \times 2$

c. $3(1 + 2^2 - 3)^2$

Examples: The Order of Operations

a. $10 - 3(1 + 2 \times 3) - 2^2 \times 4$

b. $3(1 + 2^2 - 3)^2 + 5(4 - 2 \times 3)$

c. $\frac{1}{2}(5 + 3) - 2^2(5 - 6)$

Lesson: Advanced Notes on Exponents

Multiplicative Property of Negative One

Explain what is the difference between -5^2 and $(-5)^2$ from the standpoint of the order of operations:

Example 1

Evaluate the following.

a. $(-3)^2$

d. -3^3

b. $(-3)^3$

e. -7^2

c. -3^2

f. $(-2)^5$

Example 2

What is the sign on the following problems? Do not worry about the final answer, you only need to circle if the final answer is positive or negative.

a. $(-4)^7$ Positive Negative

d. $(-13)^{72}$ Positive Negative

b. $(-58)^{12}$ Positive Negative

e. -64^{14} Positive Negative

c. -72^{57} Positive Negative

f. $(-23)^{91}$ Positive Negative

There is one final rule we need to discuss.

Zero Exponents

For any real number a ,

Example 3

Evaluate the following:

a. 5^0

b. x^0

c. 35^0

d. -6^0

e. $(-7)^0$

f. $3^0 + 7^0$

g. $-5^0 + 2^0$

h. $(-3 + \sqrt{52})^0$

Lesson: Algebraic Expressions

First we will define the terminology, which is already written out for you.

- **Constant:** any real number in an expression that is not attached to a variable
- **Variable:** a letter that can stand for any set of real numbers, and we can substitute values in for it if desired
- **Coefficient:** a number that is being multiplied by a variable
- **Term:** this is a single number, variable, or a number and a variable multiplied together, often separated by a plus + or minus – sign.
- **Factor:** a number or variable that is being multiplied with another number or variable
- **Algebraic Expression:** a mathematical string of constants and terms that can be added, subtracted, multiplied or divided together

This terminology will be used often, so it's important to understand what it means.

Example 1

Classify each part of the expressions

<u>Expression</u>	<u>Constants</u>	<u>Variables</u>	<u>Coefficients</u>	<u>Terms</u>	<u>Factors</u>
a. $5x$					
b. $2x + y$					
c. 8					
d. x					
e. $2x^2 - 4x + 7$					

Algebraic expressions are often used in everyday situations; we just don't realize it.

Example 2

You want to go to the movies, and on Tuesdays a theatre runs a deal so that all tickets cost \$8. How much would it cost for each of the following situations?

- | | |
|--|--|
| a. A ticket for only you? | c. Tickets for 5 people, a giant tub of popcorn that costs \$20. |
| b. A ticket for you and 2 of your friends? | d. Tickets for z people? |

In the last part of this example, we wrote an algebraic expression where z represented some quantity. Notice that if you input any of the previous number of tickets from examples a, b, and c into z , you would get the same answers.

To evaluate expression, you can simply plug in the desired values for a variable.

Example 3

Evaluate each expression for $x = -2$, $y = 4$, and $z = \frac{1}{3}$ as applicable:

a. $3x - 7y$

b. $6z - x^2 + 2y$

c. $x + z^2$

Lesson: The Commutative, Associative, and Distributive Properties

Your math homework and English homework are due on the same day. Does it matter which one you do first?

This situation illustrates the idea of the Commutative Law, which we can now state below:

Commutative Law

Let a and b represent any two real numbers. Now we can define the following properties:

- The Commutative Law for Addition:
- The Commutative Law for Multiplication:

Example 1

Rewrite the following using the commutative law.

a. $3 + y$

b. $3c$

c. $\frac{2}{5}k$

d. $2 - z$

You are buying chips, a drink, and a notebook. Does it matter what order the cashier rings up this items in?

These situations illustrate the idea of the **Associative Law**, which we can now state below:

Let a , b , and c represent any three real numbers. Now we can define the following properties:

- The Associative Law for Addition:

- The Associative Law for Multiplication:

Example 2

Rewrite the following at least 2 different ways using the associative law.

a. $4xz$

b. $8 + 2 + 7 + 3$

These rules are helpful when we are computing longer expressions.

How to Simplify Calculations Using the Commutative and Associative Laws

- a. Suppose we want to calculate: $2 + 3 + 8 + 7$

Using the commutative law, we more cleverly group these numbers together like this:

$$2 + 8 + 3 + 7$$

Usually we recognize 10s faster than some other quantities, so from here we can rearrange and group like this to find the final answer:

- b. How can you rearrange $\left(\frac{1}{5} * \frac{1}{3}\right) \times (10 * 6)$ to make the calculations simpler to work with?

The Distributive Law

Let a , b , and c represent any three real numbers. Then the following property holds:

The distributive property effectively gives us permission to either add or multiply certain expressions in the order that simplifies the calculation.

Example 3

Use the distributive property as needed to simplify the following calculations.

a. $10\left(\frac{2}{5} - \frac{1}{2}\right)$

b. $\frac{2}{3}(6 + 9)$

c. $20\left(\frac{1}{4} + \frac{2}{5}\right)$

d. $\frac{2}{5}(14 - 4)$

The distributive law works in the opposite direction as well. If we start with $ab + ac$, according to the distributive law what would this equal?

Another name for this process is called **factoring**.

Example 4

Use the distributive law to factor the following:

a. $km + kn$

b. $5x + 10y$

There are a few other properties we need to mention:

The Additive Identity

For any number a , $0 + a = a + 0 = a$

The Multiplicative Identity

For any number a , $1 \times a = a \times 1 = a$

The Additive Inverse (aka the opposite of a number)

For any number a , the addition of its opposite will result in zero, ie: $a + (-a) = 0$.

The Multiplicative Inverse (aka the reciprocal)

For any number a , it's inverse can be written in the form $\frac{1}{a}$ and has the property: $a \times \frac{1}{a} = 1$

It is interesting to note that there exist number systems (not discussed in this class) where these properties do **not** exist!

Lesson: Like Terms and Combining Like Terms

In algebra, we often want to simplify expressions if possible to make our lives easier. This is where the discussion of like terms comes into play.

Like Terms

Like terms are terms that share the exact same variables with the exact same exponents. Only like terms may be added.

Do like terms work in real life? I.e., can you come up with an example of like terms that could still be added in the real world?

Building off of your example, what wouldn't be like terms and could NOT be added in that scenario?

Example 1

In the following list below, classify which terms are like terms.

Discuss the two examples with the class to ensure they understand how to identify like terms. Students may color code, underline, or draw shapes around anything in the lists to indicate they are like terms.

a. $2x, 7y, 6x^2, 8y, x, -2x^2$

b. $3x^2y^2, 7x^2y, 2xy^2, 8x^2y, x^2y^2, -3x^2, 5xy^2$

The idea of like terms only comes up when we are talking about addition or subtraction. We do not worry about this when we want to multiply two things together. You may only add or subtract terms that are like terms.

Example 2

Combine like terms. The next page is also left blank if you need more space.

a. $3x + 2y - 5x + 9y$

b. $4x^2 + 2x + 6y^2 + 2x^2 + 8x$

c. $\frac{1}{2}x + \frac{2}{3}y - \frac{3}{2}x - 2\frac{1}{3}y$

d. $3x^2y^3 + 7x^2y^2 - 2x^2y^3 + x^2y^2$

e. $\frac{1}{4}x + \frac{2}{5}y - \frac{1}{2} + \frac{1}{5}y$

Lesson: Translating English into Math

As we move through this course, one major goal is to learn how to recognize and interpret certain situations as math problems. There are particular words in English that denote certain operations in mathematics, and we will classify those here.

<u>Addition</u>	<u>Subtraction</u>	<u>Multiplication</u>	<u>Division</u>

Example 1

Translate the following sentences into an algebraic expression. There is nothing to solve for you, you are merely setting up expressions.

- | | |
|--------------------------------------|---|
| a. 3 more than Lucy's height | d. Five less than three times a number. |
| b. Two less than a number | e. Two-thirds of the cost. |
| c. Three more than twice his salary. | f. Three more than twice his and her salary combined. |

Lesson: Basic Exponent Rules – Product, Power, and Quotient Rules

When working with rules of exponents, it is very important to be able to identify the base of each exponent. To begin, we will practice this.

Example 1

State the base for each exponent in the problems below.

Expression	Exponent	Base for this Exponent
5^3		
-5^4		
$-5x^6$		
$3y^7$		

Product Rule

First we want to remember what the actual definition of an exponent means.

We want to find: $2^3 * 2^2$. We will calculate this a few ways to understand the rules.

First we will work towards finding the exact answer. Evaluate each exponent below as you normally would with the **order of operations**, and then multiply the numbers together:

Now we will re-examine this result solely from the viewpoint of exponents. Write out each exponent as a multiplication problem, and notice the results you get from this exercise.

What do you notice about the exponents?

Product Rule (formal mathematical definition)

Let x be a real number, and let a and b be integers. Then:

Example 2

Use the product rule to evaluate the following.

a. $x^2 * x^4$

c. $x^3 + x^7$

b. $3x^3 * -1x^4 * 2x$

d. $\frac{2}{3}x^4y^2z^5 * 6x^3y^2 * \frac{1}{2}y^3z^4$

Power Rule

Using the order of operations, first rewrite the expression inside the parentheses without exponents:

$(2^3)^2 =$

Now rewrite this entire result just utilizing what we know about the definition of exponents:

What is the relationship between the exponents in this case?

The Power Rule (formal mathematical definition)

Let x be a real number, and let a and b be integers. Then:

Example 3

Use the power rule to evaluate the following:

a. $(x^3)^5$

c. $(3x^4y^5)^3$

b. $(x^2y^7z)^4$

d. $\left(\frac{2}{3}x^5y^2\right)^3$

Is there a difference between $(x + y)^n$ and $(xy)^n$? Explain.

The Quotient Rule

Using the order of operations, first rewrite the expression inside the parentheses without exponents:

$$\frac{2^5}{2^3}$$

First we will evaluate and find the final answer by using the order of operations below.

Notice the final answer of this question. How can we rewrite this number in its exponent form?

What relationship do you see between the exponents in the original question and the final answer?

The Quotient Rule (formal mathematical definition)

Let x be a real number, and let a and b be integers. Then:

Example 4

Use the quotient rule to simplify the following:

a. $\frac{x^7}{x^3}$

c. $\frac{15x^4y^3z^{10}}{2x^2y^2z^5}$

b. $\frac{x^5y^8z^{10}}{xyz}$

d. $\frac{12x^7y^3}{8x^3z^4}$

Examples: Exponent Rules, Basic Examples

Simplify the following using only the product, power, and quotient rules. Your final answer should not contain negative exponents.

a. $x^2y^7 * 5x^3y^5z$

f. $(-3x^5)^2$

b. $(4x^7y^3)^2$

g. $(-3x^5)^3$

c. $\frac{15x^7y^3}{10x^2y}$

h. $(2x^4y^2z^6)^2 * 3x^2z^3$

d. $\frac{2}{5}x^2y^5 * \frac{1}{3}x^7yz^3$

i. $\frac{(x^3y^7)^2}{x^4y}$

e. $\left(\frac{5}{3}x^3y^2z^3\right)^3$

j. $\frac{(x^4y^2)^3}{(x^2y)^2}$

Examples: Exponent Rules, With Numbers Only

Simplify the following using only the product, power, and quotient rules. Your final answer should be a number.

a. $3^2 * 3$

d. $\left(\frac{2}{3}\right)^3 * 3^2$

b. $(-4)^2 * 2$

e. $\frac{6^2}{2^3}$

c. $\frac{2^7}{2^4}$

f. $\left(\frac{1}{2}\right)^4 * 2^2$

Examples: Exponent Rules, Trickier and Harder Examples

Simplify the following using only the product, power, and quotient rules. Your final answer should not include negative exponents.

a. $(3x^2y^4)^2 * (x^4y^5)^3$

d. $\left(\frac{3}{5}x^5y^2z^7\right)^2 * \frac{xy^3}{z^5}$

b. $\frac{(2x^5y^2)^3}{(x^2yz^5)^2}$

e. $\frac{(3x^5y^2z^4)^3}{(6x^2yz^3)^2}$

c. $\left(\frac{2}{3}x^5y^7z^3\right)^2 * 9x^3y^3$

f. $\frac{(4x^3y^2z^5)^2}{(2xz^2)^4}$

Lesson: Negative Exponents

Simplify: $\frac{25}{125}$

If we wrote these numbers as exponents, this calculation would look like this:

$$\frac{5^2}{5^3} = \frac{1}{5}$$

Consider what would happen if we used the quotient rule on this problem:

$$\frac{5^2}{5^3} = 5^{2-3} = 5^{-1} = \frac{1}{5}$$

This problem shows us that exponents can also be negative.

Three Rules for Negative Exponents

Let x and y be a real numbers, and let n be an integer. Then:

$$x^{-n} = \frac{1}{x^n}$$

$$\frac{1}{x^{-n}} = x^n$$

$$\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n = \frac{y^n}{x^n}$$

In general your final answer cannot have a negative exponent, so it is essential that you understand what to do with it.

Example 1

Rewrite the following expressions without negative exponents.

a. 4^{-2}

c. $\left(\frac{2}{5}\right)^{-2}$

b. $\frac{1}{3^{-3}}$

d. $\left(\frac{1}{3}\right)^{-2}$

One of the biggest challenges with negative exponents is identifying how to apply them to the proper base. As a quick review, remember how we identified the bases in the last section:

Example 2

Identify the base of the following exponents:

Expression	First Base	First Exponent	Second Base	Second Exponent
3^5				
$-2x^5$				
$5x^3y^7$				

This is extremely important to remember when working with negative exponents. In general, our final answer can't have negative exponents so you need to be used to reformatting your answers properly if negative exponents come up.

Example 3

Simplify the following expressions, and write your final answer without negative exponents.

a. $\frac{a^3b^{-2}}{c^{-4}d^6}$

b. $\frac{3a^{-3}b^4c^{-5}}{d^2e^{-2}f^5}$

c. $\frac{a^5b^3c^4}{4d^{-2}e^{-4}f^{-2}}$

d. $\frac{a^{-3}b^{-2}c^{-6}}{d^4e^7f}$

With this in mind, we should now be able to work on combining all of the rules of exponents we've learned.

Example 4

Simplify the following. Your final answer cannot contain negative exponents.

a. $(3x^{-2}y^4)^3$

c. $(-2x^3y^{-2}z)^{-3}$

b. $\left(\frac{2x^{-4}y^6}{x^5y^{-3}}\right)^4$

d. $\left(\frac{2x^3y^{-4}}{3x^{-2}y^{-5}}\right)^{-3}$

Examples: Negative Exponents, Basic Examples

Your final answer should not contain negative exponents.

a. $\left(\frac{4}{5}\right)^{-2}$

e. $\frac{14x^{-3}y^5z^3}{21x^{-2}y^{-3}z^9}$

b. $3x^{-6}y^4z^{-2}$

f. $(3x^{-4}y^{-2}z^4)^2$

c. $\frac{4x^{-4}y^7}{z^{-5}}$

g. $\left(\frac{5x^2y^{-4}}{z^2}\right)^5$

d. $\frac{6x^3y^{-5}}{x^7y^2}$

h. $(x^{-3}y^2z^{-4})^{-3}$

Examples: Negative Exponents, Tricky Examples

Your final answer should not contain negative exponents.

a. $(3x^{-4}y^5z^{-2})^{-3}$

d. $\left(\frac{3x^3y^{-3}z^4}{2x^{-2}y^4z^{-2}}\right)^{-3}$

b. $(-5x^2y^{-3}z^4)^{-2}$

e. $\frac{(2x^{-4}y^5)^4}{x^{-3}y^7}$

c. $\left(\frac{4x^{-5}y^2}{x^{-3}y^{-5}}\right)^{-2}$

f. $\frac{(3x^{-3}y^2z^{-3})^{-3}}{(2x^4y^{-5}z^3)^{-4}}$

Examples: Negative Exponents versus Negative Numbers

Examples Simplify:

a. 6^{-2}

b. -6^2

c. $(-6)^2$

d. $(-6)^{-2}$

Examples Simplify:

a. $5x^{-2}$

b. $(5x)^{-2}$

c. $-5x^2$

d. $(-5x)^{-2}$

Examples Simplify

a. -7^2

b. $(-7)^{-2}$

c. 7^{-2}

d. -7^{-2}

Examples Simplify

a. $3x^{-2}$

b. $(-3x)^{-2}$

c. $(-3x)^{-3}$

Lesson: Scientific Notation

Scientific Notation

A number in scientific notation is of the form: $A \times 10^n$ where $1 \leq |A| < 10$ and n is an integer

Before we can write numbers in this form, it is best if we can identify examples that are or are not in this format.

Example 1

Identify if the following examples are in scientific notation. If not, circle and explain why.

- | | | |
|--------------------------------------|----------------------------------|-------------------------------------|
| a. 5.34×10^7 | Yes, it's in scientific notation | No, it's not in scientific notation |
| b. $-2.7832 \times 10^{\frac{1}{2}}$ | Yes, it's in scientific notation | No, it's not in scientific notation |
| c. -4.5382×10^{-8} | Yes, it's in scientific notation | No, it's not in scientific notation |
| d. 0.8×10^{23} | Yes, it's in scientific notation | No, it's not in scientific notation |

We will now examine how to rewrite the following example in scientific notation.

Example 1

Rewrite the following in scientific notation.

Example 2

Rewrite the following examples in their expanded form.

Example 3

Use the rules of exponents to multiply the following results together.

a. $(2 \times 10^6) \times (3 \times 10^8)$

b. $(2 \times 10^{17}) \times (2.1 \times 10^{23})$

c. $(8 \times 10^7) \times (2 \times 10^{-3})$

Example 4

Use the rules of exponents to divide the following results.

a. $\frac{8 \times 10^{12}}{4 \times 10^3}$

b. $\frac{6 \times 10^7}{2 \times 10^{19}}$

c. $\frac{3 \times 10^{-12}}{6 \times 10^{-5}}$

Example 5

Add the following:

a. $(5 \times 10^{78}) + (2 \times 10^{78})$

b. $(8 \times 10^6) + (2 \times 10^6)$

c. $(3 \times 10^4) + (2 \times 10^3)$

Lesson: Solving Linear Equations, A Review of the Basics

To begin, let's review some of what we've learned about working with equations and using graphs to solve them.

Linear Equation in One Variable

An equation that can be written in the form $ax+b=0$ where a and b are real numbers

Solution

A value that when substituted into the variable of an equation makes the equation true

Example 1

Determine if $x = -1$ is a solution to the following equations.

a. $5 - 4x = 9$

b. $2x + 3 = 1 - 3x$

We will now work with some basic ideas of solving linear equations. There are two key concepts we must understand to do this.

The Addition Principle

Let a , b , and c be real numbers. If $a=b$ then:

Example 1

Use the addition principle to solve the following:

Choose at least one from each category. Label the examples you choose as a, b, c, etc to keep the class organized.

a. $x - 4 = 2$

b. $x + 3 = 3$

c. $x - \frac{1}{3} = \frac{1}{2}$

The Multiplication Principle

Let a, b, c be real numbers. If $a=b$ then:

Example 2

Use the multiplication principle to solve the following:

a. $3x = 15$

b. $5x = 0$

c. $-\frac{2}{5}x = 10$

Example 3

Solve the following examples:

a. $5 - 4x = 33$

b. $3x + 2 = 2 + 4x - 8$

c. $3(4 - 2x) = 3x + 6$

Lesson: Solving Linear Equations, No Solution and Infinite Solutions

There are 2 special outcomes we have to watch out for:

An Example Where There Is No Solution

$$3(2x - 1) = 2(2x + 1) - 2(3 - x)$$

An Example Where There Are Infinite Solutions

$$4x + 5 = 3(2x + 1) - 2(x - 1)$$

A Summary of the 3 Possible Solution Types for Linear Equations

Examples: Solving Linear Equations, Basic Examples with No Fractions or Decimals

a. $3x - 5 = 7$

e. $4x + 7 + 6x = 7(1 - 2x)$

b. $6 - 2x = 6$

f. $3(5 - 2x) - 6(3 - 2x) = 5(1 + 2x)$

c. $4x + 7 = 2x - 9$

g. $4(2x - 1) - 2(6 - 3x) = 2(3x - 4)$

d. $3 - 4x - 7 + 2x = 4 - 6x - 8 + 4x$

h. $3(2x - 5) - 4x + 1 = 4(x - 3) - 2(1 - 3x)$

Lesson: Solving Linear Equations with Fractions

To begin, we will look at 3 examples to demonstrate how to clear fractions without parentheses. Take notes as shown in the video.

a. $\frac{1}{2}x + \frac{1}{5} = \frac{6}{5}$

b. $\frac{1}{2}x + \frac{1}{5} - \frac{1}{10}x = 1 + \frac{3}{10}x - \frac{2}{5}$

c. $\frac{1}{2}x + 1 + \frac{1}{4}x = \frac{3}{4} + \frac{2}{3} + \frac{1}{3}x$

Now we will look at how to clear fractions in equations with parentheses.

a. $\frac{3}{4}(3 + 2x) - \frac{1}{2}(5 - 3x) = \frac{5}{4}$

b. $\frac{1}{2}(x + 2) + \frac{1}{3}(2x - 5) = \frac{1}{3}(4 + 2x)$

Examples: Solving Linear Equations with Fractions (No Parentheses)

a. $\frac{1}{5}x - \frac{1}{3} = \frac{2}{3}$

b. $\frac{1}{3}x - 1 + \frac{1}{5}x + \frac{2}{5} = \frac{2}{5}x + \frac{1}{5}$

c. $\frac{1}{4}x - \frac{1}{10} + \frac{1}{5}x - \frac{4}{5} = \frac{3}{20}x + \frac{1}{10} + \frac{3}{10}$

d. $\frac{2}{3}x + \frac{1}{3} - \frac{1}{2}x = \frac{5}{6}x - \frac{1}{6} - \frac{2}{3}x + \frac{1}{2}$

Examples: Solving Linear Equations with Fractions (With Parentheses)

a. $\frac{1}{2}(3x - 2) - \frac{2}{5}(2x - 3) = \frac{1}{5}(x + 1)$

b. $\frac{3}{4}(4x + 5) + \frac{1}{2}(2 - 3x) = \frac{3}{4}(2x + 1)$

c. $\frac{1}{2}(2x + 3) - \frac{1}{3}(2x - 1) = \frac{1}{4}(x + 7)$

d. $\frac{3}{4}(2x - 1) + \frac{5}{8}(2 - 4x) = \frac{1}{4}(2 - 4x)$

Lesson: Solving Linear Equations with Decimals

To begin, we will look at 3 examples to demonstrate how to clear decimals without parentheses. Take notes as shown in the video.

a. $0.03x + 0.04 = 0.19$

b. $0.05x + 0.1 - 0.03x = 0.08$

Now we will look at 3 examples to demonstrate how to clear decimals with parentheses. Take notes as shown in the video.

a. $0.4(x + 2) - 0.3(2x + 1) = 0.09$

b. $0.1x + 0.03(4 - 2x) = 0.5(x + 3)$

Examples: Solving Linear Equations with Decimals (no parentheses)

a. $0.1x - 0.06 - 0.2x - 0.04 = 0.3$

b. $0.3x - 0.6 + x - 0.2 = 0.6x - 0.8 + 0.7x$

c. $1 + 0.2x - 0.7 + 2x = 0.5 - 0.4x + 1.8 + 0.6x$

Examples: Solving Linear Equations with Decimals (with parentheses)

a. $0.4(3x + 1) - 1.5(2x + 1) = 0.6(3 - 3x)$

b. $0.2(3x + 1) - 0.05(2x + 8) = 0.1(2x + 4)$

c. $0.09(2 - 4x) - 0.1(2x - 1) = 0.2(2 - x) - 0.06(2 + 6x)$

Lesson: The Basics of Applications and Word Problems

As a reminder, here is a review of some of the key words that imply certain operations in math:

<u>Words That Imply Addition</u>	<u>Words That Imply Subtraction</u>	<u>Word That Imply Multiplication</u>	<u>Words That Imply Division</u>
Add	Subtract	Multiplied by	Divided by
Increased by	Difference	Times	Quotient of
Altogether	Less than	Of	Out of
Sum	Fewer	Product of	Ratio
Longer than	Decrease	Twice	
Plus	Minus		
Above	Take away		
More than	Shorter than		

Example Juan is memorizing a list of names for a history class quiz. So far, he has memorized 8 of the names, which is one-fourth of the list. How many names are on this list?

Example Corey wants to rent a truck for the day. It costs \$30 for the base fee, plus \$0.23 for each mile driven. When he returned the truck, he owed \$41.19. How many miles did he drive?

Example A café serves both tea and coffee. On Sunday, the café sold 95 teas and coffees combined. The café sold 9 more coffees than teas. How many teas were sold on Sunday?

Example Latoya bought a new sweater and a pair of jeans for \$64. The jeans cost \$8 less than the sweater. How much was the sweater? How much were the jeans?

Example There are red, orange, and green balls in a ball pit. There are 12 more orange balls than red balls, and there are 9 more green balls than red balls. How many are there of each color?

Lesson: Consecutive Integer Word Problems

How do you represent consecutive integers with variables?

Example The sum of 2 consecutive integers is 73. Find the integers.

Example The sum of 3 consecutive integers is -33. Find the integers.

How do you represent even consecutive integers with variables?

How do you represent odd consecutive integers with variables?

What is noteworthy about how you represent even or odd consecutive integers?

Example The sum of 2 consecutive even integers is 110. Find the integers.

Example The sum of 3 consecutive odd integers is -87. Find the integers.

Example Find 3 consecutive odd integers such that their sum is seven more than four times the largest integer.

Lesson: A Review of Percentages

What is a percent? What does it represent?

Examples Convert the following percentages to decimals:

a. 34%

b. 122%

c. 4.57%

Examples Convert the following decimals to percentages:

a. 0.052

b. 0.995

c. 0.00078

Examples Convert the following fractions to percentages:

a. $\frac{3}{4}$

b. $\frac{3}{5}$

Examples

a. Find 34% of 87

b. What is 17.5% of 32?

c. 12 is what percent of 36?

d. What percent of 18 is 3?

Lesson: Applications with Percents – General Applications and \$ Applications

Example In the incoming freshman class of English majors, 52% of them were female. If there were 2,300 females majoring in English that year, what was the total number of English majors?

Example When Hector bought his house, it was worth \$232,000. After a few years, the house's value increased by 22%. What is the increase in value of the house and what is the actual current value?

Example Devin wanted to buy a shirt that normally costs \$18. When he go to the register to buy it, he found out there was a 55% discount. What is the price of the shirt now?

Example Rachel buys dresses for her clothing store at \$8 wholesale. When she gets to the store, she marks them up 23%. How much does she sell each dress for?

Example Mia is buying a new laptop for \$800. There is an 6.25% sales tax in her county. What is the final price of the laptop?

Example Rose found a shirt she wanted for \$24. When the cashier rang up the shirt, she owed \$25.98. What percentage of tax did she pay?

Lesson: Applications with Percents – Mixing with Percentages

Example Lala deposited \$500 into a simple interest account that paid 7% interest per year, and \$1500 into another account that paid 9% interest per year. After one year, how much interest did she make? What is the total percent of interest earned on the 2 deposits?

Example Steff works at a coffee shop and she is mixing coffee roasts. The Breakfast Blend contains 20% of the beans from Columbia. She mixes 3 pounds of the Breakfast Blend with 7 pounds of another roast that is 100% pure Colombian coffee. How many pounds of this mixture contain Colombian beans? What percentage of this new blend contains Colombian coffee?

Example Larry has mixture A with 55% acid and mixture B with 12% acid. He has 200 ml of mixture A. How much of mixture B did he use if his new solution is 48% acid?

Example A company owns 2 different factories to make furniture. Factory A produced 2 times as many dining chairs as Factory B. Five percent of the chairs from Factory A and 3% from Factory B were defective. How many chairs did Factory B produce if 770 of the chairs were defective?

Lesson: Solving a Linear Equation with Multiple Variables

Examples

a. Solve for x: $x + y = 7$

b. Solve for y: $4y = z$

c. Solve for a: $3a - 6b = 2c$

d. Solve for b: $4a + 8b = 16c$

e. Solve for k: $\frac{1}{3}k = m$

f. Solve for a: $3(a - 2b) = 6c$

g. Solve for b: $\frac{2}{3}(3a + 2b) = c$

h. Solve for a: $-\frac{2}{5}(2a - 4b) = 4$

Lesson: Ratios and Proportions

What is a ratio?

Example The unit price of an item is the ratio of the price of an item to the amount of the item. Find the unit price if 5oz of peanuts costs \$1.

Example Express the ratio of 12 boys to 14 girls.

What is a proportion?

Examples Solve the following proportions:

a. $\frac{x}{5} = \frac{6}{10}$

b. $\frac{x+2}{12} = \frac{x-4}{8}$

Examples: Proportions

a. $\frac{3}{x} = \frac{9}{21}$

b. $\frac{3x+10}{15} = \frac{2}{5}$

c. $\frac{2x-1}{6} = \frac{4x}{10}$

Lesson: Proportion Word Problems

It 3 cookies cost \$1.36, how much will 7 cookies cost?

At a motorcycle race, the ratio of parents to children was 5 to 3. If 500 children attended the motorcycle race, how many parents were at the race?

One euro is equivalent to \$1.18 US dollars. How many euro would you get for \$25?

Lesson: Distance, Rate, and Time Problems

Two cars leave Glen Ellyn, IL, with one driving north and the other driving south. The northbound car is traveling 8mph faster than the southbound car, and after 2 hours they are 420 miles apart. Find the speed of each car.

Marshall and Lilly live together. Marshall leaves the house at 2pm, and then Lilly leaves the house at 2:30pm. If Marshall is driving 30 mph and Lilly is driving 35 mph in the same direction, how long will it be until their cars meet?

A Lamborghini and a Ferrari pass one another at noon on a highway in California. The cars are going opposite directions. If the Lamborghini is traveling 75 mph, and the Ferrari is traveling 80 mph, how long will it take for the cars to be 310 miles away from one another?

Lesson: Money Word Problems

Sarah has five more nickels than quarters. When combined, the coins are worth \$4.75. How many nickels and quarters does she have?

Maria has \$75 worth of \$1 and \$5 bills. In total, she has 27 bills. How many \$5 bills does she have?

Tickets at the movie theater course \$15 for an adult and \$10 for a child. In the last hour, sales were \$550. How many adult tickets versus child tickets were sold?

Lesson: Number Lines, Interval Notation, and Set Notation

Show how to set up the number line, interval notation, and set notation for the following:

a. $x > 5$

d. $-3 < x < 5$

b. $x \leq 7$

e. $-7 \leq x < -1$

c. $x \geq 0$

f. $3 < x \leq 8$

Lesson: How to Solve Linear Inequalities in One Variable

Linear inequalities in one variable have the form $ax + b < 0$. Alternatively, they could use the symbol $\leq, >, \geq$.

Solving linear inequalities has some similarities and some differences when compared to solving for linear equations.

The Addition and Subtraction Properties

Let a, b, c be real numbers. If $a < b$, then:

This also holds for $\leq, >, \geq$

Examples Solve the following. Put your final answer both on a number line and in interval notation.

a. $x + 4 > 9$

b. $x - 8 \leq 12$

The Multiplication and Division Properties

Let a, b, c be real numbers. If $a < b$, then:

- if c is positive ($c > 0$):

- if c is negative ($c < 0$)

This also holds for $\leq, >, \geq$

Why is this rule different? Use the following example.

To begin, is $-2 < 4$ a true statement? True False

Now multiply both sides of the inequality by 3 and write down the new result below:

This new statement is True False

Now multiply both sides of the inequality by -3 and write down the new result below:

This new statement is True False

Now divide both sides of the inequality by -2 and write down the new result below:

This new statement is True False

Examples Solve the following. Put your final answer both on a number line and in interval notation.

a. $3x \leq 15$

b. $-2x > 8$

c. $5x < -10$

d. $-4x \geq -12$

e. $-9x \leq 27$

From here, you can start to combine the rules to solve more types of inequalities.

Examples Solve the following. Put your final answer both on a number line and in interval notation.

a. $2x - 4 < 6$

b. $3x + 7 > 5x - 9$

c. $\frac{1}{2}x + \frac{1}{5} > \frac{3}{4}x + \frac{7}{10}$

Examples: Solving Linear Inequalities – Basic Examples

Solve the following. Put your final answer both on a number line and in interval notation.

a. $2x + 7 \geq 13$

b. $4x - 6 < 6x - 10$

c. $3(5 - 2x) > 5(3 + 2x)$

d. $2(4 - 2x) - 3(4 - 4x) \leq 4(3x + 3)$

Examples: Solving Linear Inequalities – Examples with Fractions Decimals

Solve the following. Put your final answer both on a number line and in interval notation.

a. $\frac{1}{3}x - \frac{1}{5} \leq \frac{1}{3} + \frac{2}{5}x$

b. $0.03x + 0.2 > 0.02$

c. $\frac{3}{5}(2x - 1) - \frac{3}{4}(2 - 3x) < \frac{1}{10}(4 - 3x)$

d. $0.2(2x - 4) + 0.03(6 - 10x) \geq 0.06(x + 9)$

Lesson: Three-Part Inequalities

Show how to solve the following three-part inequalities. Put your final answer both on a number line and in interval notation.

a. $4 < x + 2 \leq 10$

b. $-3 \leq 2x - 1 < 11$

c. $-5 < 3x + 4 < 19$

d. $4 \leq -1 - 5x < 14$

Lesson: The Basics of Compound Inequalities

The word “and” has special implications in mathematic. One place it is often used is with sets, and we represent “and” with the intersection symbol $\cap$.

The word “or” also has special implications in mathematic. It is also used is with sets, and we represent “or” with the intersection symbol $\cup$.

Examples Given $A = \{1, 2, 3, 4, 5\}$, $B = \{2, 4, 6, 8, 10\}$, $C = \{1, 3, 5, 7\}$, find the following:

a. $A \cap B$

c. $A \cup C$

b. $A \cap C$

d. $B \cup C$

We use similar ideas with compound inequalities.

Show how to represent a solution to $x \geq -2$ and $x < 5$

Example Represent the following with a number line and interval notation.

a. $x > 0$ and $x \leq 7$

b. $x > 8$ and $x \leq -2$

Show how to represent a solution to $x \geq 4$ or $x < -3$

Example Represent the following with a number line and interval notation.

a. $x \leq 0$ or $x > 5$

b. $x > 0$ or $x \leq 5$

We can create more complicated problems where we first have to isolate x , but then we can represent in the same manner as shown above.

Example Represent the following with a number line and interval notation.

$3x - 1 > 5$ and $5x + 4 \leq 24$

Examples: Compound Inequalities

Put your final answer both on a number line and in interval notation.

a. $x - 4 > 8$ or $x + 2 < -3$

b. $2x + 1 \geq 7$ and $5x - 10 < 20$

c. $4x + 2 > 18$ or $3x - 1 < 17$

d. $2 - 3x > 8$ and $5x - 2 > 12$

e. $5x + 9 < 16$ or $4 - 3x > 14$

f. $3 - 2x > 9$ and $5 - x < -12$

Lesson: Intro to Linear Equations in Two Variables, Graphs, and Plotting Points

A **linear equation in two variables** is an equation of the form $ax+by=c$, where a , b , and c are real numbers, and both a and b are not zero at the same time.

Solutions for equations with two variables come as an ordered pair. Unless otherwise stated, you generally view the ordered pair in the alphabetical order of the equation.

Examples Decide if the following ordered pairs are solutions to the equations.

a. $\left(2, -\frac{1}{2}\right)$ for $3x - 2y = 4$

b. $(-5, -1)$ for $3a + 2b = 10$

c. $(3, 2)$ for $y = 5 - x$

Examples Complete the ordered pairs for the following equations

a. $(2, b)$ for $4a - 6b = 2$

b. $(x, -3)$ for $y = 2x + 1$

Another way to represent ordered pairs is with a table.

Examples Complete the following tables for the linear equations:

a. $y = 2x + 4$

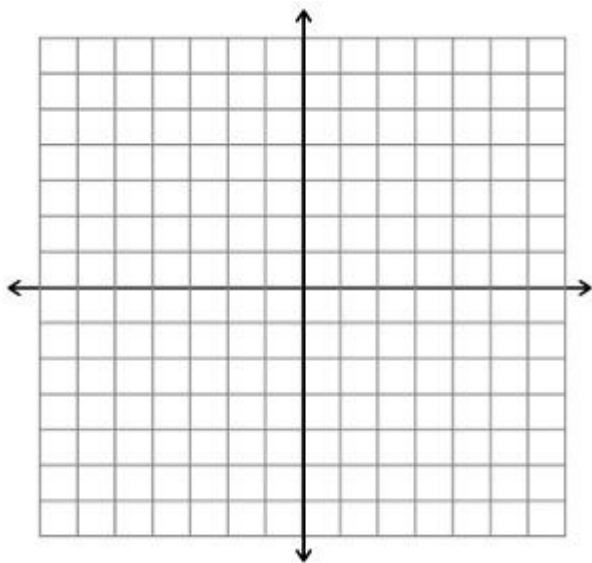
x	y
-3	
	10
0	

b. $3a - 2b = 12$

x	y
	0
0	
3	

Order pairs are often represented in coordinate systems. Below, draw the Cartesian (aka rectangular) coordinate system and label its parts.

Examples Plot the following points on the graph:



a. $(-2, 3)$

b. $(4, -1)$

c. $(1, 4)$

d. $(0, -4)$

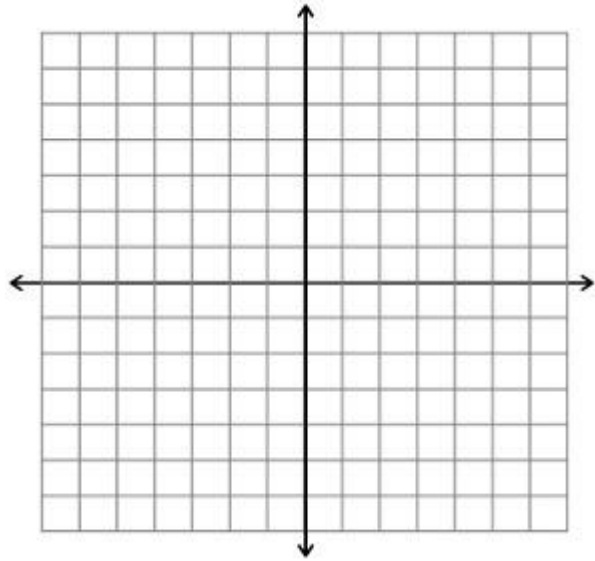
Lesson: Graphing Equations by Using a Table

One way to graph lines is by finding three points, and then plotting those points.

Examples Complete the following tables of ordered pairs, and then plot the points to form a line.

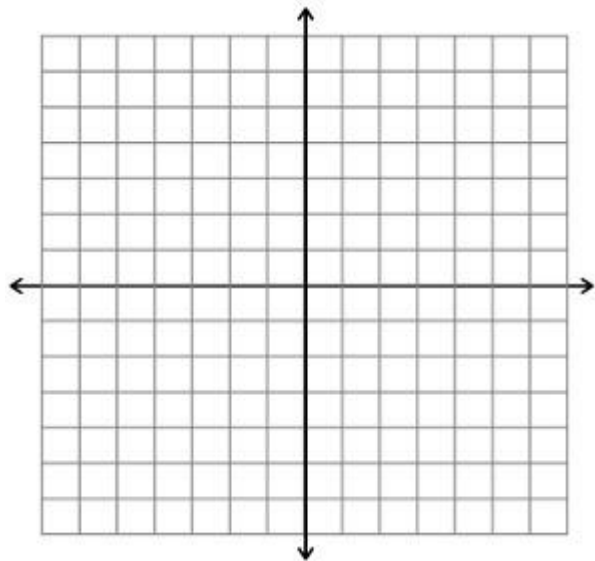
a. $y = 2x - 1$

x	y
2	
-2	
	0



b. $-2x + 6y = 12$

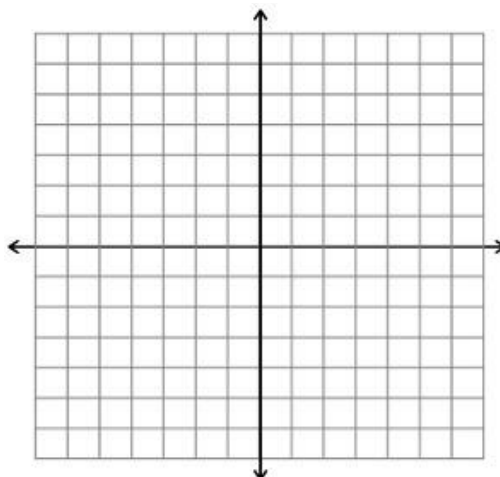
x	y
-3	
	0
3	



Examples Find three points for the following equations, then plot those points to make a line.

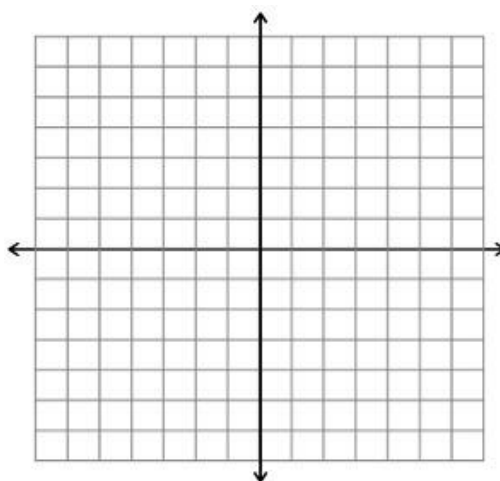
a. $y = -3x + 2$

x	y



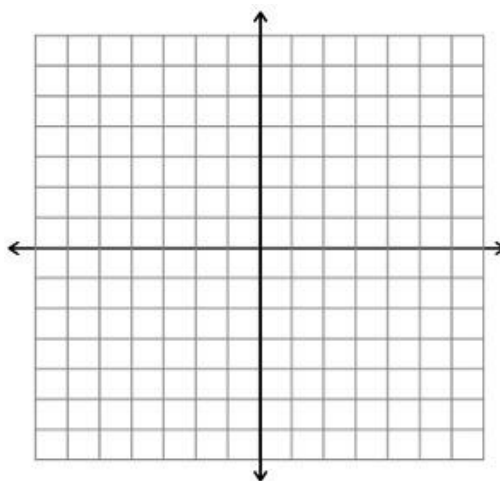
b. $y = -\frac{1}{2}x + 3$

x	y



c. $3x - 5y = 15$

x	y



Lesson: Graphing Equations by Using Intercepts

What are x-intercepts? What are y-intercepts? Draw an illustration.

How do you find x-intercepts?

How do you find y-intercepts?

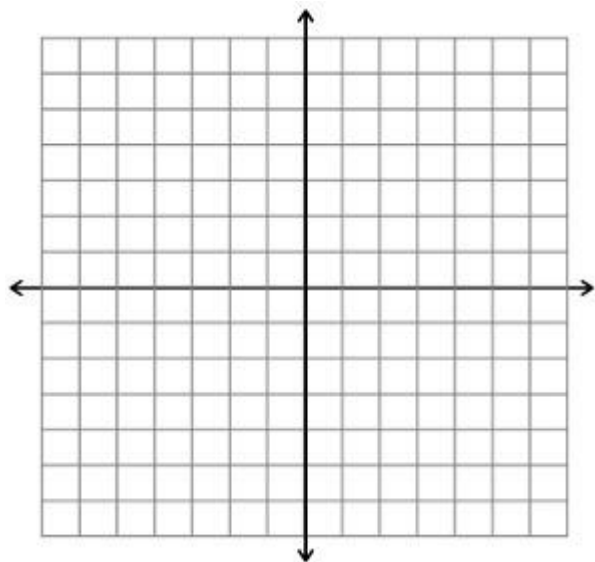
Examples Find the x- and y-intercepts of the following equations. Then find a third point. Plot these three points to make a line.

$$3x - 2y = 6$$

x-intercept

y-intercept

Third point:



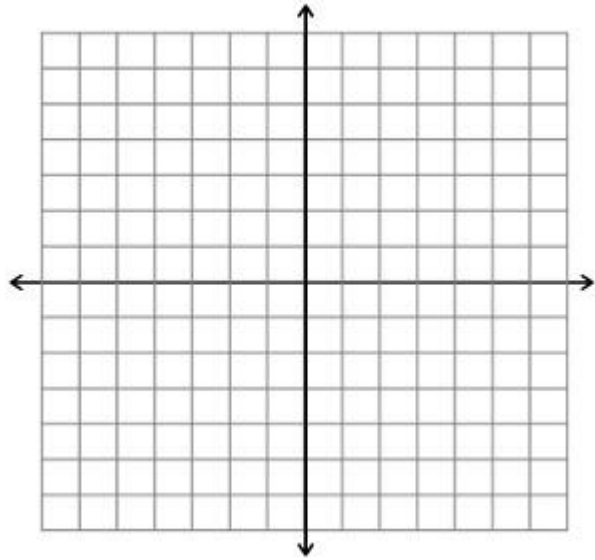
Examples Find the x- and y-intercepts of the following equations. Then find a third point. Plot these three points to make a line.

$$x - 4y = 4$$

x-intercept

y-intercept

Third point:



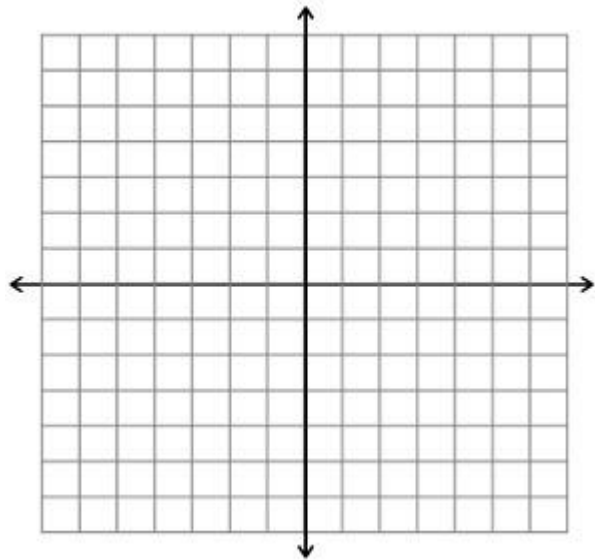
Examples Find the x- and y-intercepts of the following equations. Then find a third point. Plot these three points to make a line.

$$y = \frac{1}{2}x - 2$$

x-intercept

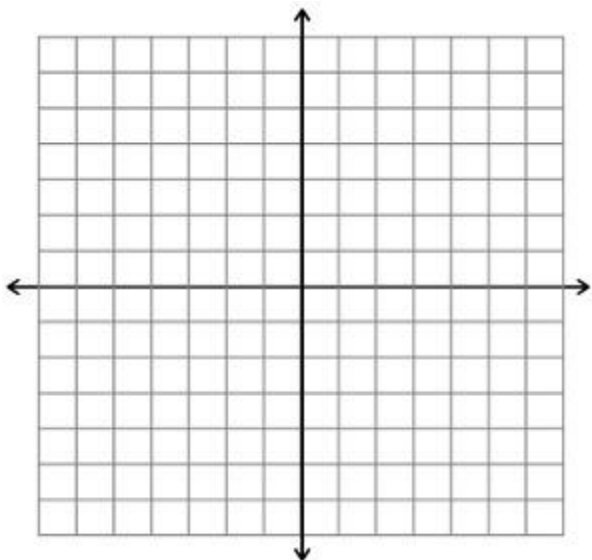
y-intercept

Third point:

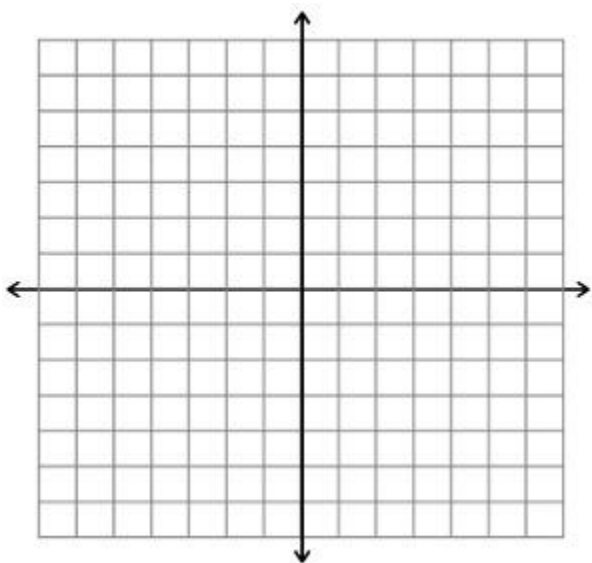


Lesson: How to Graph Vertical and Horizontal Lines

Show how to graph $y = 3$



Show how to graph $x = -2$

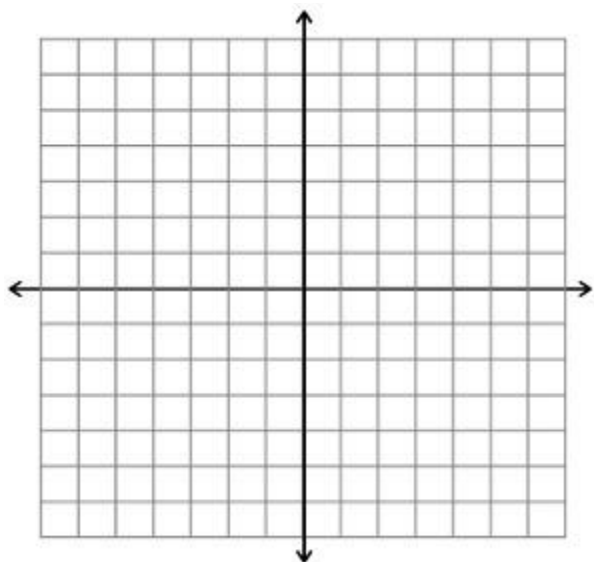


How do you know if you are working with a vertical line? How do you graph it?

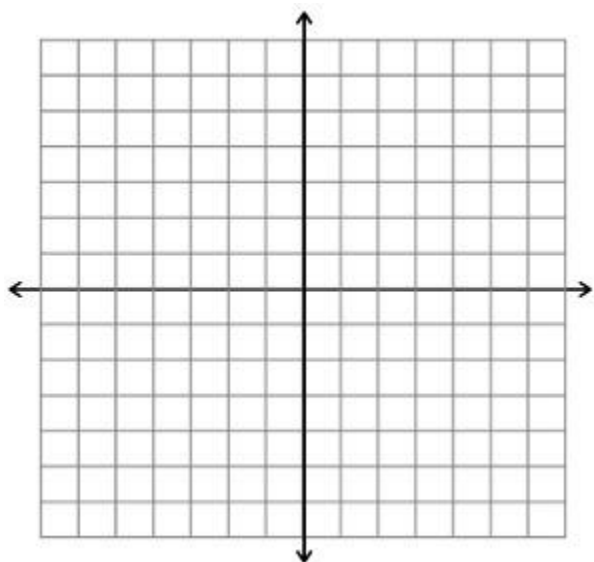
How do you know if you are working with a horizontal line? How do you graph it?

Examples Graph the following:

a. $x = 4$



b. $y = -1$



Lesson: The Slope of a Line

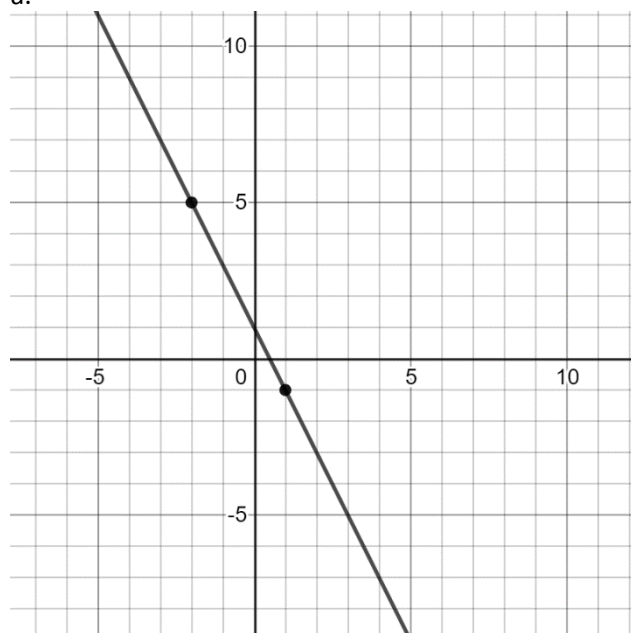
What does slope represent?

Draw the various possible slopes of lines:

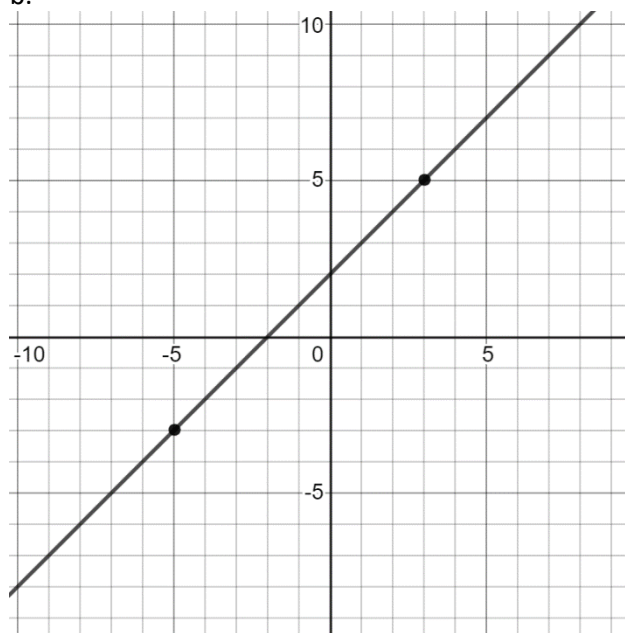
How do find the slope of a line?

Examples Find the slope of the following lines:

a.



b.



What is the slope formula?

Examples Use the slope formula to find the slope between the pairs of points:

a. $(3, 5)$ and $(4, 8)$

c. $(7, 2)$ and $(7, 4)$

b. $(6, -2)$ and $(-2, 4)$

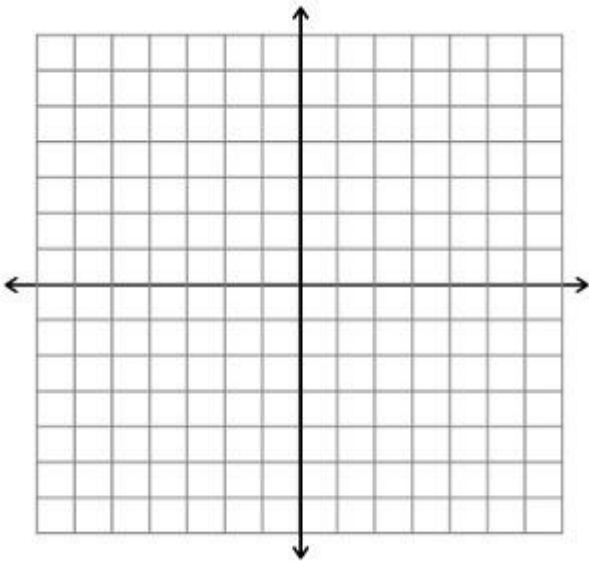
d. $(2, 4)$ and $(-1, 4)$

Lesson: Using Slope to Graph Lines; Intro to Slope-Intercept Form

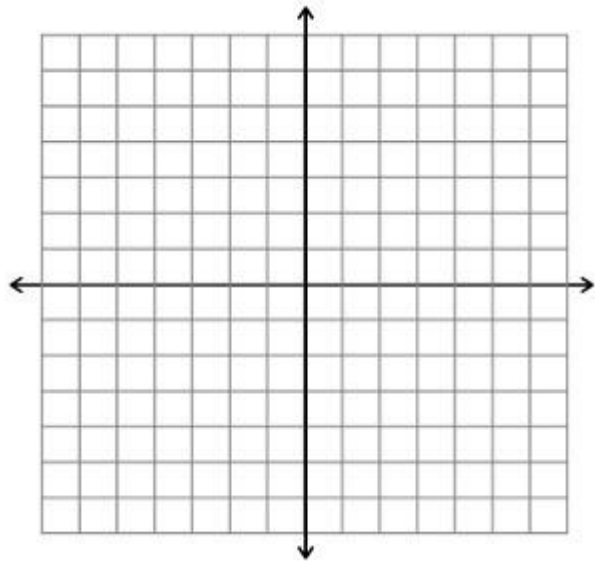
When the slope of a line and a point are known, a line can be easily graphed.

Examples Given the slope and the point, graph the following lines:

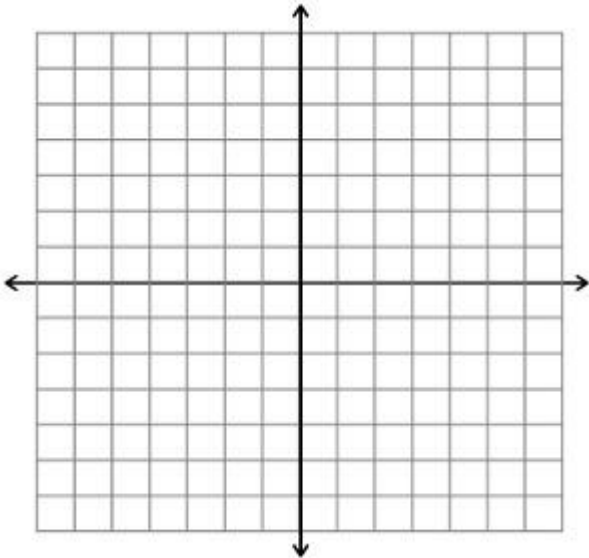
a. $m = 2$ and $(-1, 2)$



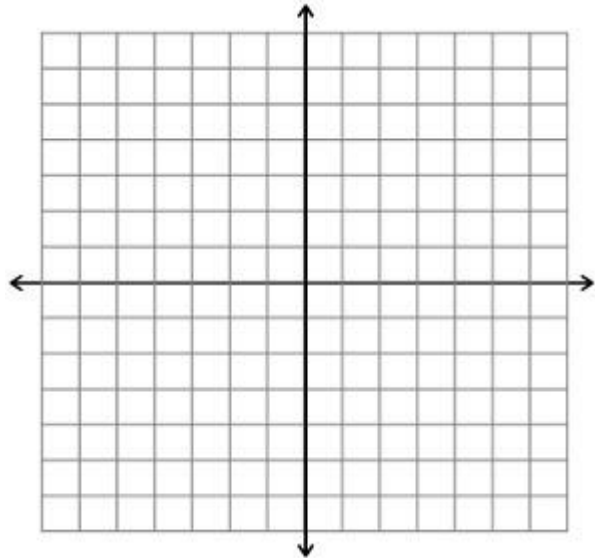
c. $m = 1$ and $(2, -1)$



b. $m = -\frac{1}{3}$ and $(-3, 4)$



d. $m = \frac{2}{3}$ and $(0, -3)$



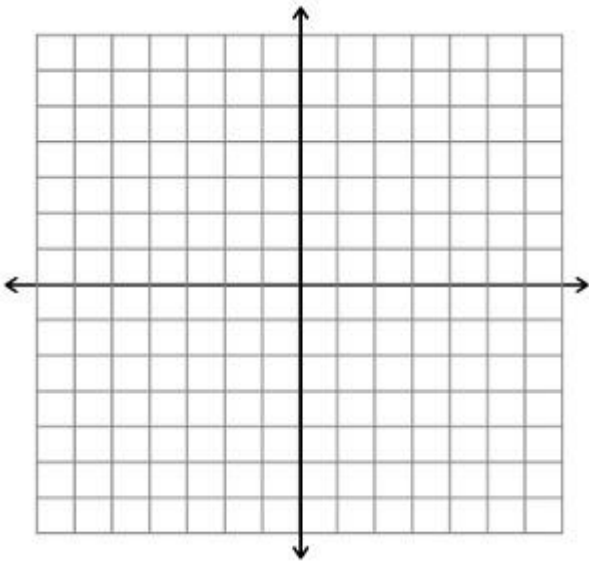
What is the slope-intercept form of a line?

Examples Identify the slope m and the y-intercept $(0, b)$ in each of the following equations:

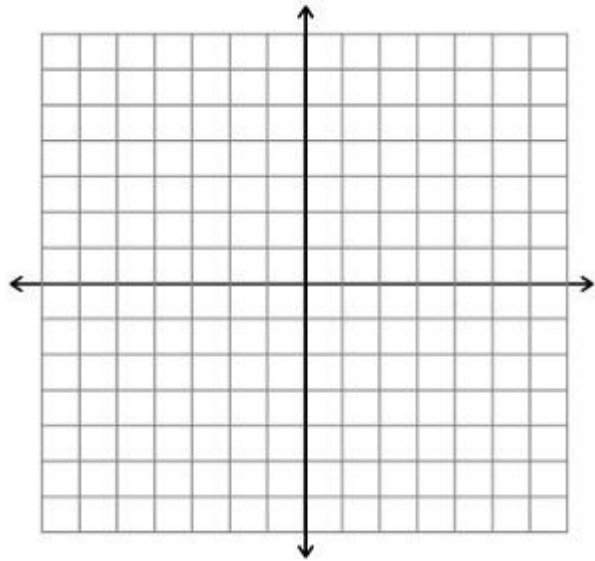
Equation	Slope, m	What stands for b	State the y-intercept (as a point)
a. $y = -2x + 3$			
b. $y = \frac{1}{4}x - 7$			
c. $y = 3x + 9$			
d. $y = x$			

Examples Graph the following lines:

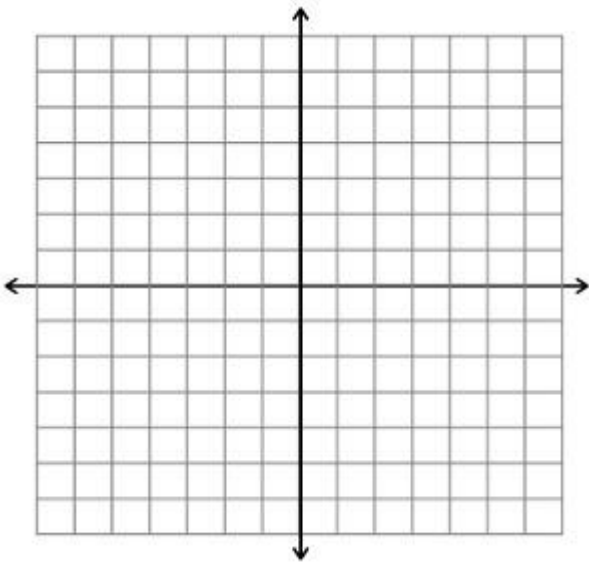
a. $y = 2x + 3$



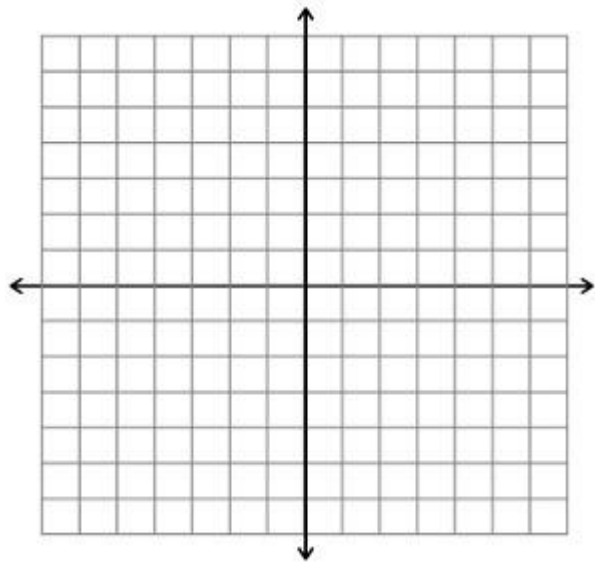
c. $y = \frac{1}{3}x - 4$



b. $y = -x + 4$



d. $y = 3x$



Lesson: Rewriting Linear Equations into Slope-Intercept Form of a Line

What is the slope-intercept form of a line?

Explain how to put the equation $2x - 4y = 8$ into slope-intercept form.

Examples Put the following equations in slope-intercept form:

a. $6y - 3x = 12$

b. $-5x + 2y = -4$

Lesson: Determining Whether Two Lines are Parallel or Perpendicular

What makes 2 lines parallel?

Examples Are the equations $y = \frac{2}{3}x + 5$ and $2x - 3y = 6$ parallel?

What makes 2 lines perpendicular?

Examples Determine the negative reciprocal of each of the following:

a. $\frac{2}{3}$

c. -7

b. $-\frac{1}{5}$

d. 1

Example Are the equations $y = \frac{3}{5}x + 2$ and $-3x + 5y = -15$ perpendicular?

Examples Are the following pairs of lines parallel, perpendicular, or neither?

a. $3x - 2y = 6$ and $4y = -6x - 8$

b. $y - 3x = 0$ and $12x - 4y = 24$

c. $y = 3$ and $x = -2$

Examples Lines A and B contain the given points. Determine whether the lines are parallel, perpendicular, or neither.

a. Line A: $(2, 11), (5, 10)$ Line B: $(1, 3), (-6, -18)$

b. Line A: $(-5, 5), (-7, -2)$ Line B: $(0, 11), (-4, 9)$

Lesson: Putting Equations in Standard Form

What is the standard form of a line?

Examples Write the following equations in standard form:

a. $y = 2x - 4$

b. $y = -5x + 3$

c. $y = -\frac{1}{2}x + 1$

d. $y = \frac{2}{3}x - 2$

Lesson: Writing Equations of Lines with The Point-Slope Form

What is the point-slope form of a line?

Examples Given the point and the slope, write the point-slope form of a line.

a. $m = -2, (1, 3)$

b. $m = -\frac{2}{3}, (5, -2)$

What is the slope-intercept form of a line?

What is the standard form of a line?

Examples Given the point and the slope, write the equation of the line in the desired format.

a. $m = -1$, $(2, -5)$ in slope-intercept form

b. $m = 3$, $(5, -4)$ in standard form

c. $m = -\frac{1}{2}$, $(6, 3)$

d. $m = \frac{2}{3}$, $(-5, -1)$

Lesson: Writing the Equation of a Line from Two Points

Write the equation of the line that contains the following 2 points in the desired format.

a. $(4, 2)$ and $(5, 5)$ in slope-intercept form

b. $(-2, 4)$ and $(-4, -2)$ in standard form

c. $(5, -3)$ and $(3, 5)$ in slope-intercept form

d. $(-1, -3)$ and $(1, 5)$ in standard form

Lesson: Writing the Equation of a Line Parallel/Perpendicular to Another Line

What makes 2 lines parallel?

What makes 2 lines perpendicular?

Examples Write the equation of a line with the given conditions.

a. Parallel to $y = 3x + 1$ that goes through $(-2, -5)$ in slope-intercept form

b. Perpendicular to $y = -\frac{2}{3}x - 5$ that goes through $(6, -2)$ in standard form

c. Parallel to $2x - 3y = 6$ that goes through $(1, -2)$ in slope-intercept form

Examples Write the equation of a line with the given conditions.

a. Perpendicular to $2x - 4y = 8$ that goes through $(-4, -2)$ in standard form

b. Parallel to $3x - y = 12$ that goes through $(-1, -5)$ in slope-intercept form

c. Perpendicular to $-2x + 3y = 12$ that goes through $(2, -5)$ in standard form

Lesson: Solving Systems of Equations by Graphing

We will now turn our attention to systems of equations.

System of Equations

A system of equations refers to 2 or more equations that share 2 or more variables.

Solution of a System of Equations

A solution of a system of equations is an ordered pair that is true in all equations.

Determine if the following pairs are solutions to the systems.

a. $(1,3)$

$$x - 2y = -5$$

$$2x - 3y = -4$$

b. $(-4,3)$

$$x + 3y = 5$$

$$2x + 3y = 1$$

There are several ways to solve systems. The first method we will focus on is how to solve by graphing.

Solving by Graphing

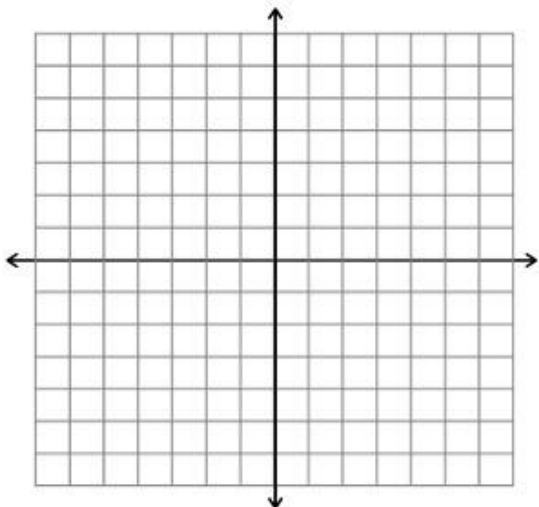
In this method, we graph both lines and determine the point of intersection if possible. We will examine all the possibilities.

Example

Graph each line and determine the point of intersection to find the solution.

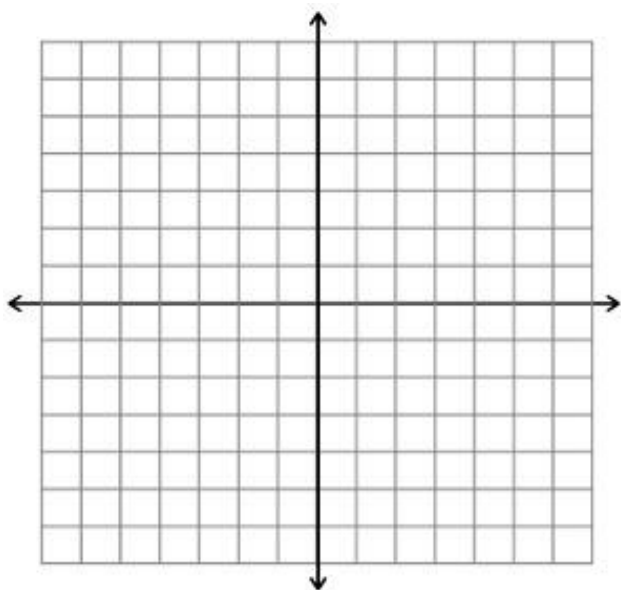
$$y = 2x + 3$$

$$y = -\frac{1}{2}x - 2$$

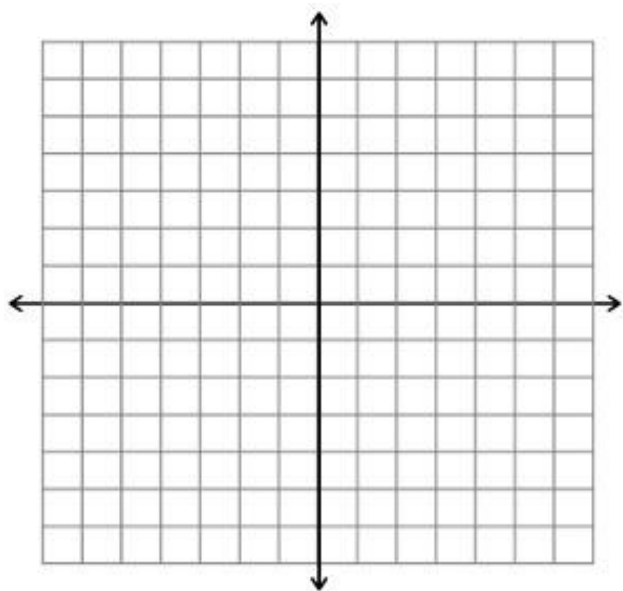


Example

a. $y = -3x + 1$
 $y = 2x - 4$



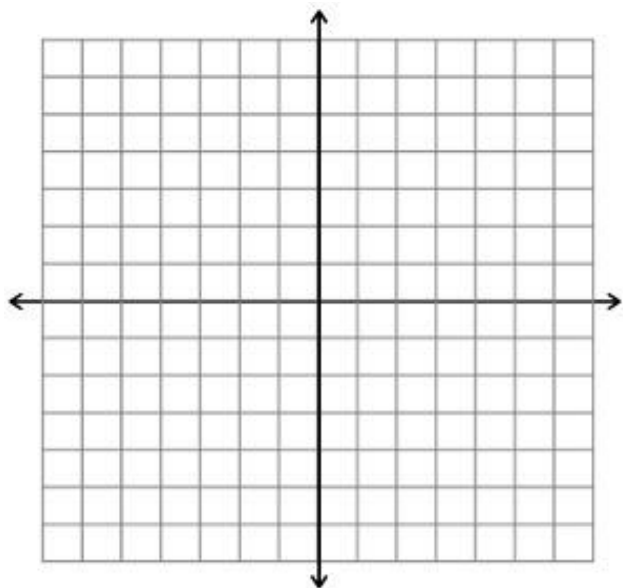
b. $y - \frac{1}{2}x = -4$
 $y + 2x = 1$



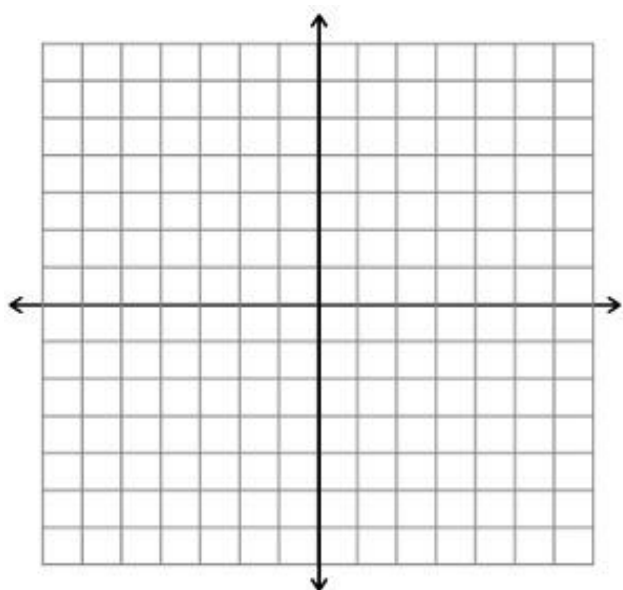
There are two special situations that can occur with graphing.

Example

a. $y = -3x + 2$
 $9x + 3y = 6$



b. $y = -\frac{1}{2}x + 4$
 $2x + 4y = 12$



Lesson: Solving Systems of Equations by Substitution

In this lesson, we will focus on the second method for solving systems: substitution.

Show how to use substitution with this example:

$$2x + y = 6$$

$$3x + 5y = 9$$

Example Solve the system using substitution

$$x - 2y = 10$$

$$5x - 7y = 20$$

Below are 2 more complicated problems using substitution. Outline the steps shown in the video:

$$-2x + 4y = 4$$

$$-2x + 6y = 6$$

$$2x - 5y = -4$$

$$8x - 9y = 6$$

Now let's see what happens when we get a problem with no solution or infinite solutions.

a. $9y - 18x = 5$

$$2x - y = 3$$

b. $x + 2y = -3$

$$4x + 12 = -8y$$

Lesson: Solving Systems of Equations by Elimination

In this lesson, we will focus on the third method for solving systems: elimination.

Show how to use elimination with this example:

$$3x - y = 8$$

$$2x + y = -3$$

Example Use elimination to solve the system.

$$4x + 2y = -6$$

$$3x - 2y = 6$$

Now let's examine a more complicated case of elimination. Write down how to approach these problems:

a. $3x - 6y = 12$

$$5x - 2y = -4$$

b. $5x - 10y = 5$

$$2x - 3y = 6$$

Now let's investigate what happens in the situation where there is no solution or infinite solutions.

a. $5x - 2y = 1$
 $-10x + 4y = 3$

b. $3x + y = 5$
 $-9x - 3y = -15$

Lesson: Solving Systems of Equations that Require Clearing of Fractions

Examples

a.
$$\begin{aligned}\frac{1}{6}x - \frac{1}{3}y &= \frac{1}{2} \\ \frac{2}{5}x - \frac{3}{5}y &= -\frac{6}{5}\end{aligned}$$

b.
$$\begin{aligned}-\frac{1}{2}x - \frac{3}{4}y &= 1 \\ \frac{1}{3}x + \frac{3}{2}y &= -\frac{2}{3}\end{aligned}$$

c. $\frac{1}{2}x + \frac{3}{5}y = 6$

$$\frac{1}{4}x - \frac{1}{2}y = -9$$

Lesson: Solving Systems of Equations that Require Clearing of Decimals

Examples

a. $0.02x + 0.1y = 0.05$
 $4x + 20y = 10$

b. $0.1x - 0.05y = 0.2$
 $0.3x - 0.25y = 0.2$

c. $-0.4x + 1.3y = 1.$
 $1.6x - 0.9y = 6.9$

Lesson: Applications of Systems

The top of a rectangular desk is twice as long as it is wide. The perimeter of the desk is 180 inches. What are the dimensions of the desk?

Sarah bought 3 shirts and a scarf for \$53. Marie bought 2 shirts and 3 scarves for \$54. Find the cost of each item.

How many milliliters of a 5% acid solution and how many milliliters of a 17% acid solution must be mixed to obtain a 60 mL solution that is 15% acid?

Lesson: Systems of Equations with 3 Equations

Show how to solve the system:

$$x + 2y + 3z = -11$$

$$3x - y + z = 0$$

$$-2x + 3y - z = 4$$

Sometimes you will have missing variables. Show how to solve the following system that is missing some variables:

$$x - 2y = -3$$

$$x + 6z = 3$$

$$2y - 4z = 6$$

Now let's look at an example where there is no solution versus infinite solutions:

$$8x + 20y - 4z = 16$$

$$-2x - 5y + z = -4$$

$$10x + 25y - 5z = 20$$

$$x + 4y - 3z = 2$$

$$2x - 3y + 2z = -5$$

$$3x + 12y - 9z = 2$$

Examples: More Examples of Systems of 3 Equations

Example Solve the system:

$$x + 3y - z = -2$$

$$-x + 3y - 2z = 5$$

$$2x + 4y + 2z = 4$$

Example Solve the following system:

$$x - 2z = 2$$

$$-4y + 2z = -2$$

$$-x + 3y = -3$$

Example Solve the following system:

$$x - 4y + 2z = -7$$

$$3x - 8y + z = 7$$

$$6x - 12y + 3z = 12$$

Lesson: A Review of Exponent Rules for Algebra Students

Power Rule: $x^m * x^n =$

Product Rule: $(x^m)^n =$

Quotient Rule: $\frac{x^m}{x^n} =$

Negative Exponents

$$x^{-n} =$$

$$\frac{1}{x^{-n}} =$$

$$\left(\frac{x}{y}\right)^{-n} =$$

Simplify the following using exponent rules. Your final answer should not contain negative exponents.

a. $3x^7y^{-4} * 5x^{-2}y^9$

e. $\frac{8x^{-3}y^4z^{-3}}{10x^5y^{-2}z^{-7}}$

b. $\left(\frac{2x^4y^5}{3z^5}\right)^3$

f. $\left(\frac{3x^{-3}y^{-2}}{6x^2y^4}\right)^{-2}$

c. $\frac{10x^4y^7}{2x^2y^3z^{-5}}$

g. $\left(\frac{2x^{-7}z^3}{3x^{-2}y^{-3}z^5}\right)^{-3}$

d. $\frac{4x^3y^{-2}}{6x^{-4}y^{-5}}$

h. $\left(\frac{5x^{-2}y^{-3}z^3}{x^3y^4z^{-7}}\right)^{-2}$

Lesson: An Intro to Polynomials

We can extend our knowledge of exponents to another algebraic topic – polynomials

Before we define what a polynomial is, it is usually easier to see a few examples of what is or is not a polynomial.

The following ARE polynomials:

- $x^2 - 3x + 8$
- $x^7 - 5x^4 + 2x^3 + x$
- 9
- $7x + \pi$
- $x^3 + \frac{2}{3}$

The following ARE NOT polynomials:

- $\sqrt{x} - 4$
- $4x^{-4} - 7x^{-3}$
- $\frac{4}{x} - \frac{2}{x+3}$

Now we can more formally define a polynomial.

Polynomial

An expression with real numbers, variables, and whole number exponents.

Descending Order

Ordering a polynomial from the largest exponent to the smallest exponent, and any freestanding number without a variable is last

Example 1

Put the polynomials in descending order.

a. $x^3 + 4x^5 - 2x^7 + 3x^2 + 5x^8$

b. $x^2 - x^3 + x^4 - x^5 - x^6 + 3$

Term

Any part in a polynomial that is separated by a “+” or “-” sign is considered to be its own term

Variables

The letters in a polynomial that represent numbers

Coefficients

The numbers attached to the variables

Constant Term

The freestanding number within a polynomial that is not attached to any variables

Example 1

Examine the polynomials, then determine the variables, coefficients, and constant terms

Polynomial	Terms	Variable(s)	Coefficient(s)	Constant
$2x + 3$				
$3x^2 - x + 5$				
5				
$2x^2y + 3y - x$				

Degree of a Term

The degree of a term is the sum of the exponents of that term.

Example 2

Identify the degree of each term

Term	Degree	Term	Degree
x^4		x^4y^3	
$3y^6$		x^7y^9	
x		$x^4y^2z^5$	
6		xy^4	

Degree of a Polynomial

The degree of a polynomial is the same as the degree of the term of highest degree within that polynomial

Leading Coefficient

The leading coefficient is the coefficient attached to the term of highest degree in a polynomial

Trinomial

A polynomial with 3 terms

Binomial

A polynomial with 2 terms

Monomial

A polynomial with 1 term

Example 3

Identify the degree of each polynomial and the type of polynomial it is.

Polynomial	Degree of Each Term, Respectively	Deg of Polynom	Leading Coefficient	Type of Polynomial
$x^5 + 4x^3 - 2x$				
$-5x^2 + 18x$				
$6x + 2x^4 - 3x^2$				
$x^5 + 2x^7 - 4$				
8				

Evaluating Polynomials

Evaluating polynomials is not unlike evaluating other expressions we've seen in this course.

Example 4

Evaluate the polynomials for $x = -2$.

a. $x^2 + 3x + 1$

b. $x^3 - x^2 - 3x + 4$

c. $\frac{1}{4}x^3 - \frac{1}{2}x^2 + 5x$

Lesson: How to Add and Subtract Polynomials

To add or subtract polynomials, combine like terms.

Examples

Perform the following operations.

a. $(2x + 4) + (5x^2 - 3x + 1)$

b. $(5x^2 - 3x + 1) - (x^3 - 2x^2 - x + 1)$

c. $(3x^3 + 2x^2 - 5x) + (4x^3 + 5x^2 + 2x)$

d. $(2x^2 + 5x - 4) - (x^3 - 2x^2 + 3x - 5)$

Lesson: How to Multiply Polynomials

To begin, let's first focus on the process of distributing a monomial with a polynomial.

Examples Multiply the following:

a. $3x(4x^2 + 5x - 7)$

b. $x^2y^4(3x^2 + 2xy - 8y^4)$

c. $\frac{1}{2}x^3y^4(4xy^3 - 2x^3y + 8xy)$

Now write down the process for how to multiply $(x + 2)(x^2 + 3x)$

Examples Multiply the following:

a. $(x + 3)(x + 2)$

b. $(3x + 1)(x - 6)$

c. $(x^2 + 4)(x - 2y)$

d. $(x + 4)(x - 4)$

e. $(x + 3)(x^2 + 4x + 5)$

f. $(2x^2 + 3x + 1)(3x^2 - 4x + 2)$

How do you multiply $(x + 5)^2$?

Examples Multiply the following:

a. $(x + 2)^2$

b. $(2x^2 + 1)^2$

c. $(3x^2 + x + 5)$

How do you multiply 3 polynomials together, such as $(x + 2)(x + 1)(x - 3)$?

Examples Multiply the following together:

a. $(x - 3)(x + 4)(x - 1)$

b. $(2x + 1)(x - 2)(x - 1)$

Lesson: The Basics of Dividing Polynomials

In this section, we will discuss how to divide polynomials that are in these forms:

$$\frac{x^2+3x}{x} \quad \text{or} \quad (4x^7y^3 + 8x^4y^2 - 12x^5y^5) \div 2x^2y$$

To divide in these cases, you can just use rules of exponents. Sometimes you will want to separate the problem into separate fractions.

Examples

a. $\frac{x^2+3x}{x}$

b. $\frac{5x^4-10x^3+15x^2}{5x}$

c. $(27x^4y^2 + 18x^3y) \div 9x^2y$

d. $(4x^7y^3 + 8x^4y^2 - 12x^5y^5) \div 2x^2y$

Lesson: Long Division for Polynomials

In this section, we will discuss how to divide polynomials that are in these forms:

$$(x^2 + 5x + 6) \div (x + 3) \quad \text{or} \quad (2x^2 + 5x + 3) \div (2x + 3)$$

In these types of problems, we need to use long division. In another video, we talked about how to divide polynomial problems like these:

$$\frac{4x^3 + 2x^2}{2x}$$

But we did not need long division. What is different about these types of problems that require long division?

Show how to use long division to divide $(x^2 + 5x + 6) \div (x + 3)$

Examples Divide the following.

a. $(x^2 + 7x + 10) \div (x + 5)$

b. $(2x^2 + 5x + 3) \div (2x + 3)$

c. $\frac{x^2 - 11x + 30}{x - 5}$

Now we will look at what to do when there is a remainder.

Divide the following: $(x^2 + 3x + 1) \div (x + 2)$

Now show how to divide when there are “missing” terms, such as with $(x^4 - 81) \div (x^2 - 9)$

Examples Divide.

a. $(5x^2 - 10x^3 + 8 - 9x) \div (x + 2)$

b. $(x^3 + 1) \div (x + 1)$

c. $(5x^3 - 4x^2 + 1) \div (x + 2)$

Lesson: The Greatest Common Factor

The Greatest Common Factor (GCF)

The Greatest Common Factor is the biggest common factor between a set of terms.

Examples

Find the greatest common factor in the following polynomials, then factor the polynomial.

a. Polynomial: $3x + 3y$

GCF:

Factor out the GCF:

b. Polynomial: $3xy + 6xz$

GCF:

Factor out the GCF:

c. Polynomial: $10x^2 - 25xy + 15xz$

GCF:

Factor out the GCF:

d. Polynomial: $3x^7y + 8x^3z^5 - 5y^4z^3$

GCF:

Factor out the GCF:

Examples

Factor the following polynomials.

a. $12xy^2 + 10x^2y^4 - 14xy^5$

b. $5xy - 10yz + 12xz$

c. $3x + 6x^2 - 12x^3$

d. $5x^4y^2z^5 + 10x^2y^4z^6 - 25x^5yz^3$

Lesson: Factoring by Grouping

Grouping can best be understood with the following example.

Factor $ax + ay + bx + by$ by grouping:

Examples

Finish the following examples where the grouping had been started.

a. $3(x - y) + 2z(x - y)$

b. $2a(3b + 2c) - 5(2c + 3b)$

c. $4x(y - z) - 3(z + y)$

d. $4z(4x - 7) - (4x - 7)$

Examples

Factor the following.

a. $5x + 10y + xz + 2yz$

b. $10x^2 + 15x^3y - 2 - 3xy$

c. $3xy + 4y - 27x - 36$

d. $3xy + 6x + 21y + 42$

e. $3x^2 - 48y + 8x - 18xy$

Lesson: The Basics of Factoring Trinomials of the Form $x^2 + bx + c$

Factoring trinomials will require us to multiply polynomials, so first we will review this skill.

Examples

Multiply.

a. $(x + 3)(x - 5)$

b. $(2x - y)(5x + 2y)$

Definition of a Trinomial

A trinomial is a polynomial with three terms.

How to Factor a Trinomial Using Trial and Error

Show how to factor $x^2 + 3x + 2$

Show how to factor $x^2 + 7x + 12$

Examples

Factor the following.

a. $x^2 + 6x + 5$

b. $x^2 + 12x + 11$

c. $x^2 - 8x + 15$

d. $x^2 - 13x + 30$

e. $x^2 + 10x - 24$

f. $x^2 - 5x - 14$

g. $2x^2 + 16x + 30$

h. $5x^4 + 55x^3 - 45x^2$

Lesson: The Basics of Factoring Trinomials of the Form $-x^2 + bx + c$

How can you factor $-x^2 - 5x - 6$?

What about $-x^2 - 15x - 36$?

Examples

Factor the following.

a. $-x^2 + 13x - 30$

b. $-x^2 + 16x - 55$

c. $-x^2 - 2x + 3$

Lesson: The Basics of Factoring Trinomials of the Form $x^2 + bxy + cy^2$

How do you factor $x^2 + 5x + 6$?

How do you factor $x^2 + 5xy + 6y^2$?

Examples Factor the following.

a. $x^2 + xy - 12y^2$

b. $x^2 - 2xy + y^2$

c. $3x^2 + 21xy + 18y^2$

Lesson: The Basics of Factoring Trinomials of the Form $ax^2 + bx + c$

In this lesson we will focus on trinomials that do not have a leading coefficient.

To begin, first let's get warmed up by factoring the following using trial and error:

a. $x^2 + 7x + 12$

b. $x^2 - 3xy + 10y^2$

How do you factor $2x^2 + 5x + 3$ using trial and error?

How do you factor $3x^2 + 16x + 5$ using trial and error?

Another way to factor $3x^2 + 16x + 5$ is by grouping. Show how to use this technique below:

Examples Factor the following:

a. $7x^2 - 11x + 4$

b. $3x^2 + 10x + 8$

c. $5x^2 - 11x - 36$

d. $3x^2 - 17xy - 6y^2$

e. $-10x^2 + 19x - 6$

Lesson: Factoring Special Trinomials

How do you factor $x^2 + 6x + 9$?

What's another way to write this factorization?

What do you call this type of trinomials?

Examples Factor the following:

a. $x^2 + 12x + 36$

b. $9x^2 - 12x + 16$

c. $x^2 - x + \frac{1}{4}$

d. $36x^2 - 60xy + 25y^2$

How do you factor $x^2 - 4$?

What is this type of factoring called?

Examples Factor the following:

a. $x^2 - 25$

b. $x^2 - 16$

c. $x^2 - 1$

d. $25x^2 - 36$

e. $16x^2 - 81y^2$

f. $16 - x^2$

g. $25y^2 - 64x^2$

h. $\frac{49}{64} - x^2$

Lesson: How to Factor The Sum and Difference of Cubes

Write down the formula for the sum of cubes

$$a^3 + b^3 =$$

Write down the formula for the difference of cubes

$$a^3 - b^3 =$$

Show how to factor $x^3 + 8$

Show how to factor $8x^3 - 27$

Examples Factor:

a. $x^3 - 1$

b. $x^3 - \frac{1}{8}$

c. $x^3 + 64$

Review: An Overview of All the Types of Factoring

We will review all of the different types of factoring by presenting an example of each type.

Greatest Common Factor

1. $4x^2 + 8xy$

2. $6x^3y^2 - 8x^2y^5 + 10x^5y$

Grouping

3. $3x + 6y - xz^2 - 2yz^2$

4. $12x^2 - 2y + 3x - 8xy$

Basic Trinomials of the Form $x^2 + bx + c$

5. $x^2 + 7x + 10$

6. $x^2 - 2x - 15$

Trickier or Special Forms of Trinomials of the Form $x^2 + bx + c$

7. $-x^2 - 3x + 28$

8. $6x^2 + 54x + 120$

9. $x^2 - 6xy + 8y^2$

10. $x^2 - 8x + 16$

11. $x^2 - 25$

12. $x^2 - \frac{9}{4}$

Trinomials of the Form $ax^2 + bx + c$

13. $2x^2 + 11x + 5$

14. $7x^2 - 15x + 8$

15. $-3x^2 - 15x^2 + 198x$

16. $16x^2 - 24xy + 9y^2$

More Practice with the Difference of Squares

17. $x^2 - 81$

18. $25x^2 - 4$

19. $\frac{1}{4}x^2 - 9y^2$

Sum and Difference of Cubes

20. $x^3 + 8$

21. $x^3 - 27$

22. $27x^3 - 1$

23. $8x^3 + 64y^3$

Lesson: How to Solve Quadratic Equations with Factoring

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, and a , b , and c are real numbers.

There are several ways that you can solve quadratic equations, but in this lesson we will focus on how to use factoring as a way to solve them.

Solving a quadratic equation by factoring requires a rule called the **Zero Product Rule**.

Write down the Zero Product Rule:

Examples Solve the following equations using the zero product rule:

a. $3x = 0$

b. $3(x + 1) = 0$

c. $(x + 3)(x + 1) = 0$

d. $x(2x + 1)(3x + 5) = 0$

Show how to use factoring to solve $x^2 + 5x + 6 = 0$.

Examples Solve the following by using factoring:

a. $x^2 + 12x - 45 = 0$

b. $x^2 - 12x + 11 = 0$

c. $2x^2 + 5x + 3 = 0$

d. $55x = -20x^2 - 30$

e. $x^2 = 25$

f. $x^2 = 9x$

Lesson: Tips and Tricks to Help with Harder Quadratics Solved by Factoring

A **quadratic equation** is an equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$, and a, b, and c are real numbers.

In the previous lesson, we talked about the basics of using factoring to solve quadratics.

In this lesson, we will look at some trickier examples. First, let's warm up by solving the following:

a. $4x = 14x^2 - 48$

b. $3x^3 = 12x^2$

Sometimes we will have to do further work to simplify the problem before we factor.

Examples Use factoring to solve.

a. $(x + 3)(x + 4) = 6$

b. $5x(x + 2) - 4x = 4(x^2 - 2)$

c. $(x + 4)^2 - 2x = 11$

What can be done to solve $\frac{1}{3}(x+1)^2 - \frac{1}{4}x = \frac{1}{6}(x-2)^2 - \frac{1}{3}$?

Examples Use factoring to solve.

a. $\frac{1}{4}(2x-3)^2 + \frac{1}{4}x = \frac{1}{4}(x-5)^2 - \frac{3}{2}$

b. $\frac{1}{4}x(2-x) - \frac{3}{4} = \frac{1}{5}x(x+1) - \frac{7}{10}$

Lesson: Applications of Quadratic Equations

The area of a rectangle is 15ft squared, and the length of the rectangle is 7ft less than twice the width. Find the dimensions of the rectangle.

An 12ft ladder leans against the side of a house. The bottom of the ladder is 6ft from the side of the house. How high is the top of the ladder from the ground? If necessary, round your answer to the nearest tenth.

A kite flying in the air has a 14ft line attached to it. Its line is pulled taut and casts a 9ft shadow. Find the height of the kite. If necessary, round your answer to the nearest tenth.

The longer leg of a right triangle is 2in longer than the shorter leg. The hypotenuse is 4in longer than the shorter leg. Find the side lengths of the triangle.

Lesson: Linear Inequalities in Two Variables

What is a linear inequality in 2 variables?

Show how to graph $y \leq 2x + 1$

Examples Graph the following:

a. $3x + 2y > -6$

b. $5x - 2y \leq 10$

c. $-3x + 4y < -12$

Lesson: Compound Linear Inequalities in Two Variables

What is a compound linear inequality?

What does “and” mean in compound inequalities?

What does “or” mean in compound inequalities?

Show how to graph $y > 2x + 1$ and $y \leq -x - 3$

Show how to graph $y \leq -2x + 1$ or $y > x + 2$

Examples Graph the following:

a. $3x - 2y > 6$ and $2x + y \leq 3$

b. $5x - 2y \leq 10$ or $2x + 6y > 12$

Examples Graph the following:

a. $y \geq 4$ and $y < -2x + 1$

b. $x < 3$ or $4x - 2y \leq 8$

c. $x > -1$ and $y \geq 2x + 2$