Lesson: An Intro to Functions

Define a relation:

Define a domain:

Define a range:

<u>Example</u> In the set of ordered pairs $\{(2,3), (4, -1), (7, 2)\}$, define the domain and range of the relation.

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Define a function:

What is the visual explanation that explains what a function is or is not?

<u>Example</u> In the set of ordered pairs $\{(2,3), (4, -1), (7, 2)\}$, is this a function?

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Example Are the following functions?

a. $\{(4,-1),(3,2),(7,-1)\}$

b. $\{(2,4),(3,5),(2,-1)\}$

What is the notation for a function? How do we describe/use this notation?

Example In the set of ordered pairs $\{(2,3),(4,-1),(7,2)\}$, what is $f(4)$?

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Suppose $f(x) = 2x + 1$ and $g(x) = x^2 - 3$. Find the value of the following:

a. $f(2)$

b. $g(1)$

c. $f(a)$

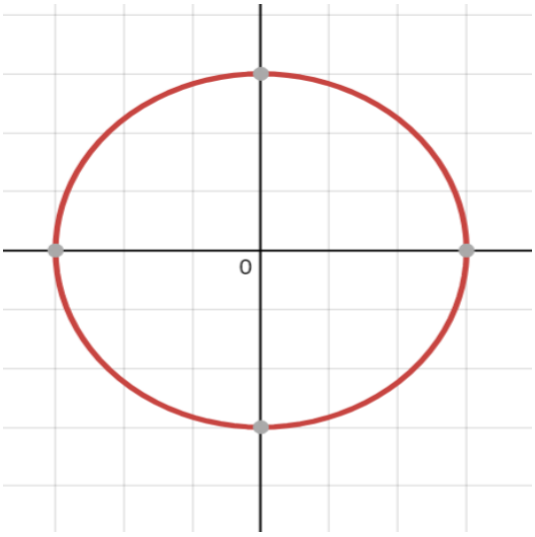
d. $g(h + k)$

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

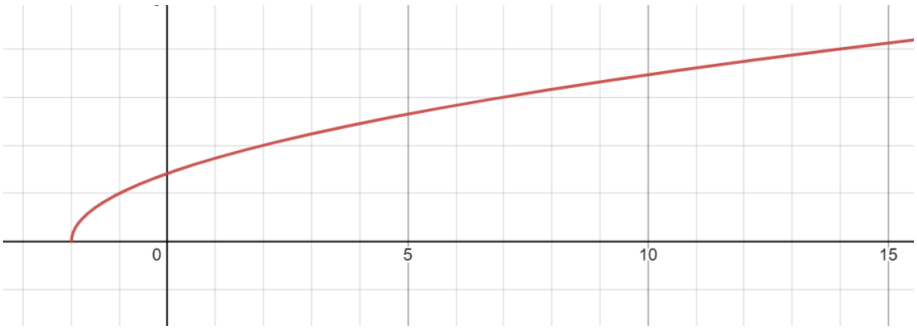
What is the vertical line test?

<u>Example</u> Do the following graphs represent a function?

a.



b.



Copyright 2023, Jennifer Hill, DivideandConquerMath.com

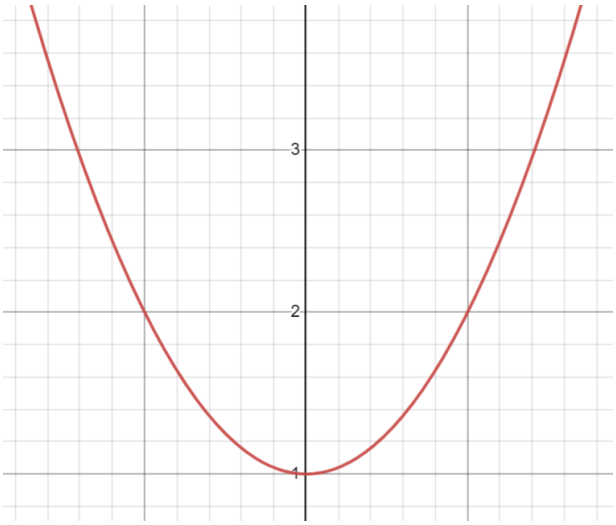
Lesson: Determining the Domain and Range of Functions In General

How do you determine the domain and range of a set of points like $\{(2,4), (5,8), (4, 6)\}$?

What is the domain and range of that set of points?

In general, it is easiest to determine the domain and range when we have the graph in front of us.

For the following graph, write out how you should determine the domain and range.



Write a sentence on how you should find the domain:

State the domain:

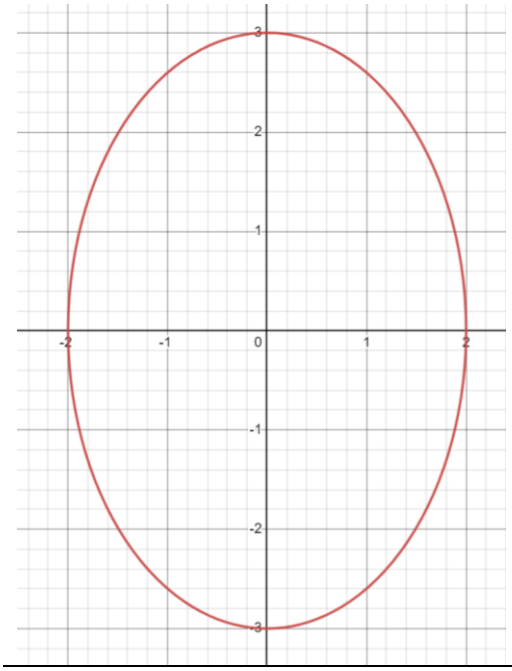
Write a sentence on how you should find the range:

State the range:

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Examples Determine if the following graphs represent functions, and then state the domain and range.

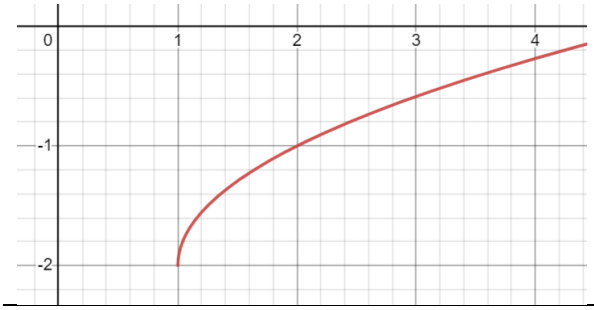
a.



Domain:

Range:

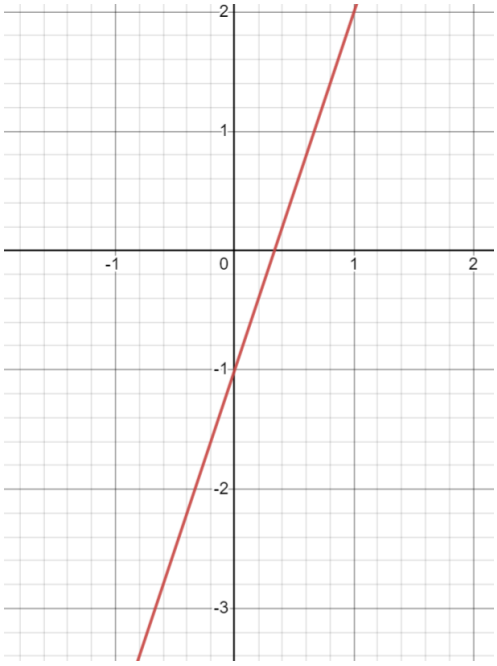
b.



Domain:

Range:

c.



Domain:

Range

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Lesson: The Domain of Some Specific Types of Functions

What is a polynomial? Write some examples.

What is the domain of any polynomial function?

What is a rational function?

What is the domain of any rational function?

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Examples Determine the domain of the following.

a. $f(x) = 3x - 1$

b. $g(x) = \frac{4}{x+2}$

c. $h(x) = 3x^3 - 2x^2 + 4x - 1$

d. $f(x) = \frac{2x+1}{x^2-9}$

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

What is a square root function?

What is the domain of a square root function?

<u>Examples</u> Find the domain of the following:

a. $f(x) = \sqrt{x - 2}$

b. $g(x) = \sqrt{4x - 1}$

c. $h(x) = \sqrt{4 - x}$

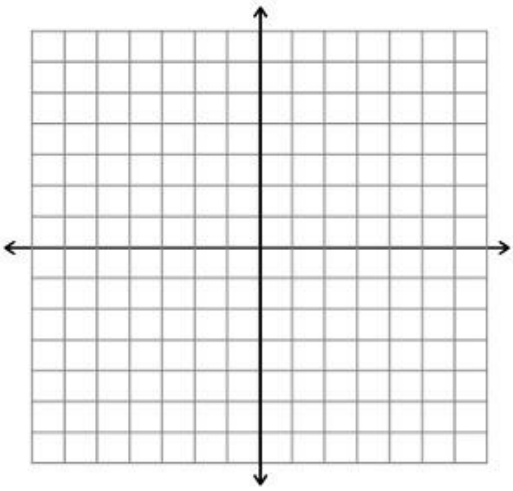
Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Lesson: How to Graph a Function

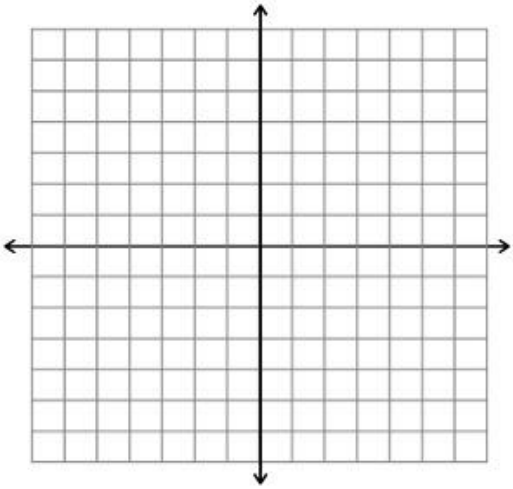
How do you graph a function?

<u>Examples</u> Graph the following functions.

a. $f(x) = 3x + 1$



b. $g(x) = 4$



Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Lesson: Review of Functions

How do you represent a function?

What is a function often interchangeable with? How can you rewrite $f(x) = 3x + 8$?

So what IS a function? What does it actually represent?

What's another handy feature of functions?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What are the domain and range?

Does domain and range only work for functions?

<u>Examples</u> Are the following functions, and what is the domain and range?

a. $\{(2,3), (5, -1), (-3, 7)\}$

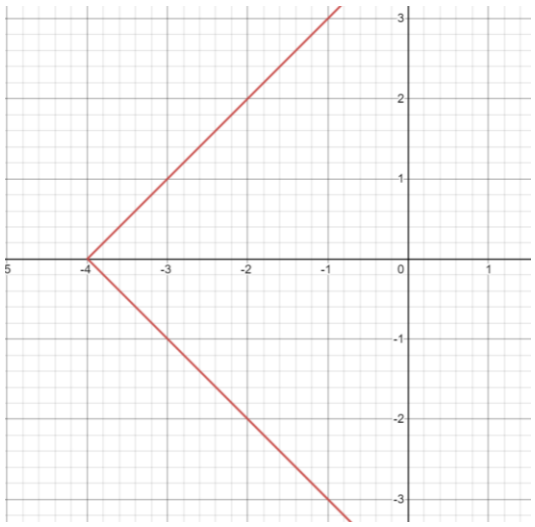
b. $\{(-1, 5), (-8, 3), (-1, 6)\}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

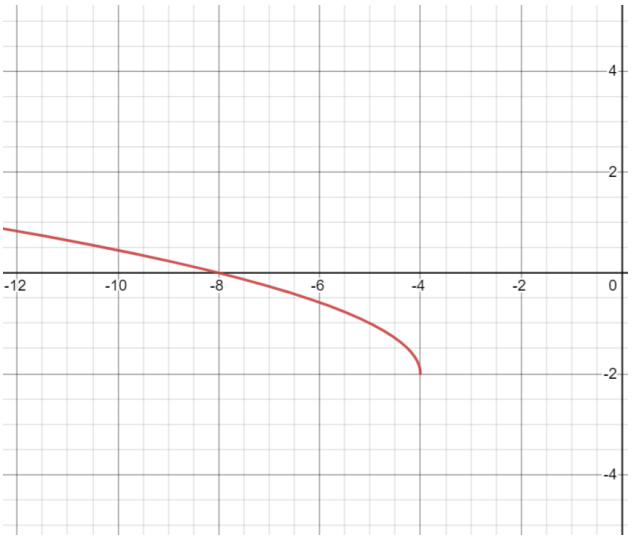
What is the Vertical Line Test, and how does it help you determine if you have a function?

<u>Example</u> State the domain and range of the following, and use the vertical line test to determine if the graph represents a function.

a.



b.



Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Determine if the following represent functions.

a. $y = 3x^2$

b. $x = y^2$

c. $y = \frac{5}{x^2}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Write the following in function notation: $3x + 2y = 6$

Example Suppose $f(x) = 3x - 5$ and $g(x) = x^2 + 4x - 1$ and $h(x) = \{(2,5),(6,-3),(-1,-4)\}$. Find the following:

a. $f(7) =$

b. $g(3) =$

c. $f(k) =$

d. $g(x + h) =$

e. $h(6) =$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Increasing, Decreasing, and Constant Functions

Below are the "formal math" definitions of whether a function is increasing, decreasing, or constant. Let I be an open interval and let x_1 and x_2 be two points in that interval.

- A function is **increasing** over I if when $x_1 < x_2, f(x)_1 < f(x_2)$
- A function is **decreasing** over I if when $x_1 < x_2, f(x)_1 > f(x_2)$
- A function is **constant** over I if when $x_1 < x_2, f(x)_1 = f(x_2)$

While these are the formal definitions, what we really want to know are the intuitive meanings of these definition.

Sketch a graph that is increasing:

Decreasing:

And constant:

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Sketch the graph in the video, and then identify where the graph is increasing, decreasing, or constant.

a.

b.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: A Few Basic Graphs

There are a few graphs that are helpful to immediately know as we move forward. Let's break these down.

The Parabola: $f(x) = x^2$

Create table of relevant points and then sketch your own graph.

Absolute Value: $f(x) = |x|$

Create table of relevant points and then sketch your own graph.

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

X cubed: $f(x) = x^3$

Create table of relevant points and then sketch your own graph.

Square Root: $f(x) = \sqrt{x}$

Create table of relevant points and then sketch your own graph.

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Cube Root: $f(x) = \sqrt[3]{x}$

Create table of relevant points and then sketch your own graph.

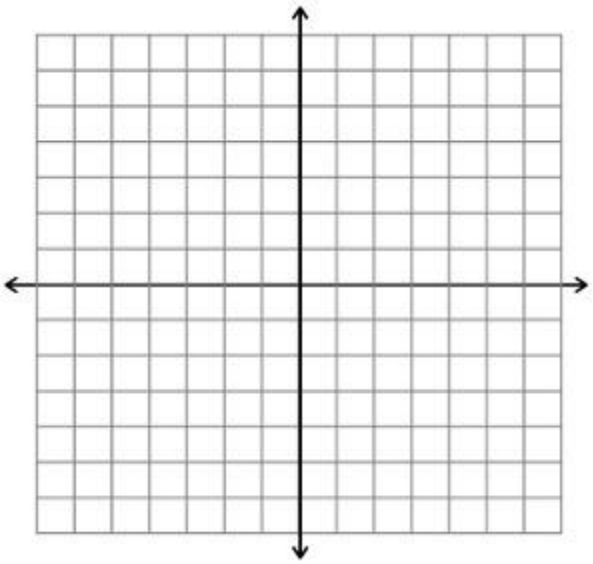
Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Lesson: An Intro to Translations and Reflections of Graphs

We are going to develop the intuition for this concept before we actually graph anything.

Vertical Translations

Draw the base graph of $f(x) = x^2$ in one color and then use some other colors to show how the translations work. Label the graphs.



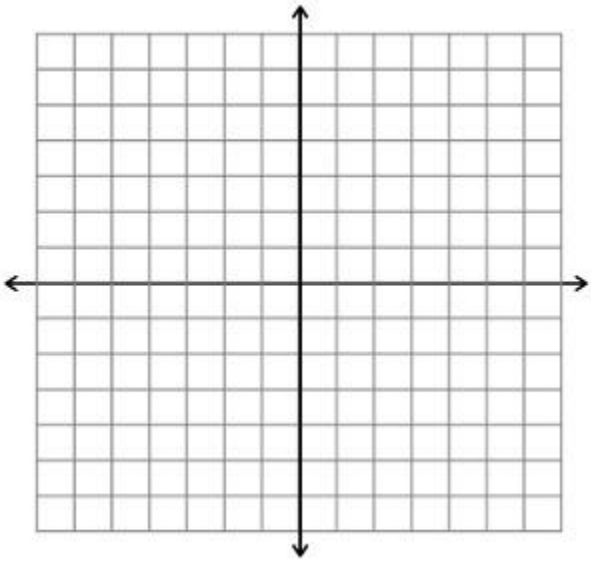
Explain the general way that vertical shifts work:

<u>Example</u> How does $f(x) = \sqrt{x} + 6$ shift?

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Horizontal Translations

Draw the base graph of $f(x) = |x|$ in one color and then use some other colors to show how the translations work. Label the graphs.



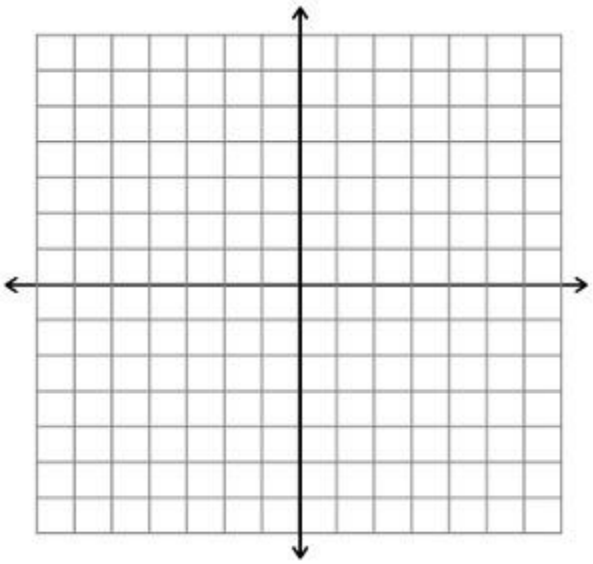
Explain the general way that horizontal shifts work:

<u>Example</u> How does $f(x) = \sqrt[3]{x+3}$ shift?

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Reflections Over the X-Axis

Draw the base graph of $f(x) = \sqrt{x}$ and then graph the reflection in another color. Label the graph of the reflection. Label the graphs.



Explain the general way that reflections over the x-axis work:

<u>Example</u> How does $f(x) = -x^2$ look?

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Example Describe the translations and reflections of the following.

a. $f(x) = -x^2 + 3$

b. $g(x) = \sqrt{x+2} - 1$

c. $h(x) = (x-5)^3 + 2$

d. $f(x) = -|x+2| + 5$

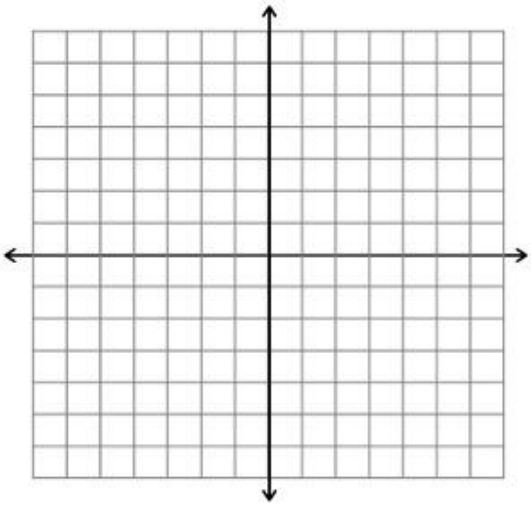
e. $g(x) = -\sqrt[3]{x-1} - 4$

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

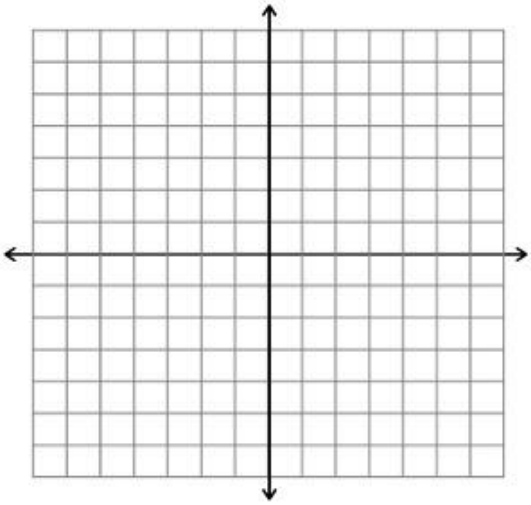
Lesson: Graphing with Translations and Reflections

<u>Examples</u> For each of the following, describe the translations and reflections. Then graph with 2-3 points.

a. $f(x) = x^2 + 1$
   Describe any translations or reflections:

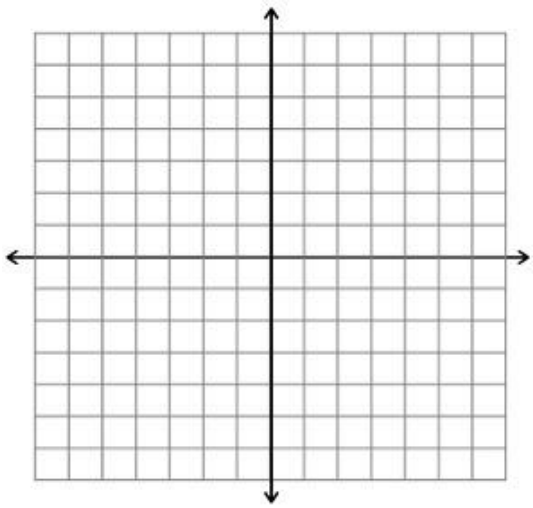


b. $g(x) = \sqrt{x - 2}$
   Describe any translations or reflections:

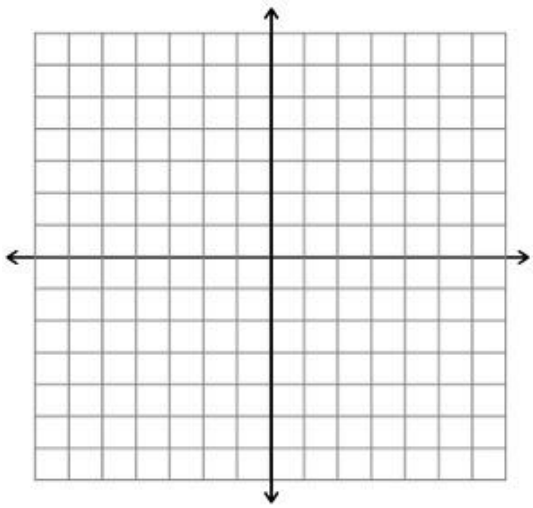


Copyright 2023, Jennifer Hill, DivideandConquerMath.com

c. $f(x) = (x + 2)^3 - 1$
Describe any translations or reflections:

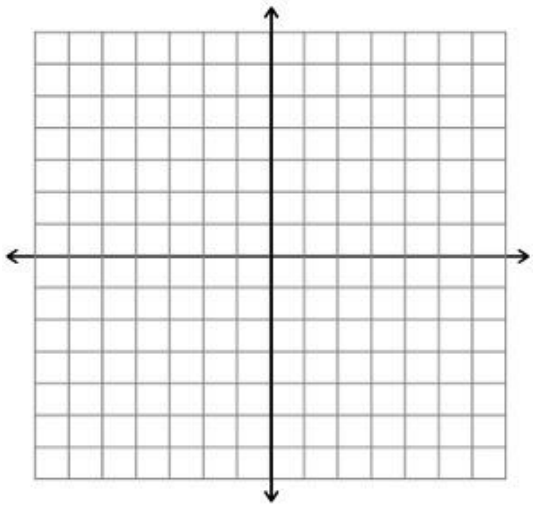


d. $h(x) = |x - 3| + 2$
Describe any translations or reflections:

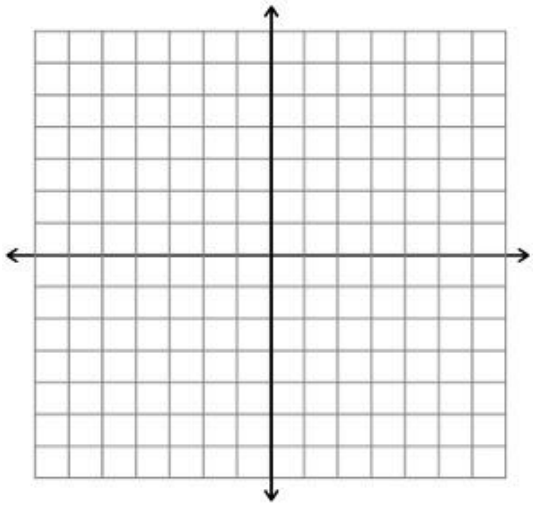


Copyright 2023, Jennifer Hill, DivideandConquerMath.com

e. $f(x) = -|x - 3| - 2$
Describe any translations or reflections:



f. $f(x) = -\sqrt{x + 3} + 1$
Describe any translations or reflections:



Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Lesson: Writing Equations of Functions with Translations and Reflections

<u>Example</u> Write the equation of a function $g(x)$ where the following transformations are performed on the graph of $f(x)$

a. $f(x) = \sqrt[3]{x}$ is shifted 3 units down and 2 units right

b. $f(x) = x^2$ is shifted 2.5 units up and π units left

c. $f(x) = e^x$ is reflected about the x-axis, shifted 4 units down and 1 unit left

Copyright 2023, Jennifer Hill, DivideandConquerMath.com

Lesson: Graphs of Basic Functions

In this section, we will review some basic graphs that are vital to know for this class and calculus. Knowing the "basic shapes" of certain graphs will make your life much easier and give you greater intuition when graphing.

Show hot to graph the following and highlight the **key points** and **key patterns.** This will come in handy for the next lesson. Take your time to sketch your own NEAT CLEAN CLEAR graphs.

$f(x) = x$

$f(x) = x^2$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

$f(x) = x^3$

$f(x) = \sqrt{x}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

$f(x) = \sqrt[3]{x}$

$f(x) = |x|$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Piecewise Functions

What is a piecewise function? Draw an example of what they look like.

<u>Example</u> Given the function: $f(x) = \begin{cases} 3x - 1, & x \geq 2 \\ \frac{1}{2}x - 4, & x < 2 \end{cases}$, evaluate the following:

a. $f(3) =$

b. $f(-1) =$

c. $f(2) =$

<u>Example</u> Given the function: $f(x) = \begin{cases} 4, & x > 3 \\ 4x - 2, & -2 < x \leq 3 \\ x^2 - 5, & x \leq -2 \end{cases}$ evaluate the following:

a. $f(4) =$

b. $f(-5) =$

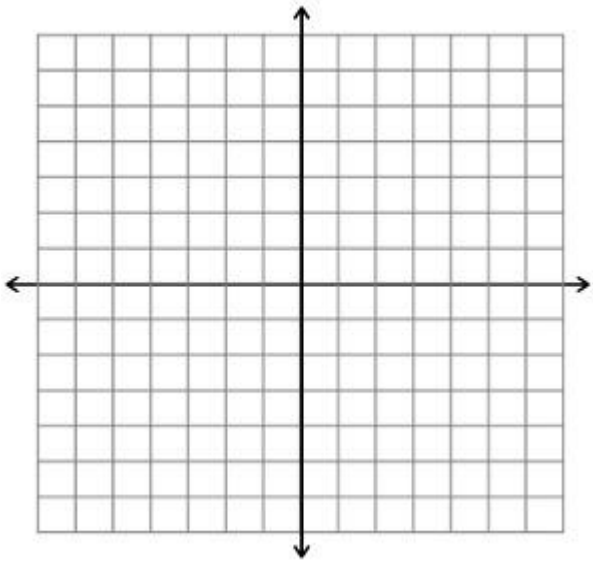
c. $f(-2) =$

d. $f(3) =$

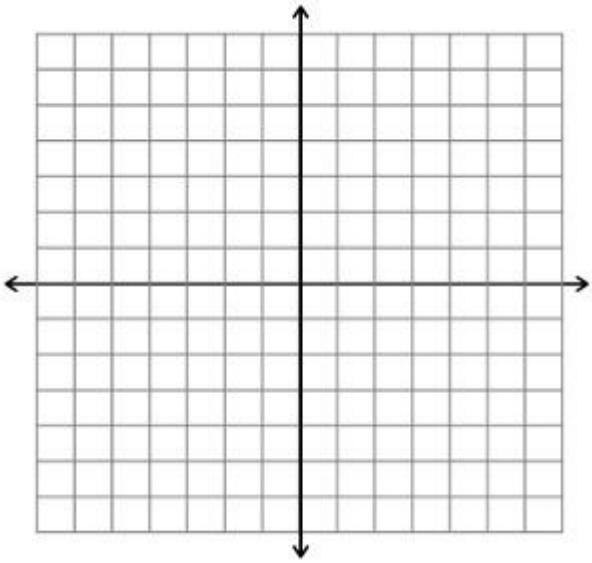
Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Show how to graph the following piecewise functions.

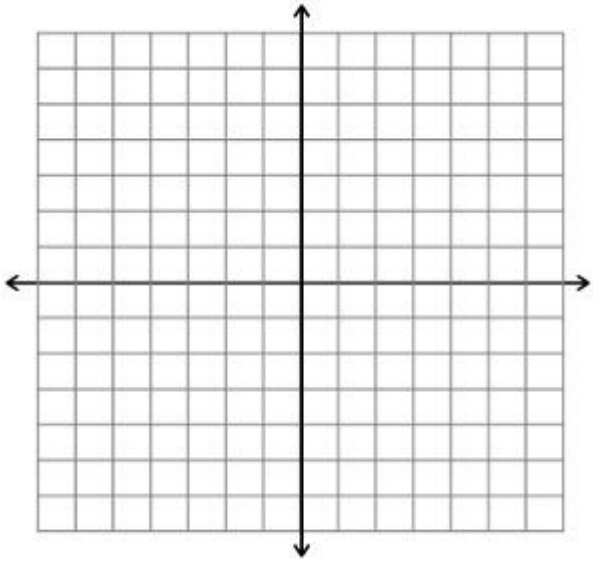
a. $f(x) = \begin{cases} 4, & x > 5 \\ -1, & x \leq 5 \end{cases}$



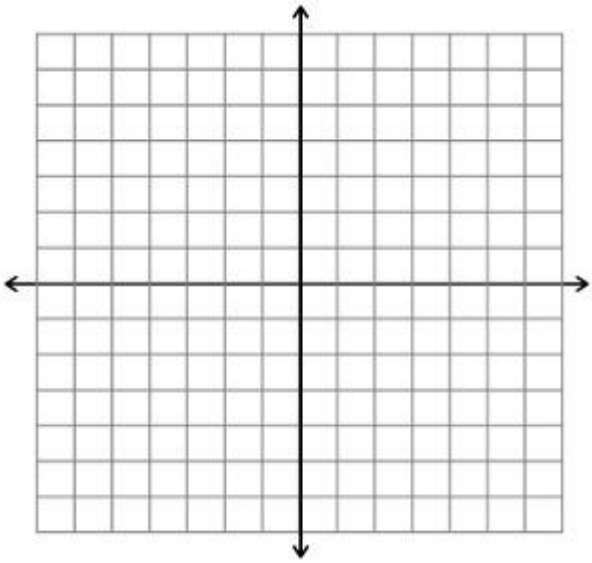
b. $f(x) = \begin{cases} 3x + 1, & x > -3 \\ -2x, & x \leq -3 \end{cases}$



c. $f(x) = \begin{cases} 3, & x > 2 \\ 2x, & -1 < x \leq 2 \\ -x + 3, & x \leq -1 \end{cases}$



d. $f(x) = \begin{cases} 3x + 1, & x \geq 1 \\ 6, & -1 \leq x < 1 \\ x^2, & x < -1 \end{cases}$



Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Greatest Integer Function

Show how to graph the greatest integer function $f(x) = [\![x]\!]$

<u>Example</u> Graph $f(x) = \left[\!\left[\frac{1}{2}x - 2\right]\!\right]$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Review of Function Operations

Given that f and g are functions, how do we define the function operations?

$(f+g)(x) =$

$(f-g)(x) =$

$(fg)(x) =$

$\left(\frac{f}{g}\right)(x) =$

<u>Example</u> Let $f(x) = 3x+1, g(x) = x^2-4, h(x) = 3x^2+7x+2$, and $k(x) = x^2+6x+9$. Find each of the following:

a. $(f+g)(x) =$

b. $(f+g)(3) =$

c. $(g-k)(1) =$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = 3x^2 + 7x + 2$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $\left(\frac{h}{f}\right)(x) =$

b. $(gh)(x) =$

c. $\left(\frac{k}{h}\right)(1) =$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Review of Composition of Functions

For two functions f and g, what is the composition $(f \circ g)(x)$?

<u>Example</u> Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = \sqrt{x}$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $(f \circ g)(x) =$

b. $(g \circ f)(x) =$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Let $f(x) = 3x + 1, g(x) = x^2 - 4, h(x) = \sqrt{x}$, and $k(x) = x^2 + 6x + 9$. Find each of the following:

a. $(k \circ f)(1) =$

b. $(g \circ h)(4) =$

c. $(h \circ k)(x) =$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Difference Quotient

Explain what the difference quotient is and draw some sketches to explain how it works. In other words, watch the video explanation and then explain it in your own words on this page.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

State the formula for the difference quotient here:

<u>Example</u> Calculate the different quotient for the following.

a. $f(x) = 4x + 5$

b. $f(x) = x^2 - 1$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Difference Quotient

Find the difference quotient.

a. $f(x) = \frac{1}{x+3}$

b. $f(x) = x^2 - 4x + 1$

c. $f(x) = -x^2 + 3$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Translations and Reflections of Functions

Using the graph $f(x) = |x|$ as the "base function," sketch how vertical translations work (summarize what was discussed in the video).

How do you describe vertical translations in function notation?

Using the graph $f(x) = x^2$ as the "base function," sketch how horizontal translations work (summarize what was discussed in the video).

How do you describe horizontal translations in function notation?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Use the graph $f(x) = \sqrt{x}$ to show how reflections over the x- and y-axis work (summarize what was discussed in the video).

How do you describe reflections over the x-axis and y-axis using function notation.

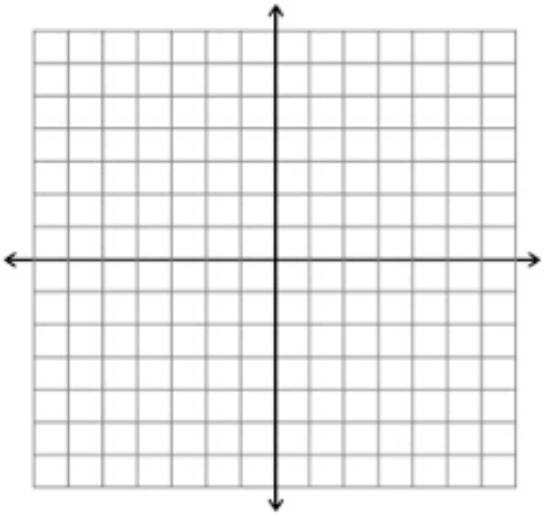
Use the graph $f(x) = |x|$ to show how stretching and shrinking works (summarize what was discussed in the video).

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

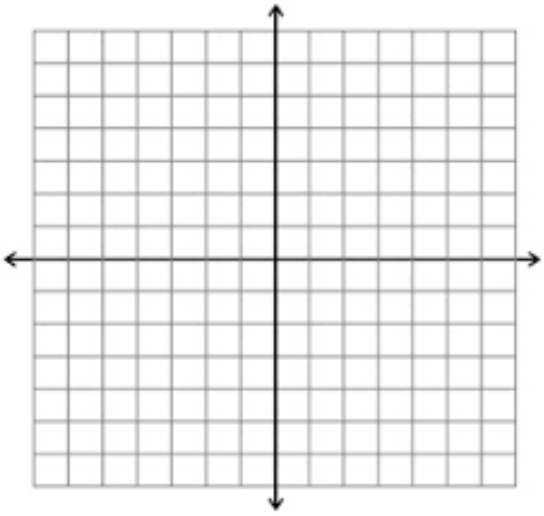
Explain how stretching and shrinking works with function notation.

<u>Example</u> Describe any translations / reflections / stretching / shrinking and sketch a graph of each of the following.

a. $f(x) = -x^2 + 3$



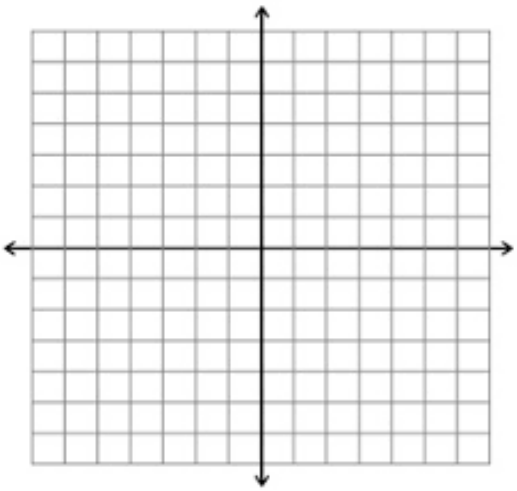
b. $g(x) = (x + 2)^3 - 3$



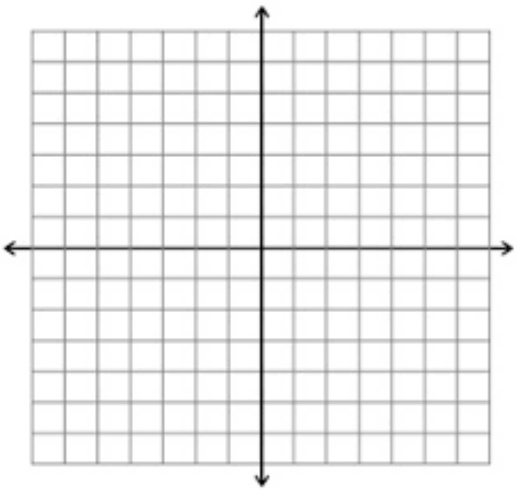
Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Describe any translations / reflections / stretching / shrinking and sketch a graph of each of the following.

a. $g(x) = -2(x+1)^2 - 5$



b. $h(x) = 2\sqrt{-x} + 1$



Examples Write the equation of the given functions with the following characteristics.

a. The function $f(x) = x^2$ moved 3 units left and 5 units down.

b. The function $g(x) = \sqrt[3]{x}$ reflected over the y-axis, translated 2.5 units right and 1.75 units up.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

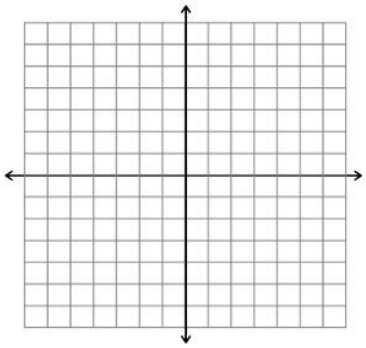
Transformations of Functions

These are looking at graphs of things like:

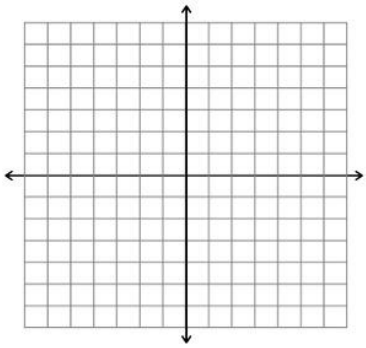
$$f(x) = x^2 - 5 \quad \text{or} \quad f(x) = |x+3| - 2 \quad \text{or} \quad f(x) = \sqrt{x-3}$$

First we need to discuss the shape of a few basic functions. Write these down and note the important points.

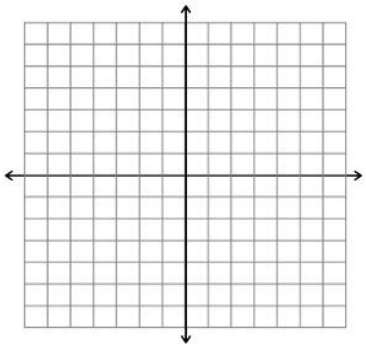
a. $f(x) = x^2$



b. $f(x) = |x|$



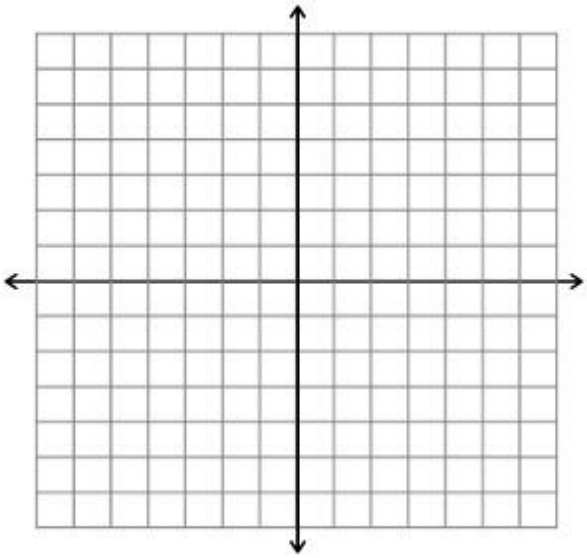
c. $f(x) = \sqrt{x}$



Using a few different colors, sketch out what the graphs of

a. $f(x) = x^2$ versus b. $g(x) = x^2 + 3$ versus c. $h(x) = x^2 - 1$

look like all on the same graph below:



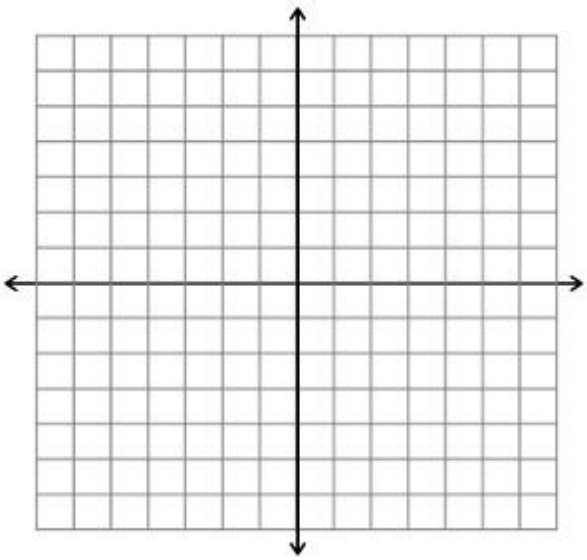
Copyright 2020, Jennifer Hill, DivideandConquerMath.com

This is an example of a vertical shift. Explain what this means in mathematical terms:

Using a few different colors, sketch out what the graphs of

a. $f(x) = |x|$ versus b. $g(x) = |x - 1|$ versus c. $h(x) = |x + 2|$

look like all on the same graph below:



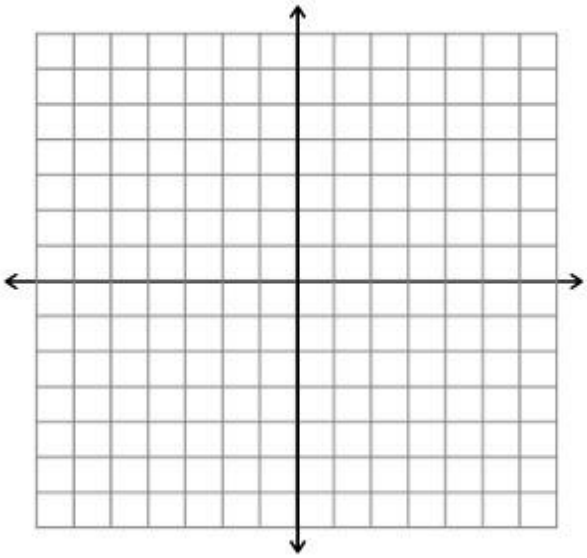
This is an example of a horizontal shift. Explain what this means in mathematical terms:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Using a few different colors, sketch out what the graphs of

a. $f(x) = |x|$ versus b. $g(x) = -|x|$

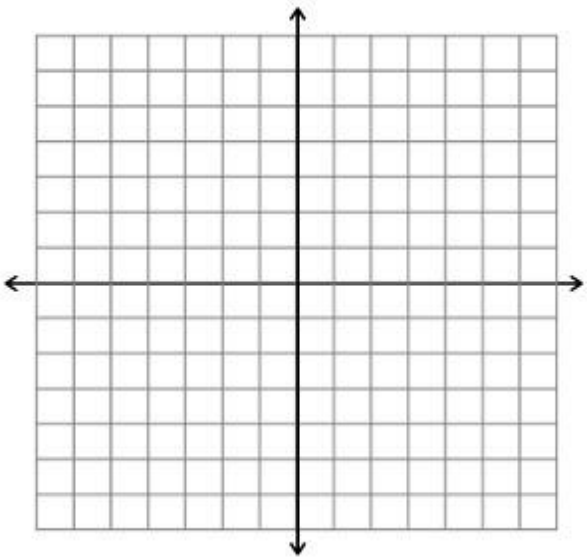
look like all on the same graph below:



This is an example of a reflection over the x-axis. Explain what this means in mathematical terms:

<u>Example</u> Explain in plain language what types of shifts/reflections are occurring, and then graph the function.

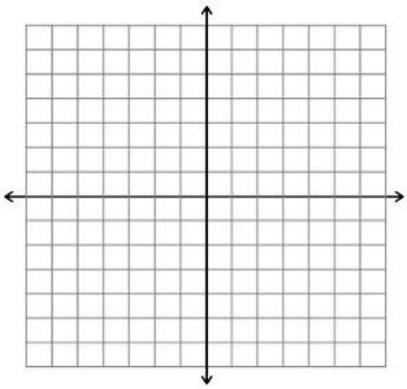
$$f(x) = |x| + 3$$



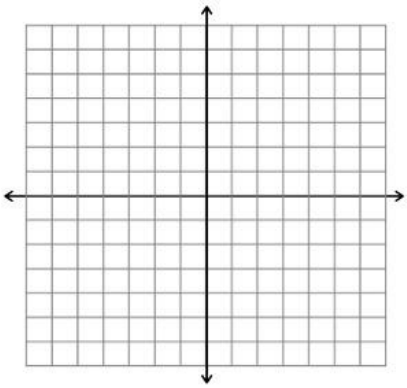
Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Explain in plain language what types of shifts/reflections are occurring, and then graph the function.

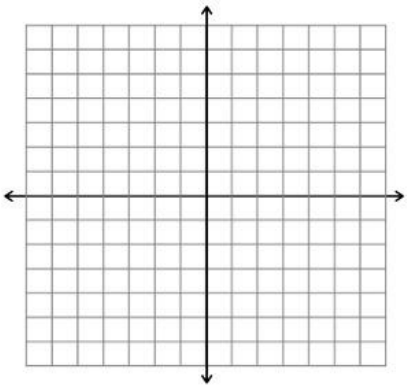
a. $f(x) = (x-1)^2$



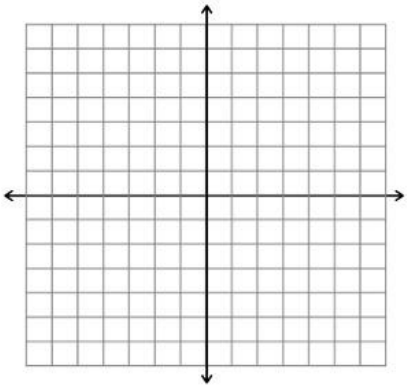
b. $g(x) = \sqrt{x} - 2$



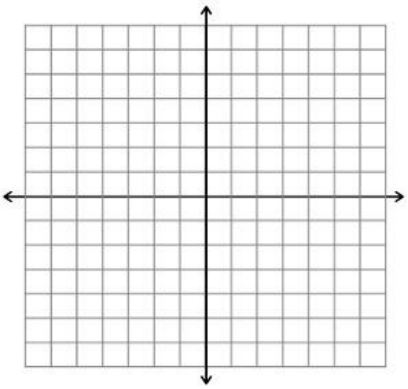
c. $h(x) = |x+1| - 2$



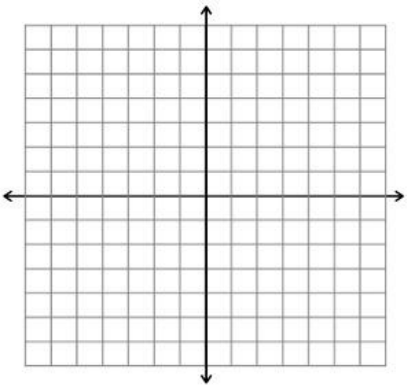
d. $k(x) = (x-2)^2 + 4$



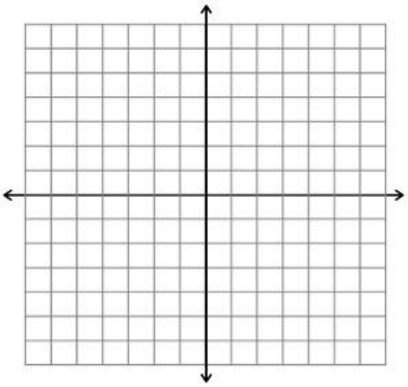
e. $f(x) = -(x+3)^2 - 2$



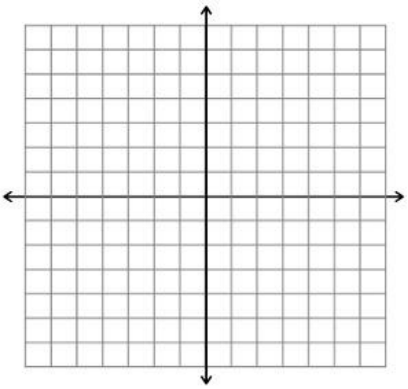
f. $g(x) = -|x+3| - 1$



g. $h(x) = -\sqrt{x-2}$



h. $k(x) = -\sqrt{x+1} - 3$



Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Write the equations that represent the listed transformations.

a. A function $f(x) = \sqrt{x}$ is shifted 7 units down.

b. A function $g(x) = x^3$ is shifted 3 units left and 2 units up.

c. A function $h(x) = |x|$ is shifted 2.5 units right and reflected over the x-axis.

d. A function $f(x) = e^x$ is shifted 4 units up and reflected over the x-axis.

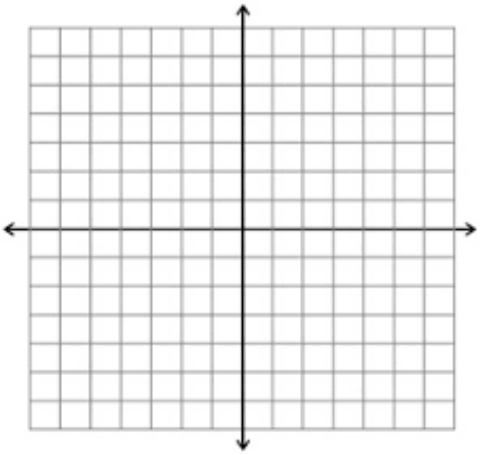
e. A function $g(x) = \sqrt[3]{x}$ is shifted 3.2 units right, 5.7 units up, and reflected over the x-axis

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

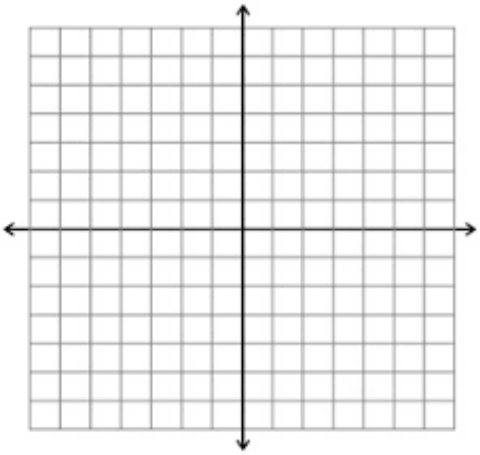
Examples: Translations and Reflections of Functions

Describe any translations / reflections / stretching / shrinking and sketch a graph of each of the following.

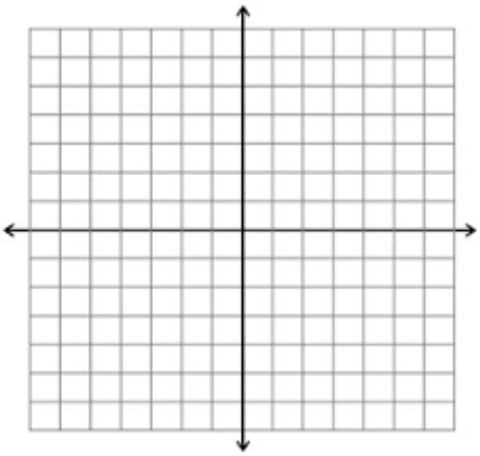
a. $f(x) = -(x+2)^2 - 3$



b. $g(x) = \frac{1}{2}(x-1)^3 + 2$



c. $h(x) = -\frac{1}{3}\sqrt{x+1} + 1$



Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Symmetry and Even or Odd Functions

Explain when a graph is symmetric with respect to the y-axis:

Explain when a graph is symmetric with respect to the x-axis:

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Explain when a graph is symmetric with respect to the origin:

What makes a function odd?

What makes a function even?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

So what is the test for even or odd functions?

<u>Examples</u> Determine what type of symmetry the following functions have:

a. $y = x^2 + 4$

b. $x^2 + y^2 = 25$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Determine what type of symmetry the following functions have:

a. $y = x^3 - 5$

b. $y = 7x^2 + 2x - 4$

c. $y = 2x^{\frac{1}{3}} - 3x$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: A Review of Absolute Value Equations and Inequalities

We can summarize how to solve each case with the following chart:

Case (k is a number)	How to Setup
$\lvert x \rvert = k$	
$\lvert x \rvert < k$ (or $\leq k$)	
$\lvert x \rvert > k$ (or $\geq k$)	

<u>Examples</u> Solve the following. Put your answer on a number line and in interval notation when you have an inequality.

a. $|x + 3| = 5$

b. $|2x - 4| \geq 6$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Solve the following. Put your answer on a number line and in interval notation when you have an inequality.

a. $|4x - 2| = |6x + 8|$

b. $|5x - 2| + 1 < 9$

c. $|2 - x| - 1 > 9$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Special Cases!

<u>Example</u> Explain what is special about each case below and how to solve the problem.

a. $|4x - 1| = -7$

b. $|4x - 1| = 0$

c. $|4x - 1| < -7$

d. $|4x - 1| > -7$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Explain what is special about each case below and how to solve the problem.

a. $|4x - 1| \geq 0$

b. $|4x - 1| \leq 0$

c. $|4x - 1| < 0$

d. $|4x - 1| > 0$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Determining if Functions are One-to-One

Before we begin, you should already be familiar with the concept of functions and the vertical line test. First let's do some review.

<u>Review</u> Determine whether the following relations are functions, then determine the domain and range.

a. $\{(2,3),(5,7),(2,4)\}$

b.

Now we will discuss a specific type of function: a one-to-one function. Define this below:

How can we visualize this?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example Determine if any of the following are i) functions and ii) one-to-one functions

a. $\{(2,4),(-1,7),(3,7)\}$

b. $\{(3,6),(-1,2),(-3,-4)\}$

Similar to how we have a test to determine if a graph represents a function, we have another test to determine if a function is one-to-one.

Describe the horizontal line test below:

Examples Determine whether the following graphs represent i) functions and ii) one-to-one functions (draw your own):

a. b. c.

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Finding the Inverse of Functions

One-to-one functions have inverses. Describe what the inverse of a function is:

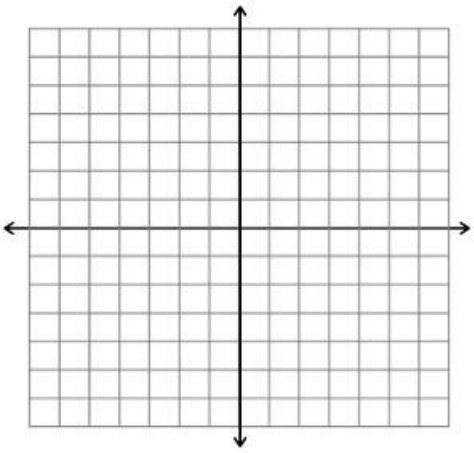
<u>Example</u> Find the inverse of the following function: $\{(1,4),(7,8),(3,-2)\}$

Describe the steps to find the inverse of a function:

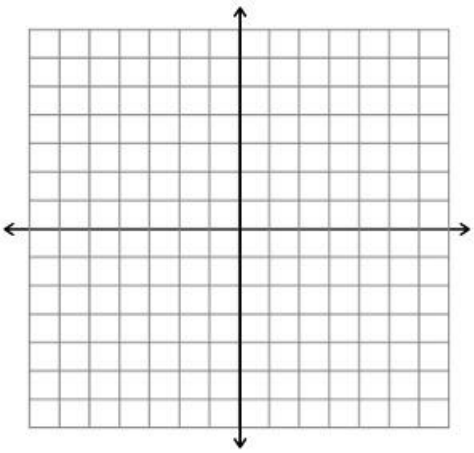
Copyright 2020, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Find the inverse of the following one-to-one functions. Then plot the original function on the graph with its inverse.

a. $f(x) = 3x + 1$



b. $g(x) = -\frac{1}{2}x + 4$



What is the relationship of a graph with its inverse?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Find the inverse of the following one-to-one functions.

a. $h(x) = \sqrt[3]{x}$

b. $f(x) = x^2, x \geq 0$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: A Review of Inverse Functions

What is a one-to-one function?

<u>Examples</u> Are the following one-to-one functions? If no, why not?

a. $f(x) = x^2$

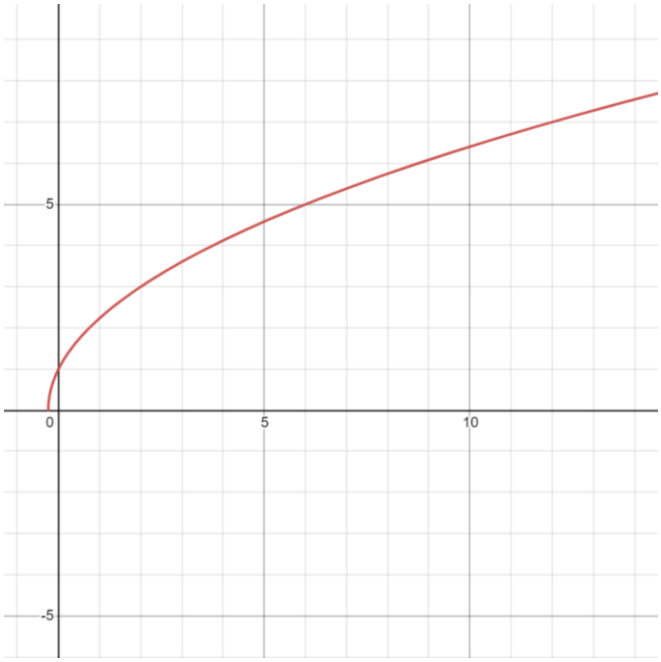
b. $g(x) = 2x + 1$

What is the horizontal line test?

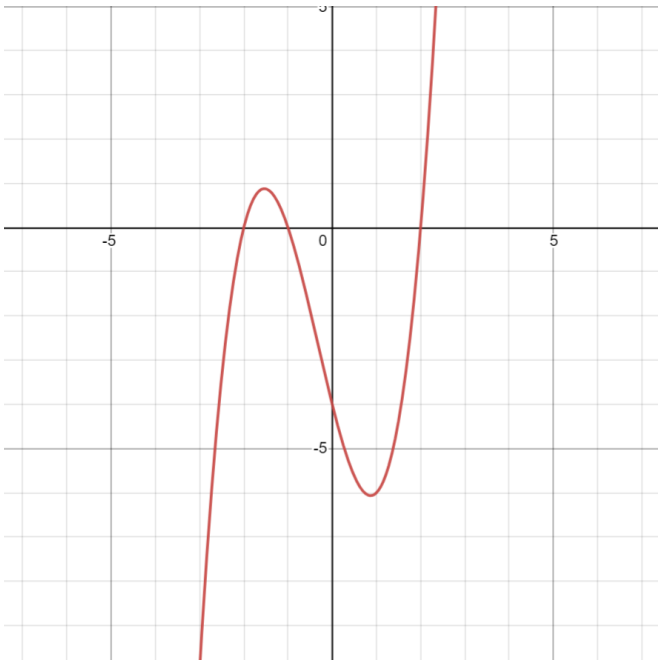
Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Use the horizontal line test to determine if the functions are one-to-one:

a.



b.



What is an inverse function?

What does an inverse function have to do with one-to-one functions?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Describe how to use composition to determine if 2 functions are inverses of one another.

Example Use composition to determine if the functions $f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x - 2$ are inverses.

Example Use composition to determine if the functions $f(x) = \frac{2}{x+6}$ and $g(x) = \frac{6+x}{2}$ are inverses.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Using $f(x) = 4x + 2$, outline the steps for how to find the inverse of a function.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Find the inverse of the following functions.

a. $f(x) = \sqrt[3]{3x+1}$

b. $f(x) = \frac{x+2}{x-1}$

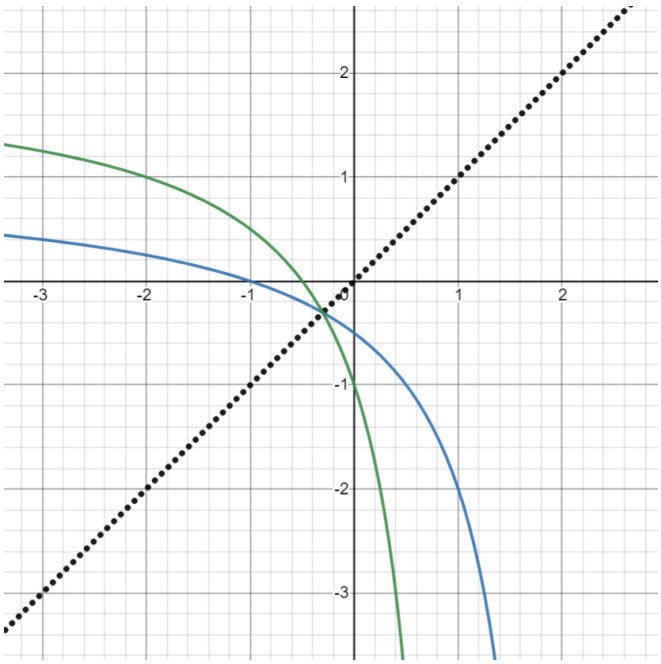
c. $f(x) = \sqrt{x-5}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

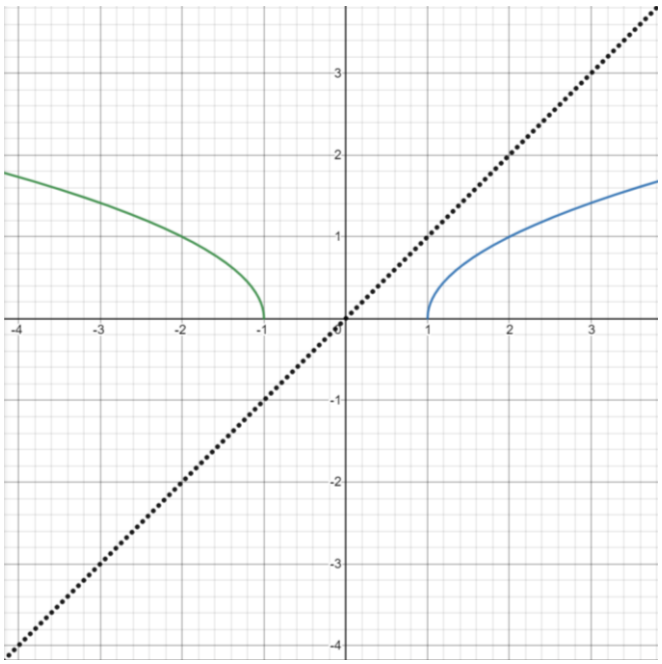
How can we tell on a graph if 2 functions are inverses of one another?

<u>Examples</u> Are the following graphs inverses of one another?

a.

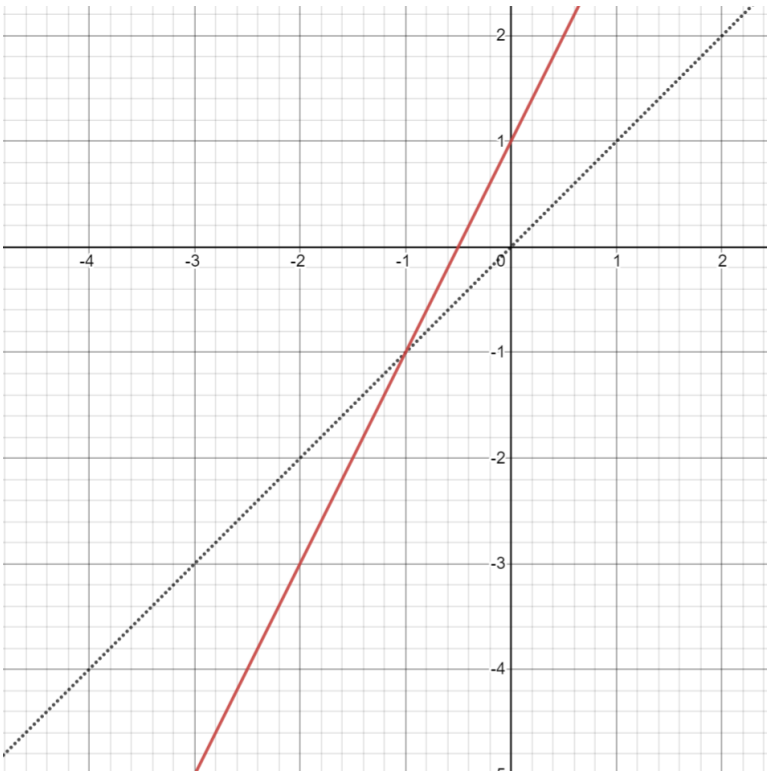


b.



Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Draw the inverse of the following graph.



Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Review of Lines and Linear Functions

What makes a function **linear?** What are 2 forms of a linear function? How do you interpret this equation?

Examples What is the domain and range of $f(x) = 4x + 1$. Show how to graph it.

Examples What is the domain and range of $f(x) = 5$. Show how to graph it.

Examples What is the domain and range of $x = -4$. Show how to graph it.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What is the **standard form** of a line?

<u>Examples</u> Give the domain and range of $3x - y = 6$. Show how to graph it.

How do you find the **slope** of a line?

What is the slope of a vertical line?

What is the slope of a horizontal line?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Find the slopes between the following sets of points.

a. $(5, 6)$ and $(-3, 2)$

b. $(-4, 7)$ and $(2, 7)$

c. $(6, 2)$ and $(6, -1)$

Example Find the slope of the following lines:

a. $4x - 2y = 8$

b. $-5x + 3y = -15$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Review of Writing Equations of Lines

What is the point-slope form of a line?

<u>Example</u> Write the equation of a line through the point $(4, 3)$ with slope $m = -2$.

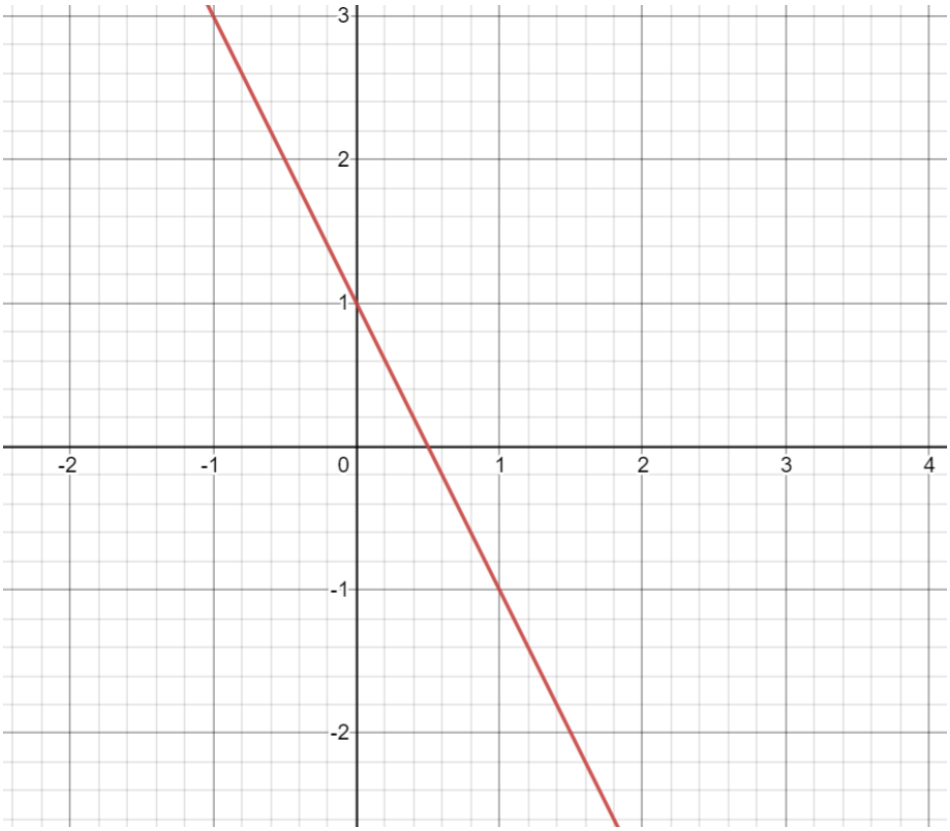
<u>Example</u> Write the equation of a line that goes through the points $(2, 4)$ and $(-4, 10)$. Put your answer in standard form.

What is the slope-intercept form of a line?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Show how to write $4x - 6y = 12$ in slope-intercept form.

Example Find the slope, x- and y-intercept, and equation of the line:



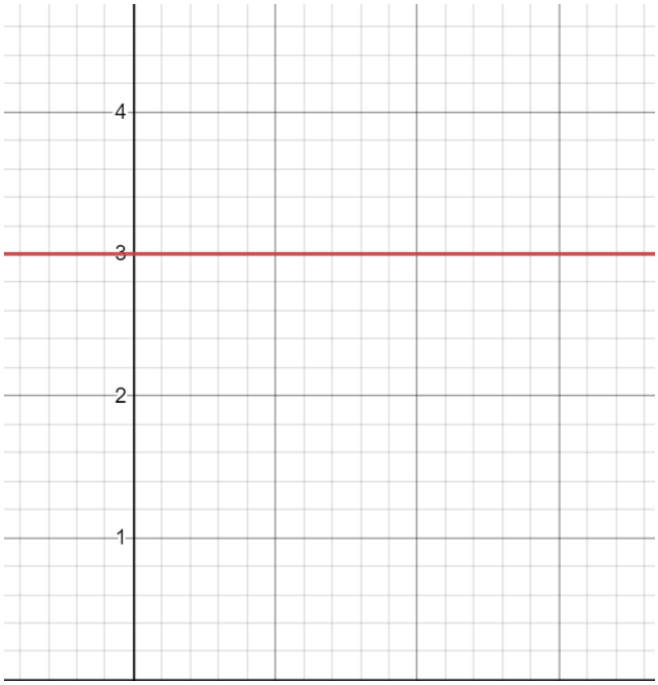
Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What is the form of a vertical line?

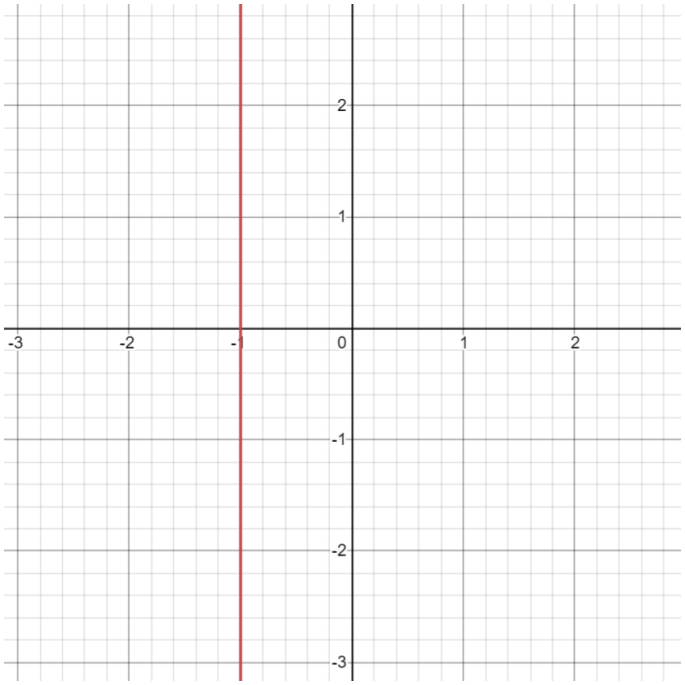
What is the form of a horizontal line?

<u>Example</u> Determine the equation of the following lines.

a.



b.



What makes 2 lines parallel?

What makes 2 lines perpendicular?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Write the equation of a line that goes through $(-2, -5)$ that satisfies the conditions:

a. parallel to the line: $3x - 2y = -6$

b. perpendicular to the line: $x - 2y = -4$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Quick Reference of Line Formulas

Slope-Intercept Form

Standard Form

Point-Slope Form

Slope Formula

Form of a Horizontal Line

Form of a Vertical Line

What makes 2 lines parallel?

What makes 2 lines perpendicular?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Writing the Equation of a Line Through 2 Points

<u>Example</u> Write the equation of a line through the given 2 points. Put your answer in standard form, and your standard form should not include any fractions or decimals.

a. $(4, 5)$ and $(2, 9)$

b. $(2, 7)$ and $(-4, 7)$

c. $(-4, 2)$ and $(2, 5)$

d. $(3, 5)$ and $(3, 8)$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Writing the Equation of a Line Through 2 Points

<u>Example</u> Write the equation of a line through the point $(2, 3)$ that satisfies the given conditions. Your final answer should be in standard form with no fractions or decimals.

a. Parallel to $5x - 2y = 10$

b. Perpendicular to $4x - 2y = 1$

c. Parallel to $y = 5$

d. Perpendicular to $x = 7$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Slope of a Line

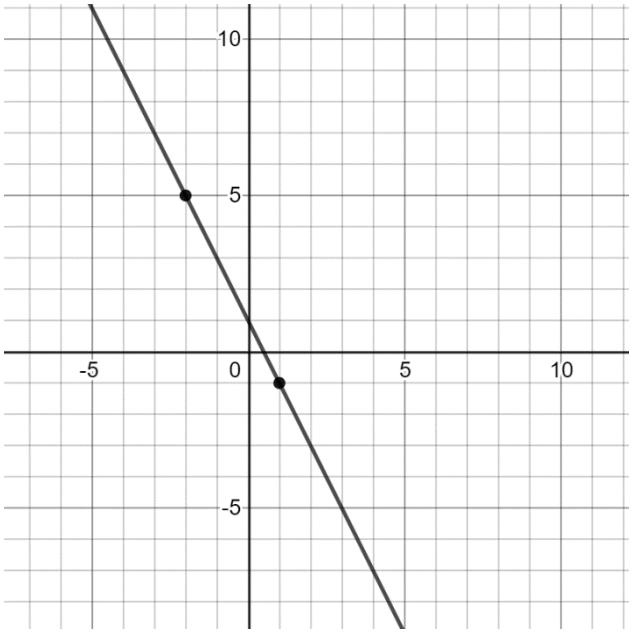
What does slope represent?

Draw the various possible slopes of lines:

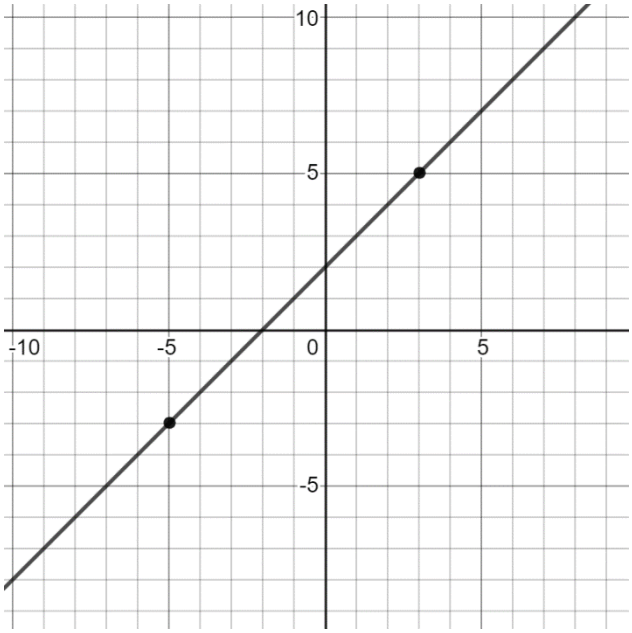
How do find the slope of a line?

<u>Examples</u> Find the slope of the following lines:

a.



b.



Copyright 2020, Jennifer Hill, DivideandConquerMath.com

What is the slope formula?

<u>Examples</u> Use the slope formula to find the slope between the pairs of points:

a. $(3, 5)$ and $(4, 8)$

b. $(6, -2)$ and $(-2, 4)$

c. $(7, 2)$ and $(7, 4)$

d. $(2, 4)$ and $(-1, 4)$

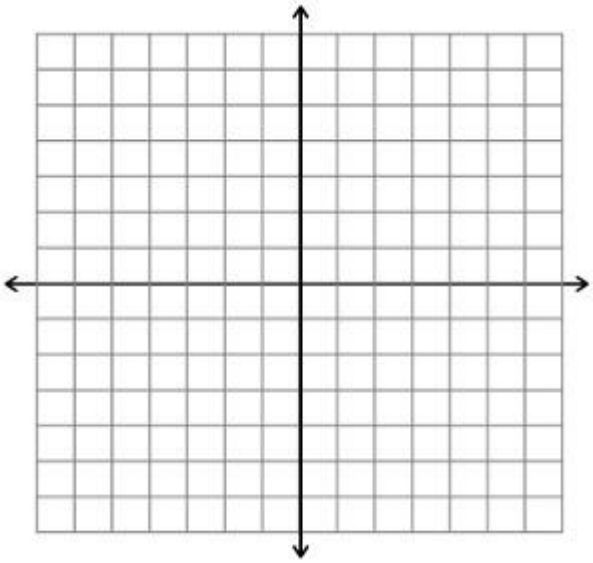
Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Using Slope to Graph Lines; Intro to Slope-Intercept Form

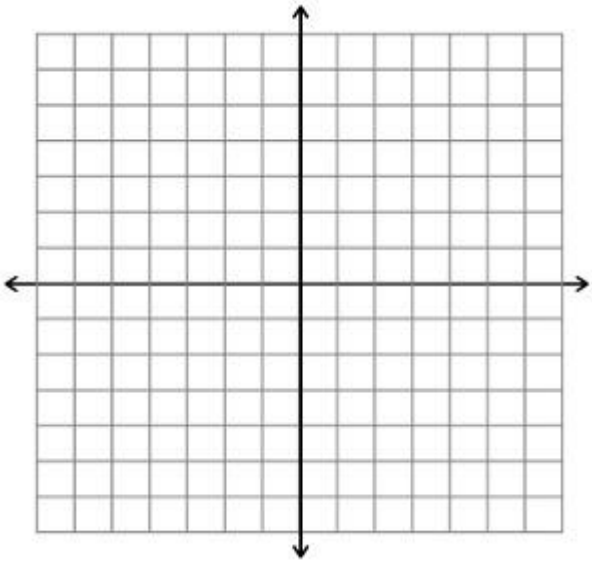
When the slope of a line and a point are known, a line can be easily graphed.

<u>Examples</u> Given the slope and the point, graph the following lines:

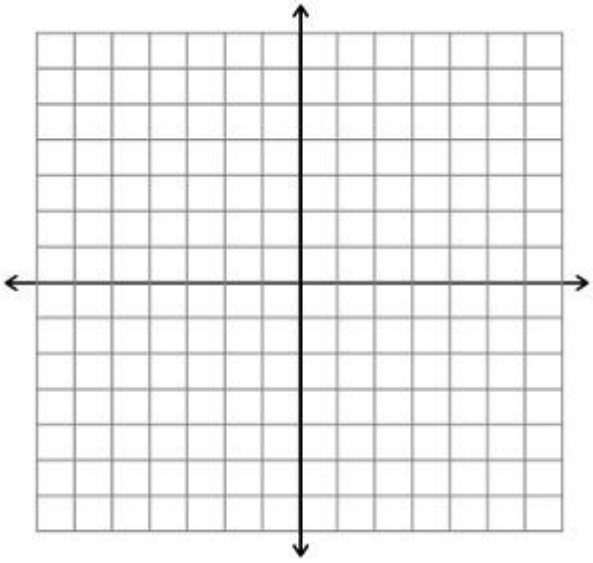
a. $m = 2$ and $(-1, 2)$



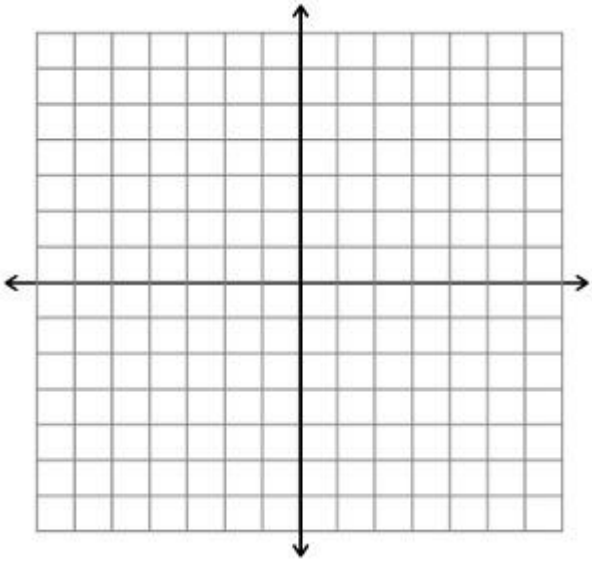
c. $m = 1$ and $(2, -1)$



b. $m = -\frac{1}{3}$ and $(-3, 4)$



d. $m = \frac{2}{3}$ and $(0, -3)$



What is the slope-intercept form of a line?

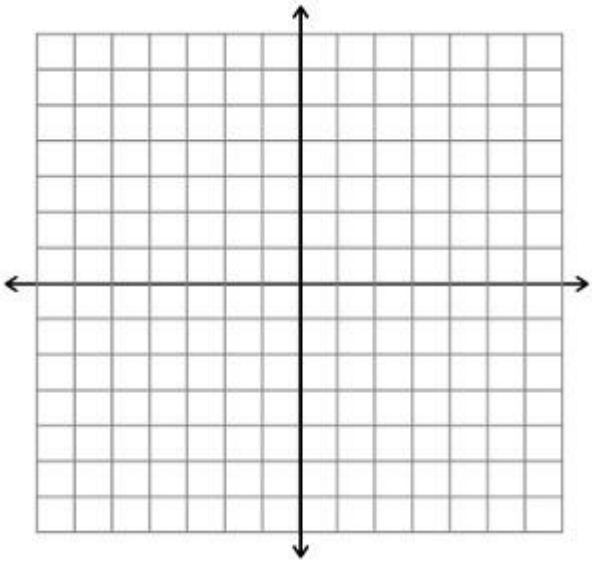
Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Identify the slope m and the y-intercept $(0, b)$ in each of the following equations:

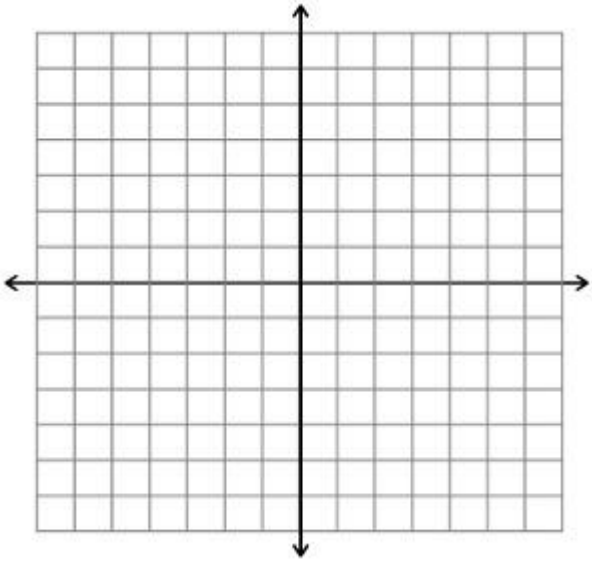
Equation	Slope, m	What stands for b	State the y-intercept (as a point)
a. $y = -2x + 3$			
b. $y = \frac{1}{4}x - 7$			
c. $y = 3x + 9$			
d. $y = x$			

Examples Graph the following lines:

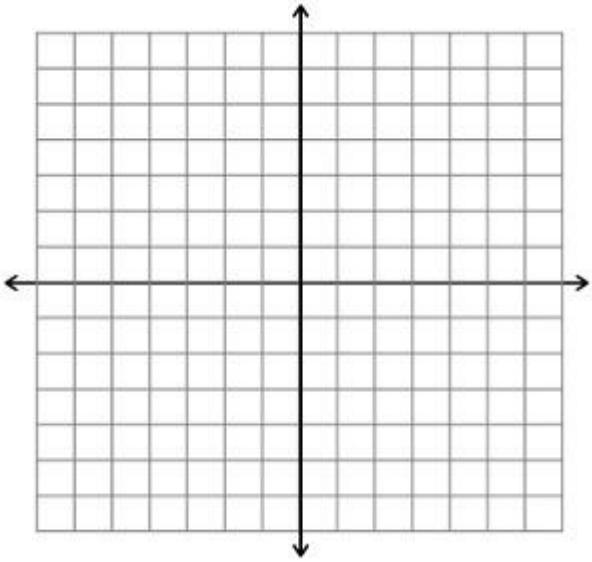
a. $y = 2x + 3$



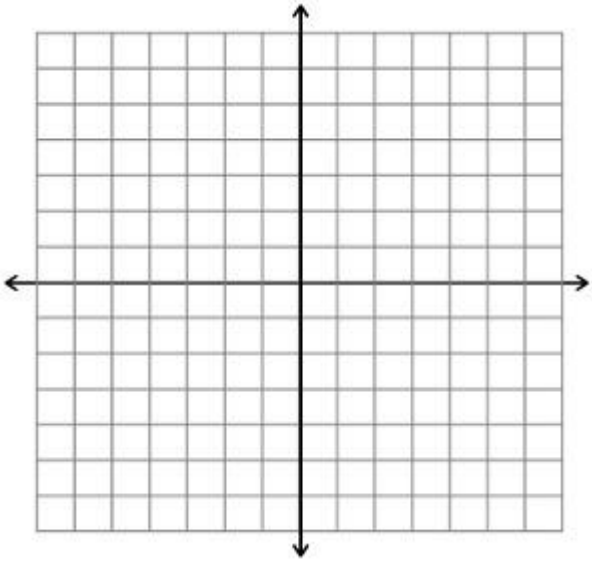
c. $y = \frac{1}{3}x - 4$



b. $y = -x + 4$



d. $y = 3x$



Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Rewriting Linear Equations into Slope-Intercept Form of a Line

What is the slope-intercept form of a line?

Explain how to put the equation $2x - 4y = 8$ into slope-intercept form.

<u>Examples</u> Put the following equations in slope-intercept form:

a. $6y - 3x = 12$

b. $-5x + 2y = -4$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Determining Whether Two Lines are Parallel or Perpendicular

What makes 2 lines parallel?

<u>Examples</u> Are the equations $y = \frac{2}{3}x + 5$ and $2x - 3y = 6$ parallel?

What makes 2 lines perpendicular?

<u>Examples</u> Determine the negative reciprocal of each of the following:

a. $\frac{2}{3}$

b. $-\frac{1}{5}$

c. -7

d. 1

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example Are the equations $y = \frac{3}{5}x + 2$ and $-3x + 5y = -15$ perpendicular?

Examples Are the following pairs of lines parallel, perpendicular, or neither?

a. $3x - 2y = 6$ and $4y = -6x - 8$

b. $y - 3x = 0$ and $12x - 4y = 24$

c. $y = 3$ and $x = -2$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Lines A and B contain the given points. Determine whether the lines are parallel, perpendicular, or neither.

a. Line A: $(2, 11), (5, 10)$ Line B: $(1, 3), (-6, -18)$

b. Line A: $(-5, 5), (-7, -2)$ Line B: $(0, 11), (-4, 9)$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Putting Equations in Standard Form

What is the standard form of a line?

<u>Examples</u> Write the following equations in standard form:

a. $y = 2x - 4$

b. $y = -5x + 3$

c. $y = -\frac{1}{2}x + 1$

d. $y = \frac{2}{3}x - 2$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Writing Equations of Lines with The Point-Slope Form

What is the point-slope form of a line?

<u>Examples</u> Given the point and the slope, write the point-slope form of a line.

a. $m = -2, (1, 3)$

b. $m = -\frac{2}{3}, (5, -2)$

What is the slope-intercept form of a line?

What is the standard form of a line?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Given the point and the slope, write the equation of the line in the desired format.

a. $m = -1, (2, -5)$ in slope-intercept form

b. $m = 3, (5, -4)$ in standard form

c. $m = -\frac{1}{2}, (6, 3)$

d. $m = \frac{2}{3}, (-5, -1)$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Writing the Equation of a Line from Two Points

Write the equation of the line that contains the following 2 points in the desired format.

a. $(4, 2)$ and $(5, 5)$ in slope-intercept form

b. $(-2, 4)$ and $(-4, -2)$ in standard form

c. $(5, -3)$ and $(3, 5)$ in slope-intercept form

d. $(-1, -3)$ and $(1, 5)$ in standard form

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Writing the Equation of a Line Parallel/Perpendicular to Another Line

What makes 2 lines parallel?

What makes 2 lines perpendicular?

<u>Examples</u> Write the equation of a line with the given conditions.

a. Parallel to $y = 3x + 1$ that goes through $(-2, -5)$ in slope-intercept form

b. Perpendicular to $y = -\frac{2}{3}x - 5$ that goes through $(6, -2)$ in standard form

c. Parallel to $2x - 3y = 6$ that goes through $(1, -2)$ in slope-intercept form

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Write the equation of a line with the given conditions.

a. Perpendicular to $2x - 4y = 8$ that goes through $(-4, -2)$ in standard form

b. Parallel to $3x - y = 12$ that goes through $(-1, -5)$ in slope-intercept form

c. Perpendicular to $-2x + 3y = 12$ that goes through $(2, -5)$ in standard form

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

How to Evaluate Square Roots that Have Imaginary Number Answers and Dealing with Negatives in Square Roots

In this video, we will look at how to evaluate things like this:

$\sqrt{-4}$ or $\sqrt{-12}$ or $\sqrt{-3} * \sqrt{-6}$

What is an imaginary number?

What is a complex number?

How do you evaluate something like $\sqrt{-9}$?

<u>Examples</u> Compute the following:

a. $\sqrt{-25}$

b. $\sqrt{-81}$

c. $\sqrt{-12}$

d. $\sqrt{-7}$

e. $\sqrt{-\frac{25}{16}}$

f. $\sqrt[3]{-8}$

g. $\sqrt[4]{-16}$

h. $\sqrt{-32}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

How do you evaluate problems like $\sqrt{-3} * \sqrt{-2}$?

<u>Examples</u> Evaluate the following:

a. $\sqrt{-6} * \sqrt{-5}$

b. $\sqrt{-6} * \sqrt{-2}$

c. $\sqrt{-10} * \sqrt{-15}$

d. $\frac{\sqrt{-6}}{\sqrt{-2}}$

e. $\frac{\sqrt{-50}}{\sqrt{-2}}$

f. $\frac{\sqrt{-7}}{\sqrt{-2}}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Operations with Imaginary Numbers

In this video, we will look at how to evaluate things like this:

$2i(3i-5)$ or $(4i+1)-(5i+3)$ or $\frac{4}{3+2i}$

Adding and Subtracting with Imaginary Numbers:
Use the example $(2i+3)+(5i-6)$ to show how to add or subtract with imaginary numbers.

<u>Examples</u> Simplify:
a. $(3i-2)-(4i-9)$

b. $(5i+3)+(2i-1)$

What can we say about i^2?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

With that in mind, explain what is unique about multiplying: $2i(3i+1)$

<u>Examples</u> Simplify:

a. $4(2-3i)$

b. $6i(3i+2)$

c. $5i(2-8i)$

d. $(5i+1)(2i-3)$

e. $(4i-3)(6i+7)$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Simplify:

a. $(5i + 2)^2$

b. $(3i - 5)^2$

Why do you need to rationalize $\frac{3}{2i+1}$? Explain how to do this.

Examples Rationalize the following:

a. $\frac{4}{2i+6}$

b. $\frac{7}{i+3}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Rationalize the following:

a. $\frac{3i+1}{5i-2}$

b. $\frac{7i+3}{2i+1}$

c. $\frac{i-3}{5i+3}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Evaluating Higher Powers of Imaginary Numbers

In this video, we will look at how to evaluate things like this:

i^{21} or i^{54}

First we need to investigate the powers of i and the patterns with it. Write this down below:

Now that we can see a pattern, explain how to evaluate the following:

i^{15}

i^{22}

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Evaluate the following:

a. i^{36}

b. i^{43}

c. i^{52}

d. i^{33}

e. i^{25}

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: A Review of Imaginary Numbers

Please note: Knowing imaginary numbers if one of the "prerequisite" topics for this course. Ie – you should already know this (though I also realize you probably need a review!). If you have *never* seen imaginary numbers before, then I strongly suggest that you watch my series on imaginary numbers (because, yes, I have one). So if you felt like this went too fast and you want more details, please take responsibility for your education and go back to review this!

How do we represent an imaginary number?

What is i^2?

What is a complex number?

When are 2 complex numbers equal?

What does $\sqrt{-a}$ mean?

Examples Write using the definition of $\sqrt{-a}$

a. $\sqrt{-12}$

b. $\sqrt{-25}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Complete the following operations.

a. $\sqrt{-5} * \sqrt{-3}$

b. $\frac{\sqrt{-12}}{\sqrt{-18}}$

c. $\frac{\sqrt{-50}}{\sqrt{9}}$

To add or subtract complex numbers, we can add/subtract the real parts and the imaginary parts similar to working with like terms.

Examples Find the sum or difference.

a. $(2 - 6i) + (8 + 2i)$

b. $(5 - 7i) - (-4 + 2i)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

When multiplying complex numbers, recall that $i * i = i^2 = -1$!

<u>Examples</u> Multiply.

a. $(2 + 4i)(3 - 5i)$

b. $(4 - 5i)(3 - 2i)$

c. $(3 + 2i)(3 - 2i)$

Recall, the conjugate is found by using the opposite side for the second term!

<u>Examples</u> What is the conjugate of the following?

a. $4 + 2i$

b. $-3 - 4i$

c. i

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

To find the quotient of 2 complex numbers, we multiply by the conjugate of the denominator.

Examples Find each quotient. Write your answer in *standard form,* ie $a + bi$ form

a. $\frac{3-4i}{2+i}$

b. $\frac{5}{i}$

How do the powers of i cycle through?

Examples Show how to simplify the following powers of i. Write your work in great detail so that you remember how to do this!

a. i^{17}	c. i^{-14}
b. i^{28}	d. i^{-5}

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> The following quadratic equations have imaginary solutions. Show how to solve them.

a. $x^2 = -25$ using the **square root property**

b. $x^2 + 75 = 0$ using the **square root property**

c. $2x^2 - 3x = -6$ using the **quadratic formula**

d. $x^4 - 16$ using **factoring** and also the **square root property**

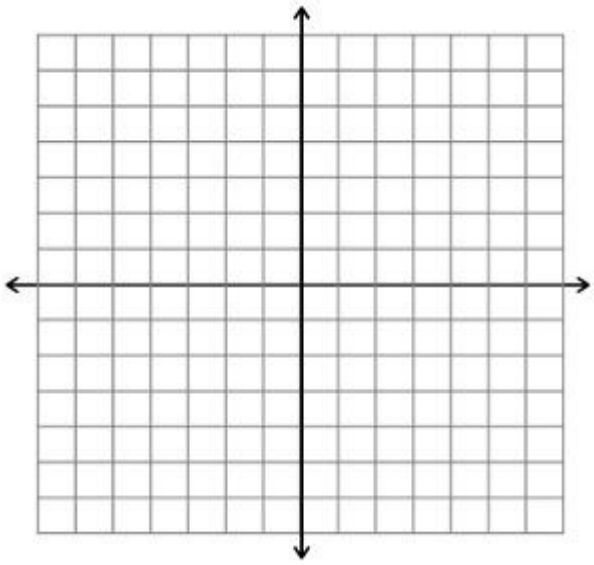
Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Understanding the Graphs of Quadratic Functions

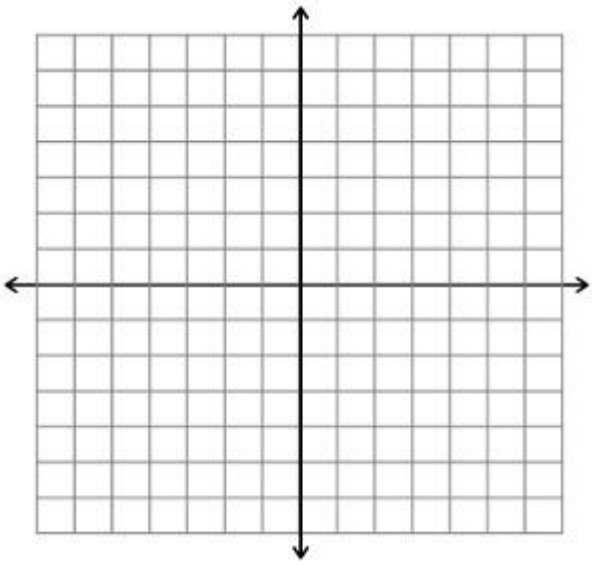
Quadratic functions are functions of the form _____, where $a \neq 0$. But in previous lessons we worked on graphing quadratic functions, ie parabolas in another way. Let's review that first.

<u>Examples</u> Graph the following:

a. $f(x) = (x+2)^2 - 3$

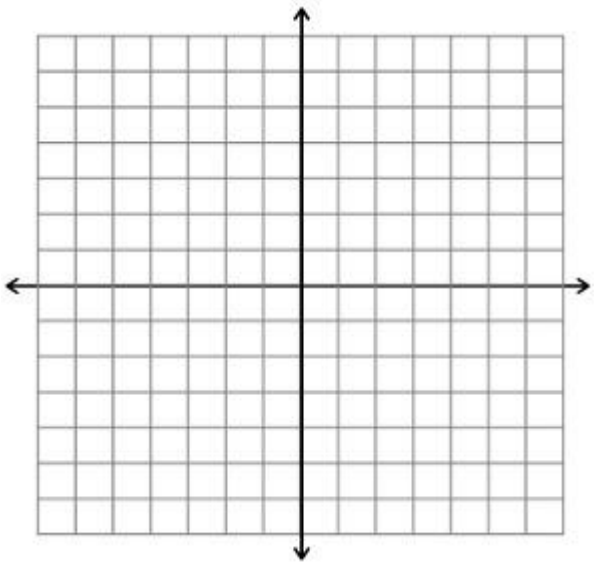


b. $g(x) = -(x-3)^2 + 1$

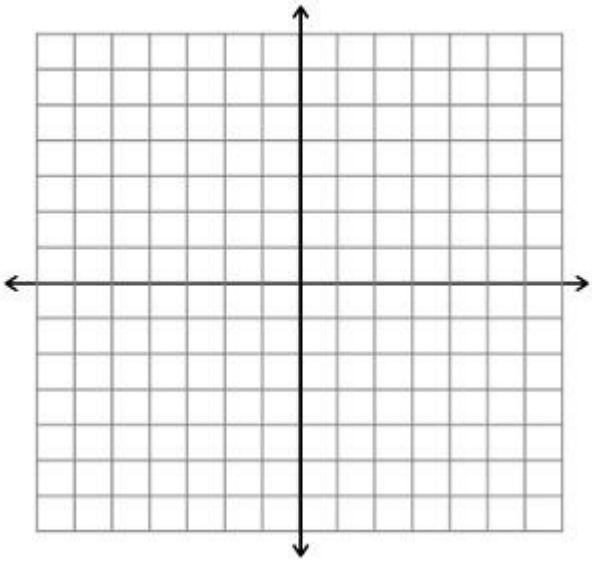


To begin, we will add on one more way to manipulate the graph of a parabola. Explain how to graph the follow:

a. $f(x) = 3x^2$
How do you interpret the 3?



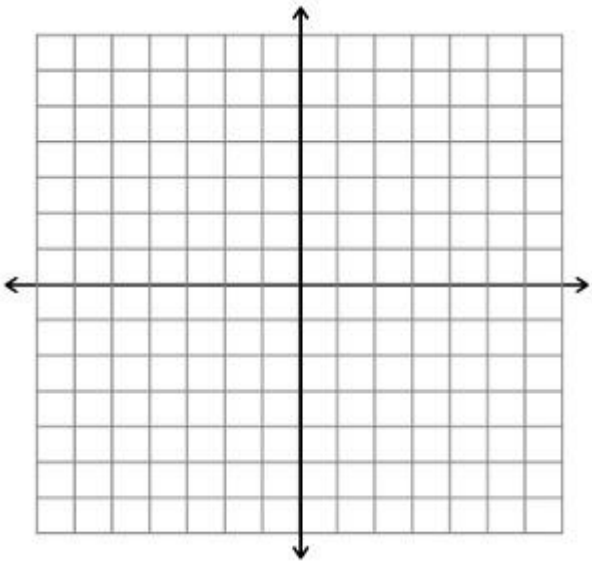
b. $g(x) = -\frac{1}{2}(x-2)^2 + 1$
How do you interpret the $-\frac{1}{2}$?



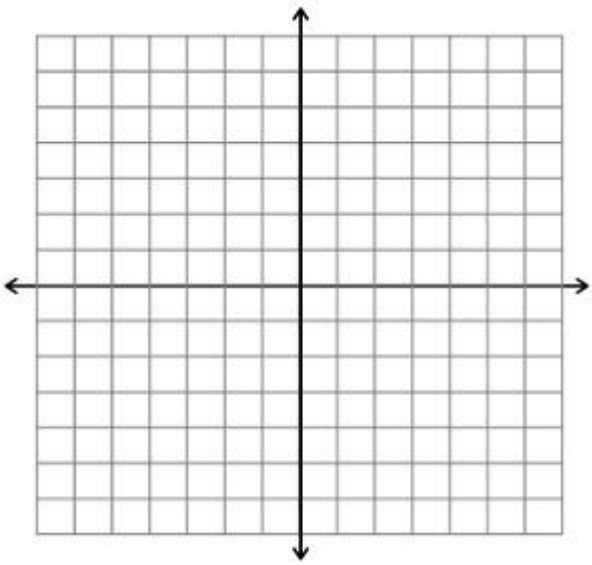
Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Graph the following:

a. $f(x) = -2(x+1)^2 + 3$



b. $g(x) = \frac{1}{3}(x-2)^2 - 2$



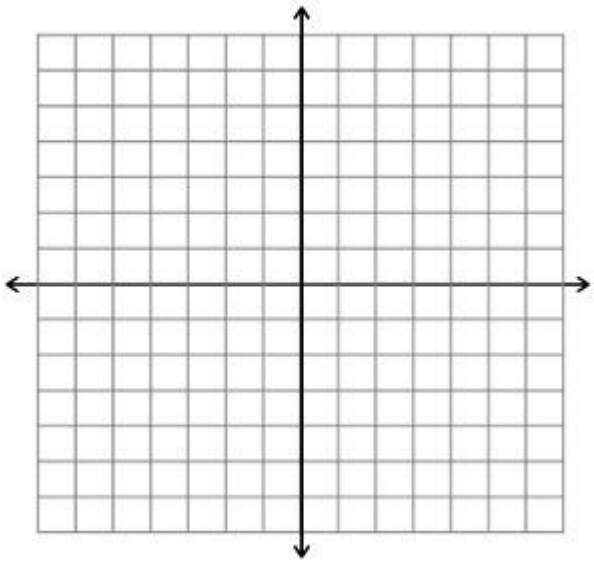
Using all of the examples we have discussed, what is another equation form for the parabola?

Explain some of the characteristics of a parabola that we get from this format:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x-3)^2 - 4$



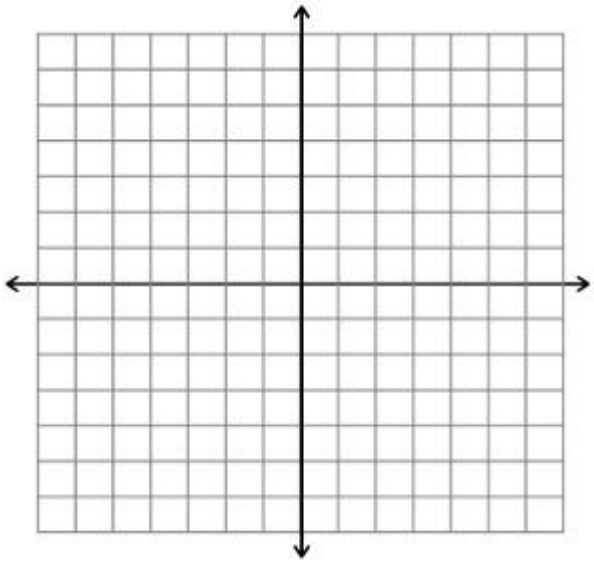
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2 + 8$



Vertex:

Axis of Symmetry:

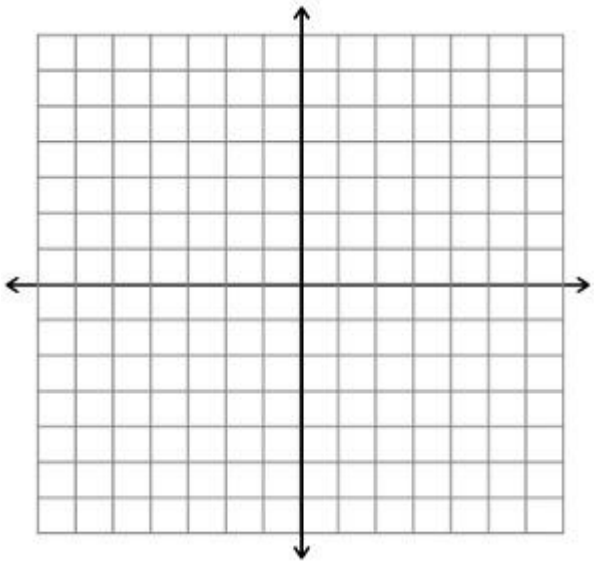
X-intercepts:

Y-intercepts:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{4}(x+2)^2 - 3$



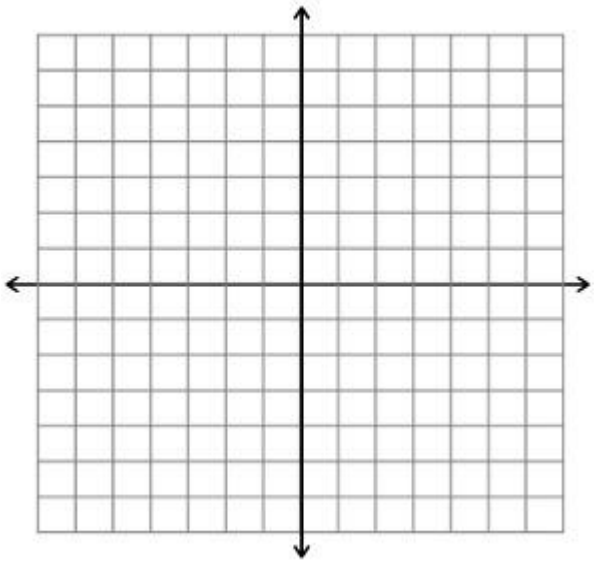
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -\frac{1}{3}(x-1)^2 + 3$



Vertex:

Axis of Symmetry:

X-intercepts:

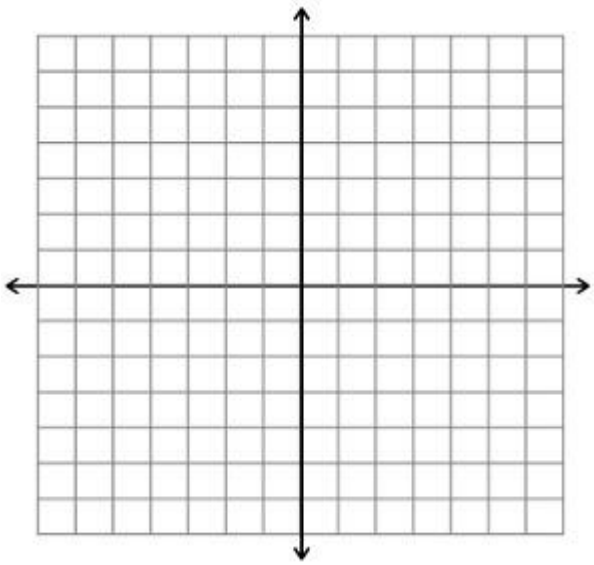
Y-intercepts:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Understanding How Many X-Intercepts There Are

<u>Examples</u> For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = \frac{1}{2}(x+1)^2 - 8$



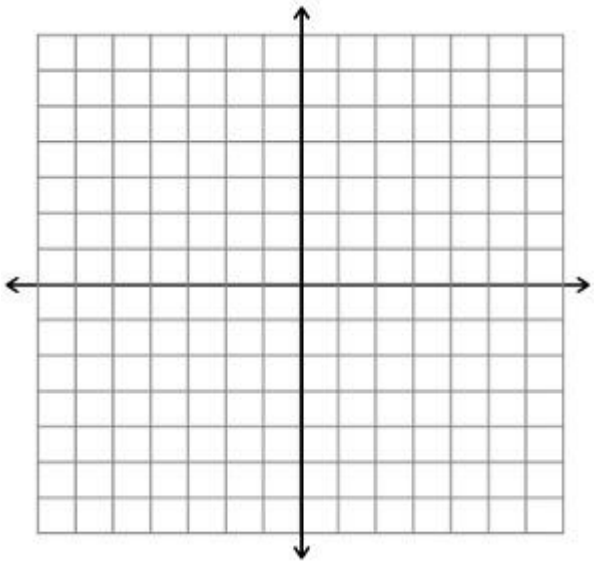
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -2(x-3)^2$



Vertex:

Axis of Symmetry:

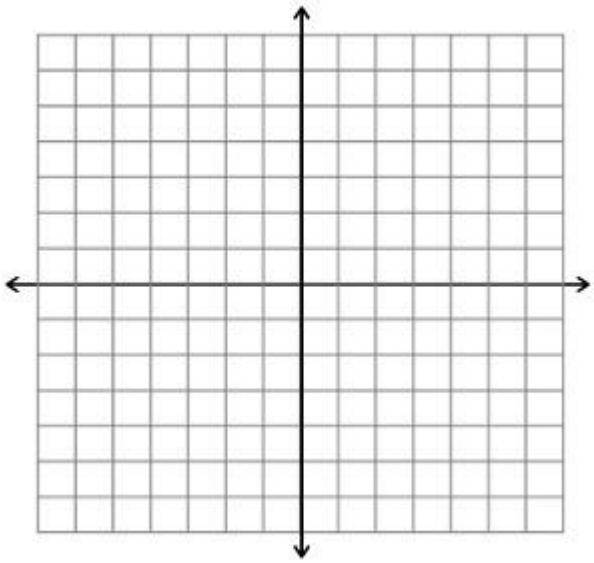
X-intercepts:

Y-intercepts:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples For each of the following functions, find and state: (i) the vertex (ii) axis of symmetry (iii) x-intercepts (iv) y-intercepts (v) and then graph the function

a. $f(x) = (x-2)^2 + 1$



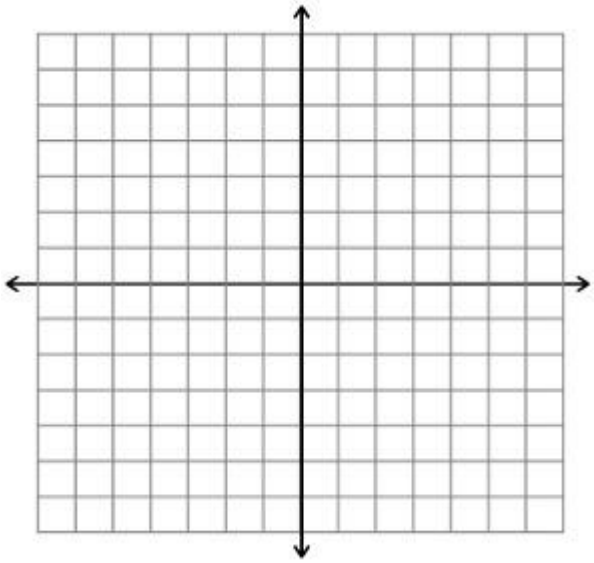
Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

b. $f(x) = -(x+3)^2 - 1$



Vertex:

Axis of Symmetry:

X-intercepts:

Y-intercepts:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Finding the Vertex of Quadratic Equations in the form $\boldsymbol{f(x) = ax^2 + bx + c}$

What is the vertex formula?

Show how to use the vertex formula for $f(x) = 3x^2 + 6x - 2$

<u>Examples</u> Find the vertex of the following:

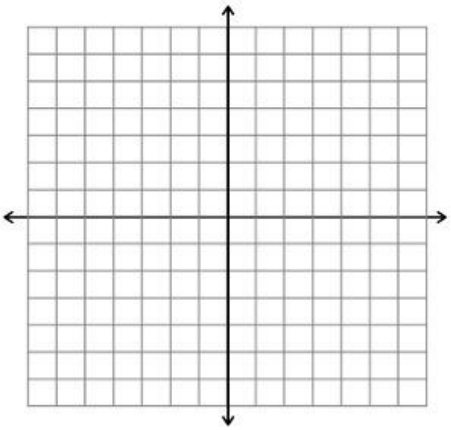
a. $f(x) = -x^2 + 4x - 2$

b. $f(x) = -\frac{1}{2}x^2 + 3x - 1$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Putting Quadratics into the $f(x) = a(x-h)^2 + k$ Form

Before you begin this lesson, you should already feel comfortable graphing an equation like $f(x) = 2(x-1)^2 + 3$. Graph this below:



Now show all the steps to put $f(x) = 2x^2 - 8x + 2$ into $f(x) = a(x-h)^2 + k$ form:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = x^2 + 4x - 2$

b. $g(x) = x^2 + 6x + 10$

c. $h(x) = -x^2 + 2x - 4$

d. $y = 3x^2 + 6x - 12$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Putting Quadratics into the $f(x) = a(x-h)^2 + k$ Form

<u>Examples</u> Put the following quadratic functions into $f(x) = a(x-h)^2 + k$ form:

a. $f(x) = \frac{1}{2}x^2 + x - 3$

b. $g(x) = -\frac{1}{3}x^2 + 4x - 2$

c. $y = x^2 + 3x + 2$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Determining Whether a Quadratic Function will have a Minimum or a Maximum

Using the form $f(x) = ax^2 + bx + c$, explain what determines whether the parabola opens up or down:

<u>Examples</u> Determine whether the following quadratics open upwards or downwards, and then determine their vertex:

a. $f(x) = -2x^2 + 4x - 2$ upwards downwards

b. $g(x) = -\frac{1}{2}x^2 - 2x + 2$ upwards downwards

c. $h(x) = x^2 + 6x + 1$ upwards downwards

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Applications of Quadratic Functions 1

A ball is thrown upwards from a cliff 24 feet about the ground. The height h of the ball, in feet, t seconds after it is thrown is given by:

$$h(t) = -16t^2 + 32t + 24$$

How long does it take the ball to reach its maximum height?

What is the maximum height attained by the ball?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Review of Quadratic Functions

A polynomial function of degree n is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Where $a_n, a_{n-1}, \ldots, a_1, a_0$ are any complex or real number, $a_n \neq 0$, and n is a nonnegative integer.

What do we call n in this equation?

What do we call a_n in this equation?

What do we call $a_n x^n$ in this equation?

<u>Example</u> What is the degree, leading coefficient, and dominating term in the following?

a. $f(x) = 4x^4 - 5x^2 + x^7 + 100$

b. $g(x) = x\sqrt{3} - 5$

c. $h(x) = 7$

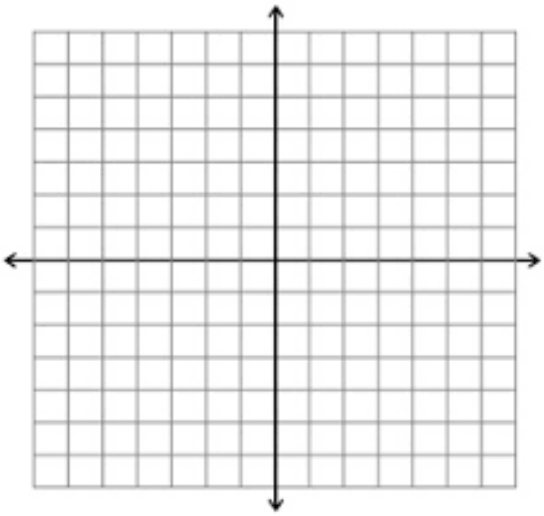
Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What is a quadratic function?

What do we call the graph of a quadratic function?

Draw and label the parts of a parabola.

<u>Example</u> Sketch the graph of $f(x) = -(x+4)^2 - 2$



What is the **vertex form** of the equation of the parabola?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Show how to use completing the square to put $f(x) = x^2 + 6x - 4$ in vertex form. Write out clear directions.

Example Use completing the square to write $f(x) = 3x^2 + 5x + 2$ in vertex form, then sketch the graph.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Find the intercepts of $f(x) = 3x^2 + 5x + 2$

How many x-intercepts can a parabola have? Draw a little sketch of each situation.

How can you tell how many intercepts a parabola will have if the parabola is in the form: $f(x) = ax^2 + bx + c$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What is the vertex formula for a parabola in the form $f(x) = ax^2 + bx + c$?

<u>Example</u> Use the vertex formula to find the vertex of the following: $f(x) = 2x^2 - 4x + 1$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Finding Intercepts, Vertices, Etc of Quadratic Functions

Find the vertex, axis of symmetry, domain, range, intercepts, and intervals of increasing and decreasing. If the function is in $f(x) = ax^2 + bx + c$ form, put it in vertex form. Sketch the graph. If the function is in $f(x) = a(x-h)^2 + k$ form, put it into standard form.

a. $f(x) = -\frac{1}{2}x^2 - 3x - \frac{1}{2}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

b. $f(x) = 2(x-1)^2 + 3$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Graphs of Basic Polynomial Functions and Translations of the Basic Graphs

Explain the graphs of $f(x) = x^2, f(x) = x^4, f(x) = x^6$, etc.

Explain the graphs of $f(x) = x^3, f(x) = x^5, f(x) = x^7$, etc.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

If we multiply the polynomial like so: $f(x) = ax^n$, what happens to the graph?

What happens if we have $f(x) = a(x-h)^n + k$?

<u>Example</u> Sketch the graphs:

a. $f(x) = (x-5)^5 + 3$

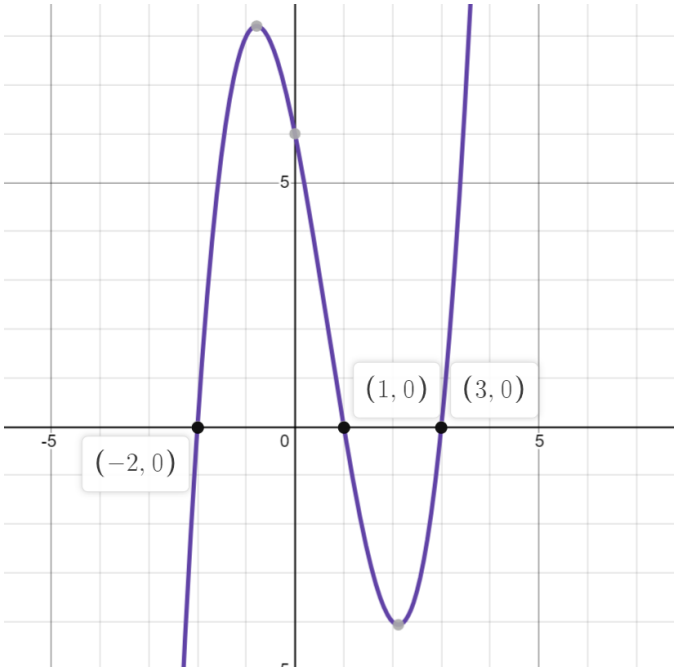
b. $f(x) = -2(x+1)^6 - 4$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

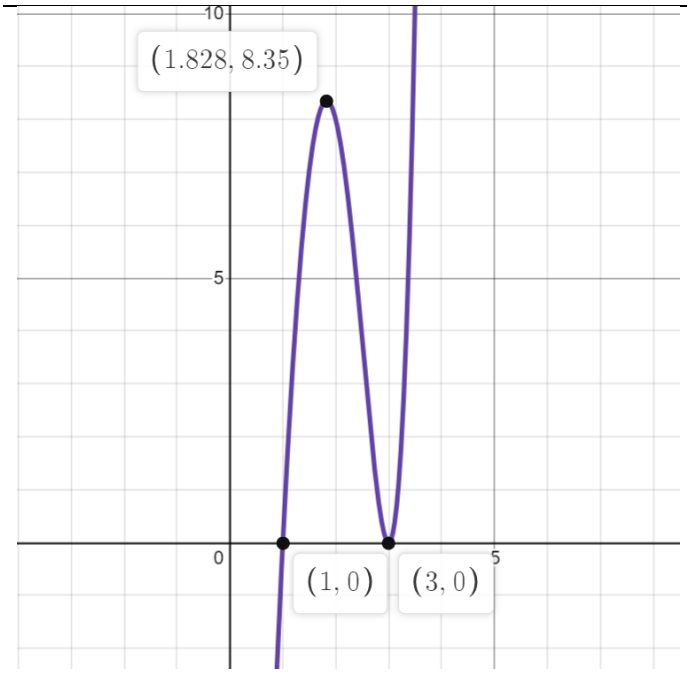
Lesson: Graphs of Polynomial Functions, Turning Points, and End Behavior

In the last video, we discuss the graphs of basic polynomials and translations. In this video, we will look at more complicated possibilities.

Below are the graphs of a few polynomials. What patterns do you notice?

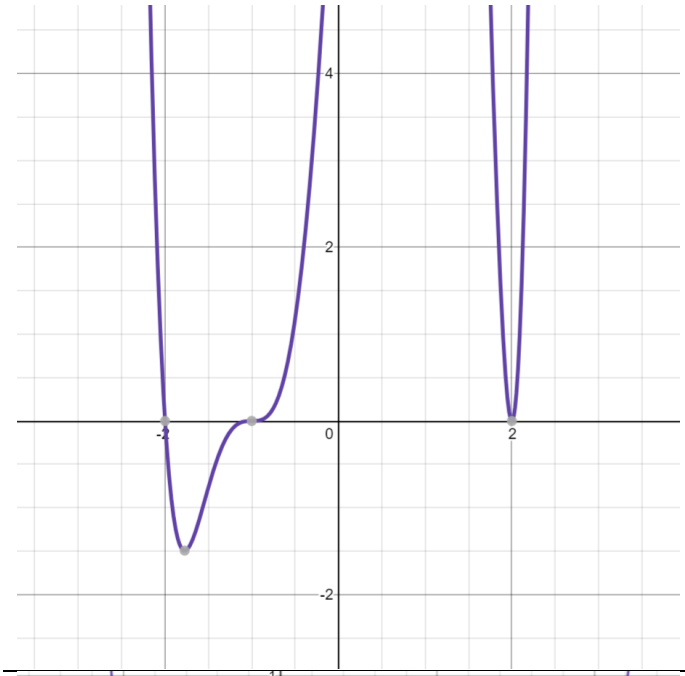


$$f(x) = x^3 - 2x^2 - 5x + 6 = (x-1)(x+2)(x-3)$$

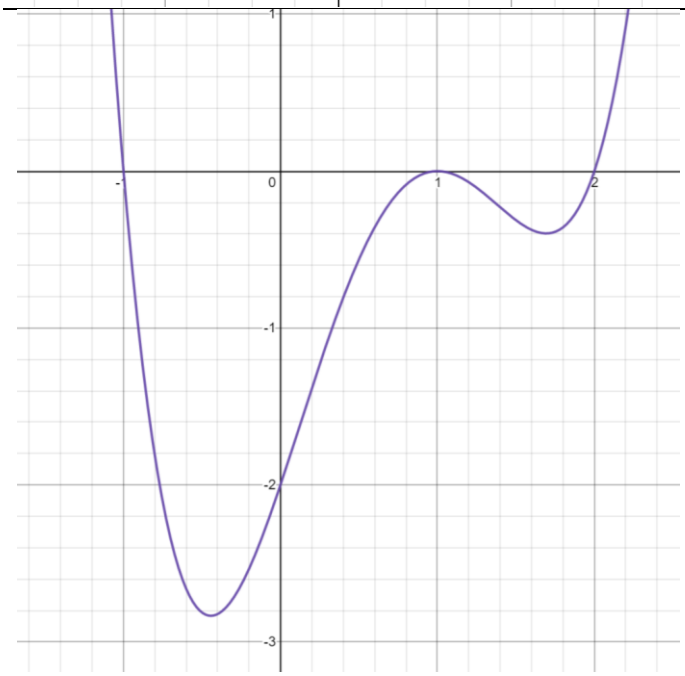


$$f(x) = x^5 - 7x^4 + 19x^3 - 37x^2 + 60x - 36$$
$$= (x-3)^2(x-1)(x-2i)(x+2i)$$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com



$f(x) = (x-2)^2(x+1)^3(x+2)$



$f(x) = (x-1)^2(x+1)(x-2)$

What do you notice about the behavior of these graphs at zeros? Summarize this in your own words, it was discussed as a general pattern in the video.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What are turning points and how many will a degree n polynomial have?

What is the end behavior of a polynomial and how do you determine it?

<u>Example</u> What are the number of turning points of the following and what is the end behavior of each?

a. $f(x) = x^2 + 3x + 2$

b. $f(x) = (x+1)^2(x-2)$

c. $f(x) = -\frac{1}{2}x(x-3)^4(x+1)^3(x+2)$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

List the steps required to graph a polynomial:

<u>Example</u> Graph $f(x) = x^3 - x^2 - 4x + 4$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Graph $f(x) = -(x+1)(x-2)^2(x-3)$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What is the relationship between x-intercepts, zeros, solutions, and factors of polynomials????

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Graphs of Polynomial Functions, Turning Points, and End Behavior

Graph the following polynomials:

a. $f(x) = x^3 + 5x^2 + 2x - 8$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

b. $f(x) = 3x^4 + 5x^3 - 2x^2$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

c. $f(x) = (x-1)^3(x+2)^2$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Intermediate Value Theorem and the Boundedness Theorem

State the Intermediate Value Theorem

<u>Example</u> Using synthetic division to show $f(x) = -x^3 + 2x^2 - 4x + 5$ has a real zero between 1 and 2.

<u>Example</u> Using synthetic division to show $f(x) = x^4 - 4x^3 - x + 3$has a real zero between 0.5 and 1.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

State the Boundedness Theorem

<u>Example</u> Show that $f(x) = x^4 + 5x^2 + 3x - 7$ has no real zero less than -3.

<u>Example</u> Show that $f(x) = 2x^5 - x^4 + 2x^3 - 2x^2 + 4x - 4$ has no real zero greater than 2

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Synthetic Division and the Remainder Theorem

The outcome of division can be restated using the division algorithm.

State the division algorithm:

Show how we use polynomial long division with $(x^2 - 3x - 1) \div (x + 2)$, and then restate the result using the format of the division algorithm.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Now show how to do the operation $(x^2 - 3x - 1) \div (x + 2)$ with synthetic division:

Restate this result using the division algorithm:

<u>Example</u> Use synthetic division to perform: $\frac{4x^3-3x^2-7}{x+1}$. Rewrite the result using the division algorithm.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

So what are 3 keys about using synthetic division to remember?

What is the remainder theorem?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Use the remainder theorem to find the desired functional values: $f(x) = -3x^4 + 5x^3 - 30x + 20, f(2)$. What is the equivalent division problem associated with this? Write the result of your division using the division algorithm.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Use the remainder theorem to find the desired functional values: $f(x) = 3x^2 + 5, k = 3 - i$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Synthetic Division and Potential Zeros of Polynomials

What is a “zero” of a polynomial?

How does the remainder theorem help us to determine if we have a zero?

What are some other terms that we use for “zero” of a polynomial?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Determine if the given k is a zero of the polynomial. If it is not, then what is the value of $f(k)$?

a. $f(x) = x^2 - 4x - 5; k = 5$

b. $f(x) = 2x^3 - 3x^2 + 5; k = 1$

c. $f(x) = x^2 - 2x + 2; k = 1 + i$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Factor Theorem

What does the factor theorem say?

<u>Example</u> Determine whether $(x-1)$ is a factor of each polynomial:

a. $f(x) = 9x^4 - 13x^2 + 4$

b. $f(x) = 4x^3 + 10x^2 + 2x - 4$

Given the above, what can we say is also a zero for example a?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Factor $f(x) = 3x^3 + 4x^2 - 17x - 6$ into linear factors given that -3 is a zero.

Example Factor $f(x) = 2x^3 - x^2 - 8x + 4$ into linear factors given that $\frac{1}{2}$ is a zero.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Rational Zeros Theorem

The rational zeros theorem is a way to determine *possible* candidates for *rational* zeros of a polynomial function.

State the Rational Zeros Theorem:

<u>Example</u> Find all the possible rational zeros for $f(x) = 2x^3 - x^2 - 8x + 4$ and then factor into linear factors.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Find all the possible rational zeros for $f(x) = 5x^4 - 19x + x^2 + 31x + 6$ and then factor into linear factors.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Fundamental Theorem of Algebra and the Number of Zeros Theorem

What does the Fundamental Theorem of Algebra state?

What is the number of zeros theorem?

<u>Example</u> What is the maximum number of zeros that each polynomial could possibly have?

a. $f(x) = x^2 - 3x + 1$

b. $f(x) = x^{17} - 3x + 1$

c. $f(x) = 9$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Find a polynomial $f(x)$ that has zeros $-3, 3$, and 1, where $f(2) = -10$

Example Find a polynomial $f(x)$ that has zeros $6, -3$, and -2, where $f(1) = 20$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Conjugate Zeros Theorem

Before we can discuss the theorem, let's review a few properties of conjugates.

What is the conjugate of the following:

a. $3 + 2i$ b. $5 - 3i$ c. $-7 + i$

What is another way that we denote conjugates? Summarize the explanation in the video. Give an example of how this notation works.

What are some properties of conjugates?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What is the conjugate zeroes theorem?

<u>Example</u> Find a polynomial function of least degree with only real coefficients and zeros 2 and $(3 - i)$.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Suppose $k = -i$ is one zero of $f(x) = x^4 + 26x^2 + 25$. What are the other zeros?

Example Factor $f(x) = 2x^3 + (3 - 2i)x^2 + (-8 - 5i)x + (3 + 3i)$ into linear factors given $k = 1 + i$ is a factor.

Before you begin, what is unique about this problem?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Finding Zeros of a Polynomial Function

In previous videos we discussed:

- The factor theorem
- The rational zeros theorem
- The fundamental theorem of algebra
- The number of zeros theorem
- The conjugate zeros theorem

Using all of these theorems, we can combine them to find the zeros of *any* polynomial.

What can we say about a polynomial function of odd degree with real coefficients?

What can we say about a polynomial function of even degree with real coefficients?

What if a polynomial doesn't have only real coefficients?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Find all zeroes of the following polynomials.

a. $f(x) = x^3 - 21x - 20$

b. $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Finding Zeros of a Polynomial Function

Find all the zeros of the polynomials.

a. $f(x) = x^4 + 13x^2 + 36$

b. $f(x) = 2x^4 - 11x^3 + 4x^2 + 29x + 12$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

c. $f(x) = x^4 - 7x^3 + 17x^2 - x - 26$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Descartes' Rule of Signs

Let $f(x)$ be a polynomial function with real coefficients and a nonzero constant term, with terms in descending powers of x. Explain how to use Descartes' Rule of Signs:

<u>Example</u> Determine the number of different possibilities for the number of positive, negative, and nonreal complex zeroes:

a. $f(x) = 3x^5 + 4x^4 - x^3 + 2x - 1$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

b. $f(x) = x^6 + x^5 - x^4 + x^3 + x^2 - x + 1$

c. $f(x) = 3x^5 - 7x^3 + 6x + 8$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Basic Graphs of Rational Functions

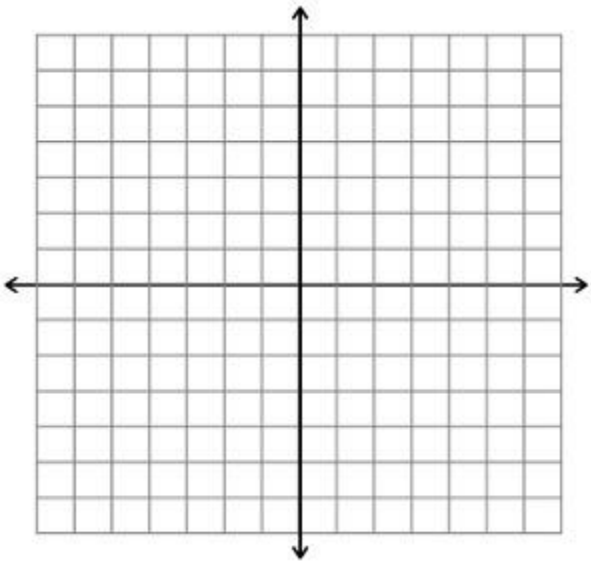
A rational function is a function of the form:

$\frac{f(x)}{g(x)}, g(x) \neq 0$

Where $f(x)$ and $g(x)$ are polynomials.

There are 2 graphs that we view as the most basic version of rational functions.

Make a table and graph $f(x) = \frac{1}{x}$



What is the domain and range?

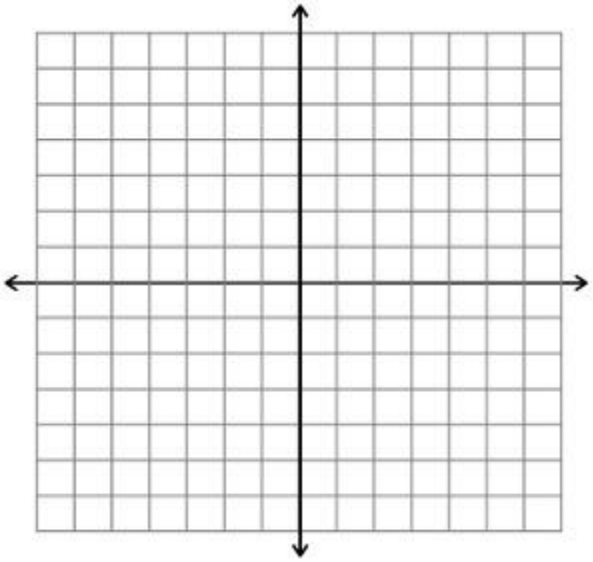
Where does the graph not exist (ie where is it discontinuous)

What are the y-axis and x-axis for this graph?

What kind of symmetry does this graph have?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Make a table and graph $f(x) = \frac{1}{x^2}$



What is the domain and range?

Where does the graph not exist (ie where is it discontinuous)

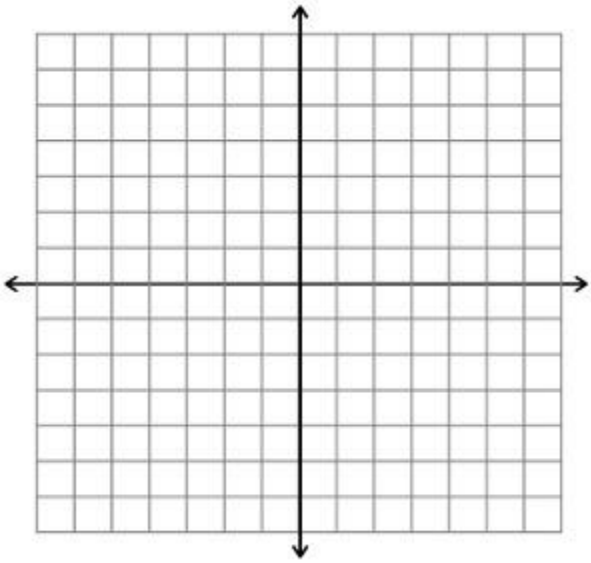
What are the y-axis and x-axis for this graph?

What kind of symmetry does this graph have?

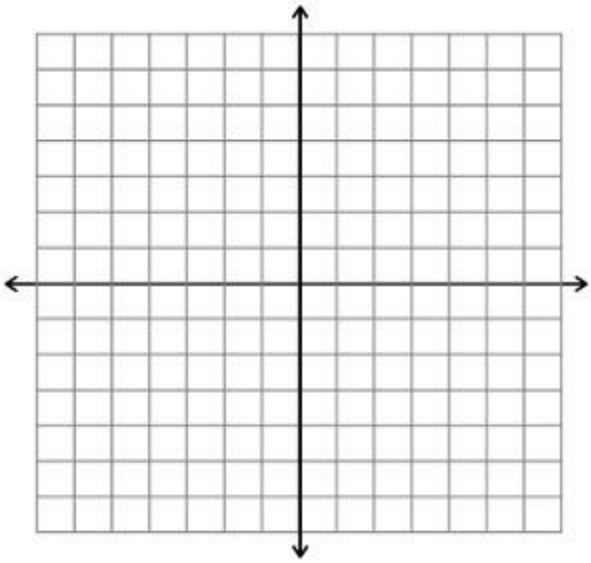
Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Describe the translations for the following graphs, and then draw an approximate sketch of the graph.

a. $f(x) = \frac{1}{(x+1)^2} - 2$



b. $g(x) = -\frac{2}{x} + 1$



Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Sketch what happens for odd powers of n for: $f(x) = \frac{1}{x^n}$, ie just draw a sketch of $y = \frac{1}{x^3}, y = \frac{1}{x^5}, y = \frac{1}{x^7}$

Sketch what happens for higher even powers of n for: $f(x) = \frac{1}{x^n}$, ie just draw a sketch of $y = \frac{1}{x^4}, y = \frac{1}{x^6}, y = \frac{1}{x^8}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Asymptotes of Rational Functions

What is an informal definition of asymptotes?

Is it possible that a graph would graph an asymptote?

Here are the formal definitions of asymptotes:

A graph $f(x) = \frac{p(x)}{q(x)}$ $(q(x) \neq 0)$ has a

- **Vertical asymptote** if $|f(x)| \to \infty$ as $x \to a$. The asymptote is x=a.
- **Horizontal asymptote** if $f(x) \to b$ as $|x| \to \infty$. The asymptote is y=b.

What is super important about asymptote?

Where do you typically find vertical asymptotes?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

How can you determine horizontal asymptotes? Describe the two ways, and also explain how oblique asymptotes. (In other words, explain what we talked about in the video).

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Find any vertical, horizontal, or oblique asymptotes.

a. $f(x) = \frac{x+2}{x^2+7x+12}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

b. $f(x) = \frac{3x+1}{4x-2}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

c. $f(x) = \frac{x^2+3x+2}{x^2-9}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

d. $f(x) = \frac{x^2+x}{x+3}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Finding Oblique Asymptotes

Find all the asymptotes for the following functions.

a. $f(x) = \frac{x^2+2x}{2x+1}$

b. $f(x) = \frac{x^2+2}{x+1}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Graphing Rational Functions

To learn how to graph rational functions, we will outline the steps and then graph. **Clearly write directions for each step as we go through this process!!!**

Graph $f(x) = \frac{x+3}{x^2+7x+10}$. Outline all the relevant steps discussed in the video.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Graph $f(x) = \frac{x+1}{x-3}$. Outline all the relevant steps discussed in the video.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Graph $f(x) = \frac{x^2+1}{x-3}$. Outline all the relevant steps discussed in the video.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Graphing Rational Functions

Graph $f(x) = \frac{(x+2)^2}{(x-1)(x+3)}$. Outline all the relevant steps discussed in the video.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Graph $f(x) = \frac{2x^2-3x+1}{x-2}$. Outline all the relevant steps discussed in the video.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Equations of Rational Functions

Copy the graphs from the video and then describe how to find the equations of those graphs.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

The Basics of Roots
Video 2: Higher Roots

How would you evaluate the following? Explain the rationale for how to evaluate these:

$\sqrt[3]{8}$

$\sqrt[4]{81}$

$\sqrt[5]{32}$

The most general symbol for a root is $\sqrt[n]{a}$. Explain what this symbol means and once again, label it.

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Now we can create a list of the most likely roots that we will encounter. You will very likely want to create another list of this on a page you can easily access.

Squares	**Cubes**	**Other**

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Evaluate the following. Be careful when you evaluate.

a. $\sqrt[3]{125}$

b. $\sqrt[3]{-125}$

c. $\sqrt[4]{16}$

d. $\sqrt[4]{-16}$

d. $-\sqrt[3]{8}$

e. $-\sqrt[3]{-8}$

f. $\sqrt[5]{-32}$

g. $\sqrt[6]{-64}$

Now let's generalize. For all roots of the form $\sqrt[n]{a}$, what can we say if….

1. The index (n) is an **odd** number?

2. The index (n) is an **even** number and a is a **negative** number?

3. The index (n) is an **even** number and a is a **positive** number?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Simplifying Radicals
The Basics

We will look at how to simplify radicals like this:

$$\sqrt{50} = 5\sqrt{2} \quad \text{or} \quad \sqrt[3]{250} = 5\sqrt[3]{3}$$

You might want to have the list of squares / cubes / quartics ready for this exercise. If you don't, pause the video and write them all down on a separate sheet of paper so that you can have it handy for this entire lesson.

First we look at **the multiplication property of radicals:**

<u>Examples</u> Multiply the following:

a. $\sqrt[3]{2} * \sqrt[3]{3}$

b. $\sqrt{7} * \sqrt{5}$

c. $\sqrt[3]{3} * \sqrt[3]{9}$

d. $\sqrt[3]{6} * \sqrt[3]{9}$

How are examples c and d related?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Being able to multiply radicals together is powerful, because it also allows us to break apart radicals and simplify them. Using the example $\sqrt{12}$, write down how to simplify:

<u>Examples</u> Simplify the following:

a. $\sqrt{50}$

b. $\sqrt{20}$

c. $\sqrt[3]{54}$

d. $\sqrt[4]{32}$

e. $\sqrt[3]{24}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

At this point, we should probably define when a radical is actually simplified.

A simplified radical will ……

<u>Examples</u> Explain why the following radicals are not simplified:

a. $\sqrt{24}$

b. $\frac{5}{\sqrt{7}}$

c. $\sqrt[4]{\frac{3}{5}}$

d. $\sqrt[3]{8x^8y^2}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Simplify the following:

a. $\sqrt{24}$

b. $\sqrt[3]{32}$

c. $\sqrt[4]{64}$

d. $\sqrt[5]{64}$

e. $\sqrt{250}$

f. $\sqrt[3]{250}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Variation
Video Notes

The term "variation" refers to a particular way of describing how certain things relate to one another.

There are three types of variation we will look at. Write down the definitions for each, you will remember it better!

1. Direct Variation:

2. Inverse Variation

3. Joint Variation

It's also possible to combine different types of variation.

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

The first thing we have to do is learn how to interpret and write variation equations.

Examples Write the variation equation:

a. P varies directly with Q

b. J varies inversely with L

c. H varies jointly with A and B

d. Z varies inversely with the square of R

e. M varies directly as the square root of N

f. Y varies jointly with the cube of M and the square root of L

g. M varies directly as L and inversely as N

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Now we can take a look at how to solve general variation equations.

Example Write down the detailed steps / directions for this example.

J varies directly with L. If J = 12 when L = 4, find J when L = 7.

Step 1:

Step 2:

Step 3:

Step 4:

Step 5:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Using the steps outlined from the previous example, try the following:

a. M varies inversely as the square of L. If M = 12 when L = 2, find M when L = 4.

b. A varies jointly with B and C. If A = 30 when B = 2 and C = 5, find A when B = 3 and C = 4.

c. L varies directly as M and inversely as N. If L = 15 when M = 5 and N = 2, find L when M = 7 and N = 3.

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example Jada's weekly income varies directly with the amount of hours that she works. Last week, she earned $570 for working 38 hours. This week she worked 36 hours, how much will she earn?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: A Review of Variation

Describe the 3 types of variation and outline any patterns that might help you remember.

Direct Variation

Inverse Variation

Joint Variation

Combined Variation

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example If y varies directly with x, and y=15 when x=3, find y when x=6

Example If a varies jointly with b and c, and a=40 when b=2 and c=5, find a when b=3 and c=2.

Example If p varies inversely with m and directly with the square of t, and p=32 when t=2 and m=4, find p when t=3 and m=9

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Rewrite the following situations as variation.

a. $C = 2\pi r$, when C is the circumference of a circle and r is the radius.

b. $r = \frac{d}{t}$, where r is the speed when traveling d miles over t hours.

c. $f = \frac{mv^2}{r}$, where f is the centripetal force of an object of mass ma moving along a circle with radius r at velocity v.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Applications of Variation

The circumference of a circle varies directly as the radius. A circle of radius 8in has a circumference of 50.27in. Find the circumference of the circle if the radius is 5in.

The current in a simple electrical circuit varies inversely as the resistance. If the current is 30 amps when the resistance is 10ohms, find the current if the resistance if 5ohms.

The volume of a gas varies inversely as the pressure and directly as the temperature. If a gas occupies a 2.6L volume at 600 kelvins (kelvin is a temperature unit) and a pressure of 24 newtons, find the volume at 240 kelvins and a pressure of 32 newtons.

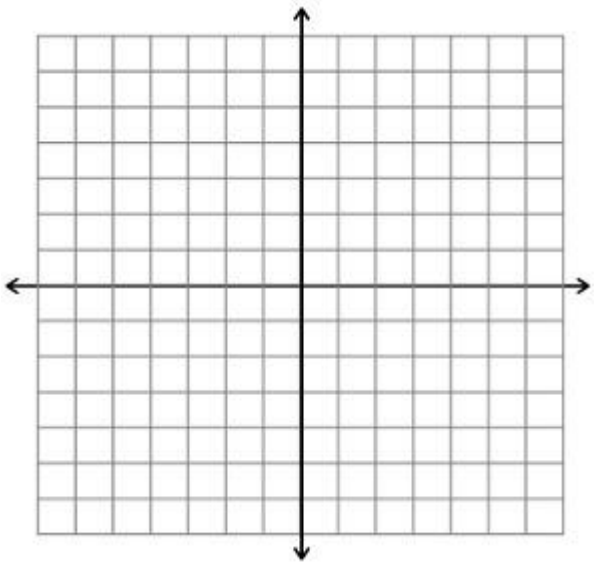
Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Exponential Functions

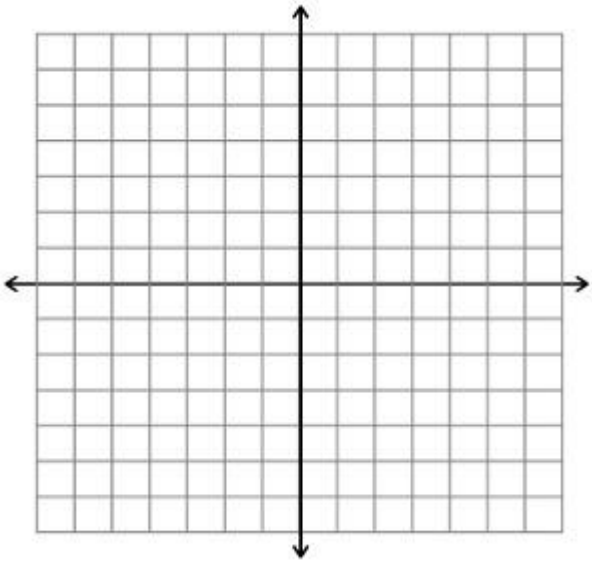
An exponential function is a function of the form _____ where $a > 0, a \neq 1$.

<u>Examples</u> Graph the following functions, then determine their domain and range.

a. $f(x) = 2^x$



b. $g(x) = \left(\frac{1}{2}\right)^x$

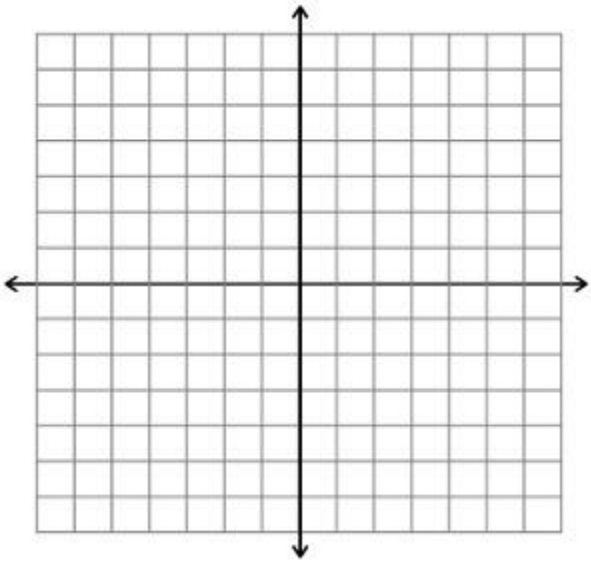


Using the graphs above, summarize some of the characteristics of these graphs:

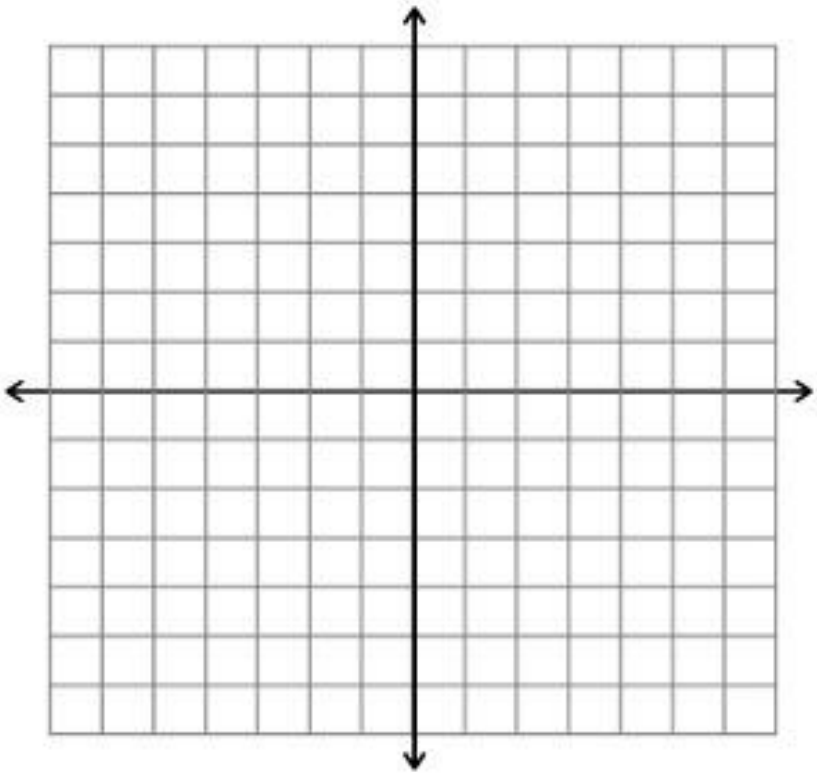
The number e is an irrational number that is frequently used in math. What is an approximation of e?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Graph $f(x) = 2^x, g(x) = e^x, h(x) = 3^x$ all on the same graph below for comparison:



Translations work similarly to other graphs we have discussed. Using the graph of $f(x) = 2^x$ as a "base graph", draw the following graphs together on the same graph: $f(x) = 2^x, g(x) = 2^x + 3, h(x) = 2^{x-1}$



Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Exponential Equations

In this lesson, we will solve equations that look like this:

$$3^x = 27 \qquad \text{or} \qquad 4^{x+3} = 8^{x-2}$$

To do this, we will write directions around the following example: $2^{3x} = 8^{x+1}$

<u>Examples</u> Solve:

a. $49^{x-3} = 7^{2x}$

b. $9^{4x} = 27^{x+4}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Solve:

a. $5^{x-2} = \frac{1}{25}$

b. $\left(\frac{3}{5}\right)^{2x} = \left(\frac{27}{125}\right)^{x+2}$

c. $27^{3x+1} = 9^{4x+2}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: A Review of Exponential Functions and their Graphs

What is the form of a general exponential equation?

<u>Example</u> Let $f(x) = 3^x$ and $g(x) = \left(\frac{1}{2}\right)^x$. Find the following.

a. $f(-2)$

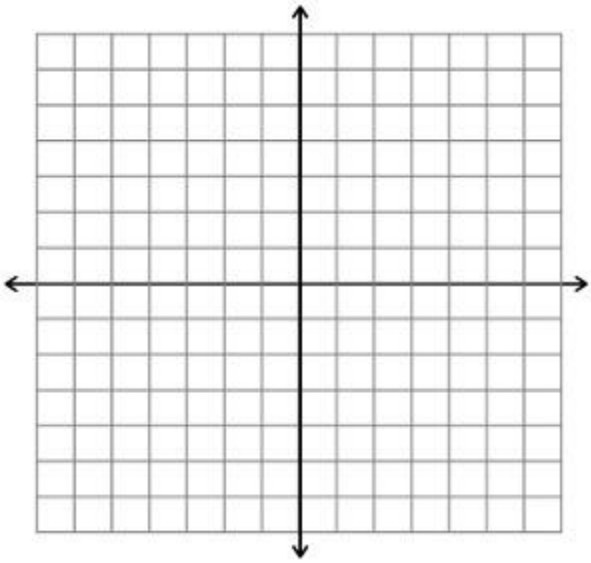
b. $f\left(\frac{3}{2}\right)$

c. $g(-3)$

d. $g(3.2)$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Sketch the graph of $f(x) = 2^x$ and create a table to go with it.



When will an exponential function $f(x) = a^x$ take this shape?

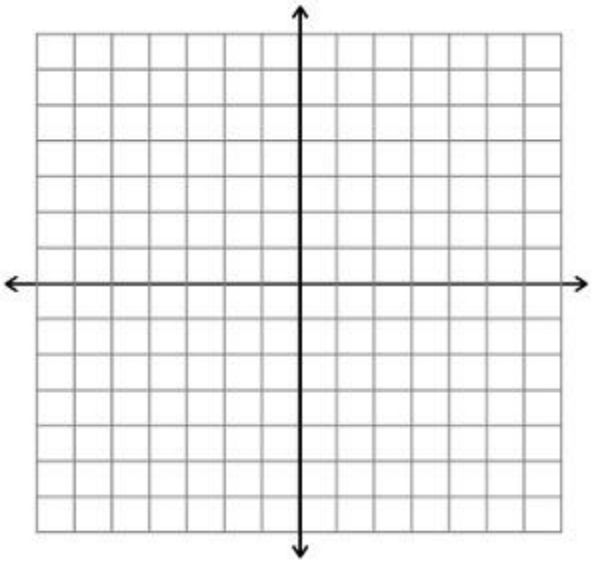
What is the x-axis for this graph?

What are 3 standard points to note here?

What is the domain and range?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Sketch the graph of $f(x) = \left(\frac{1}{2}\right)^x$ and create a table to go with it.



When will an exponential function $f(x) = a^x$ take this shape?

What is the x-axis for this graph?

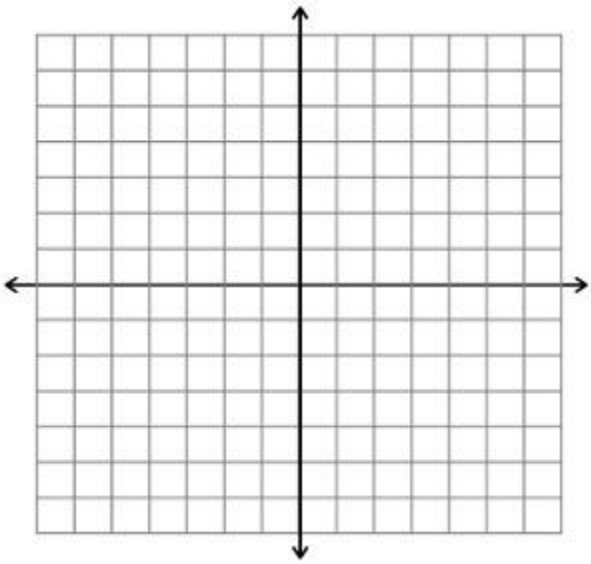
What are 3 standard points to note here?

What is the domain and range?

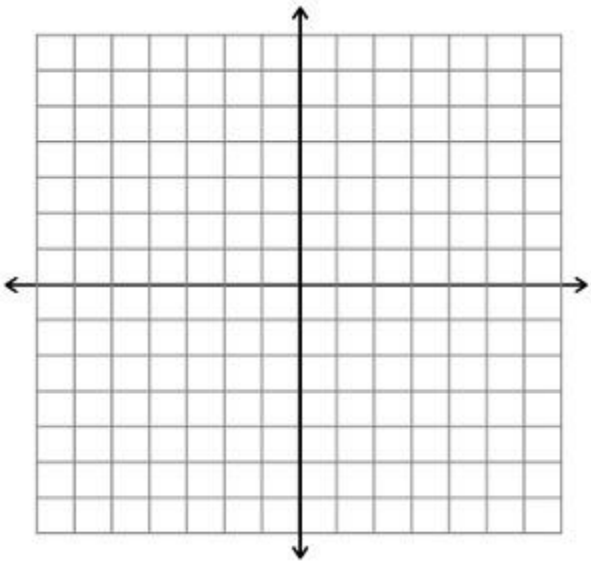
Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Describe the translations and sketch the following graphs.

a. $f(x) = -3^x + 1$



b. $f(x) = \left(\frac{1}{2}\right)^{x+1} - 2$



Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Basic Exponential Equations

Outline the general steps to solve $3^{x+1} = 81$.

Outline the general steps to solve $4^{x-2} = 8^{x+1}$.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Solve the following.

a. $27^{2x} = 9^{x-1}$

b. $e^{3-x} = (e^2)^{-x}$

c. $\left(\sqrt{3}\right)^{x+3} = 9^x$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Solve the following.

a. $x^{\frac{3}{2}} = 8$

b. $x^{\frac{5}{2}} = 4$

c. $x^{-4} = \frac{1}{16}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Compound Interest

What is the formula for compound interest and what does it mean?

<u>Example</u> Suppose $2000 is invested into an account paying 2.5% interest per year compounded monthly. How much is in the account after 10 years, and how much interest is earned in that time?

What else do we call the A from the compound interest formula?

What else do we call the P from the compound interest formula?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example In 5 years, you want to have $8000 in cash to buy a car (including taxes). Let's use savings to help us reach this goal.

a. You find an account with a 3.25% interest rate compounded biannually (2x a year). What amount would you have to deposit today to ensure you have $8000 in 5 years?

b. Let's pretend that as of today, you have $5000. What interested rate would you need to ensure that this will grow to $8000 in 5 years?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What is the number e?

What is the formula for continuous compounding?

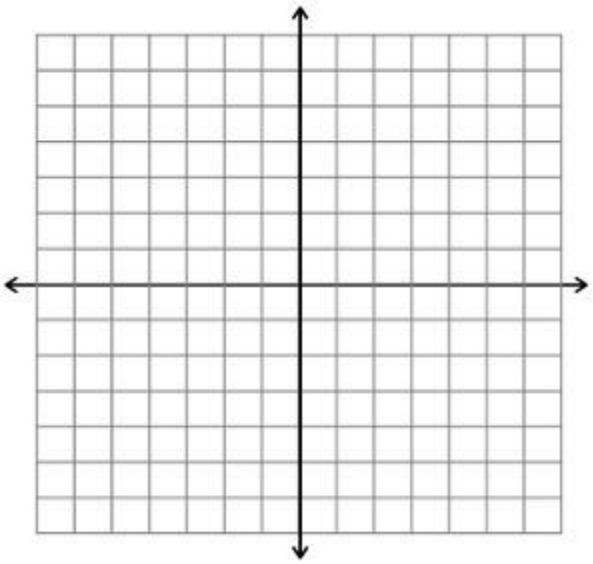
<u>Example</u> Suppose $2000 is invested into an account paying 2.5% interest per year compounded continuously. How much is in the account after 10 years, and how much interest is earned in that time?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Intro to Logarithmic Functions

First, we want to review how to function inverses of functions. Find the inverse of: $f(x) = 2x + 8$

Graph the exponential function $f(x) = 3^x$ below:



Recall the exponential functions are one-to-one, which means that they have inverses. Attempt to find the inverse of $f(x) = 3^x$ below:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

As we can see, there is an issue with finding the inverse – we have no way to solve for y! As such, we need new mathematics to help solve this.

Write down the definition of a logarithm:

Examples Write the following in exponential form:

a. $\log_3 81 = 4$

b. $\log_2 \frac{1}{8} = -3$

c. $\log_5 125 = 3$

d. $\log_9 9 = 1$

Examples Write the following in logarithmic form:

a. $2^4 = 16$

b. $3^0 = 1$

c. $6^{-2} = \frac{1}{36}$

d. $8^{\frac{1}{3}} = 2$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Now we can finally determine what is the inverse of $f(x) = 3^x$:

There are a few common logarithms:

- $\log x$ means:

- $\ln x$ means:

There are also 2 key properties of logarithms:

1. $\log_a a = 1$: explain why this makes sense

2. $\log_a 1 = 0$: explain why this makes sense:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Basics of Solving Logarithmic Equations and Evaluating Logarithms

We want to solve equations that look like:

$$\log_3 x = 4 \qquad \text{or} \qquad \log_x 36 = 2$$

Before you begin, you should already be able to do the following:

a. Put 3^5 in logarithmic form:

b. Put $\log_5 5 = 1$ in exponential form:

Now we can explain how to solve logarithmic equations using the example $\log_3(2x + 1) = 3$

<u>Example</u> Solve: $\log_x 16 = 4$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example Solve:

a. $\log_9 x = 2$

b. $\log_6(2x + 4) = 2$

Now we can also discuss how to evaluate logarithms. Explain how to evaluate: $\log_3 27$:

Examples Evaluate:

a. $\log_5 \frac{1}{25}$

b. $\log 100$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples Evaluate:

a. $\ln e$

b. $\log_3 27$

c. $\log_{25} 5$

d. $\log \frac{1}{10}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Review of the Basics of Logarithmic Functions

Explain how to find the inverse of $f(x) = 3^x$, and explain how this relates to logarithms.

Define a logarithm:

<u>Example</u> Write the following in logarithmic form.

a. $3^2 = 9$ b. $5^0 = 1$ c. $4^{-2} = \frac{1}{16}$

<u>Example</u> Write the following in exponential form.

a. $\log_3 1 = 0$ b. $\log_5 125 = 3$ c. $\log_3 81 = 4$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Show how to solve the following basic logarithmic equations.

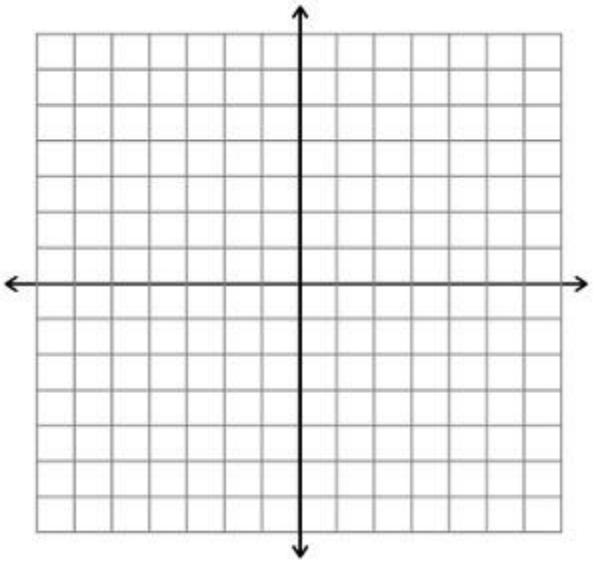
a. $\log_x 8 = 3$

b. $\log_3 x = -2$

c. $\log_{49} 7 = x$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Create a table and show how to graph $f(x) = \log_2 x$



What 3 points can you easily find on the graph?

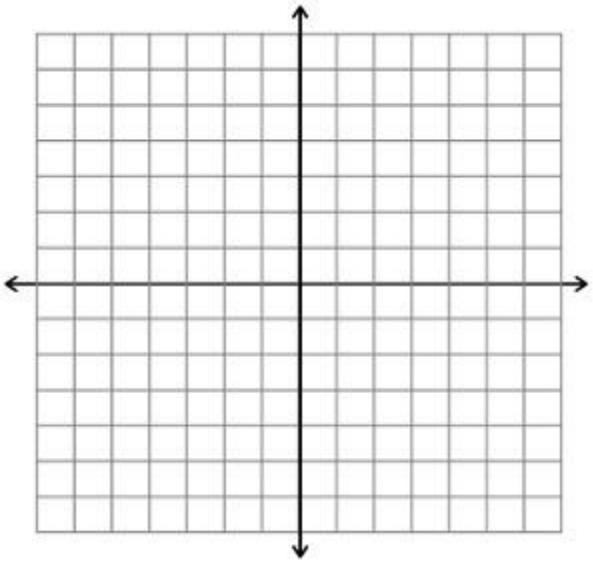
When does the graph take a shape like this for $f(x) = \log_a x$?

What is the y-axis for this graph?

What are the domain and range?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Create a table and show how to graph $f(x) = \log_{\frac{1}{2}} x$



What 3 points can you easily find on the graph?

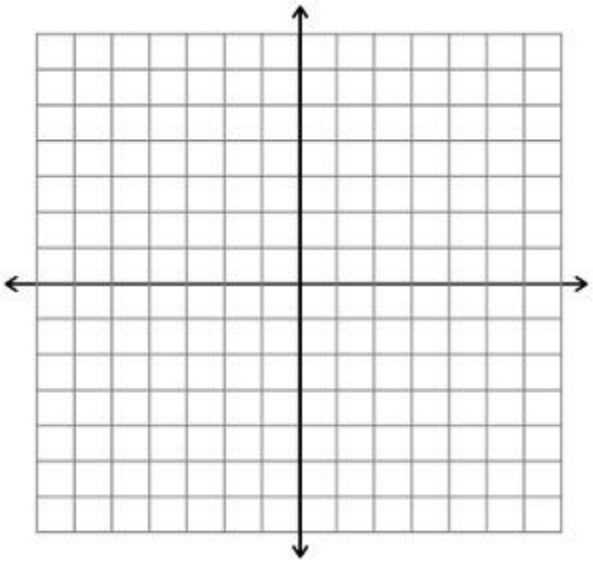
When does the graph take a shape like this for $f(x) = \log_a x$?

What is the y-axis for this graph?

What are the domain and range?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Graph the following: $g(x) = \log_2(x-1) + 1$



Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: How to Graph a Logarithm

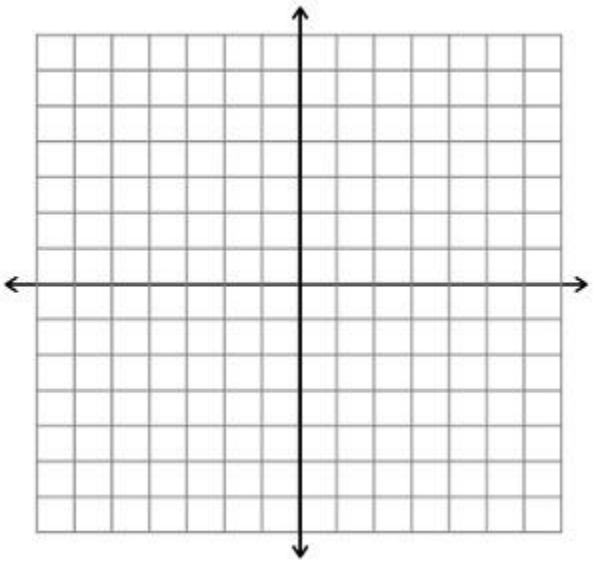
Before we begin, you should be able to do the following:

a. Convert to logarithmic form: $25 = 5^2$

b. Convert to exponential form: $\log_2 8 = 3$

c. Find the inverse of $y = 2^{\mathrm{x}}$

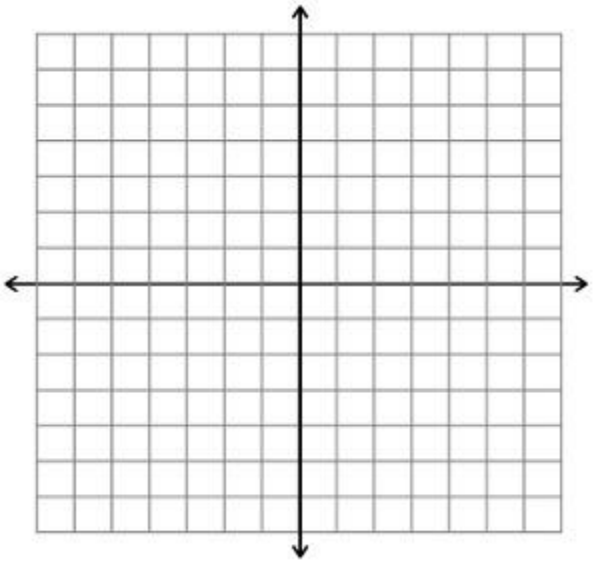
Now we can explain how to graph $y = \log_3 x$



For comparison, also graph $f(x) = 3^x$ on the same graph above.

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Graph $y = \log_{\frac{1}{2}} x$ below:



Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Review of the Basics of Properties of Logarithsm

List the major properties of logarithms.

Explain what the common logarithms mean:

$\log x =$

$\ln x =$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Expand the following as much as possible.

a. $\log_3\left(\frac{3x^3y^7}{\sqrt{z}}\right)$

b. $\ln\left(\sqrt[3]{x^2y}\right)$

c. $\log_3\sqrt{6}$

d. $\log_5\frac{25x^3y^2}{p^7r^3}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Write as a single logarithm.

a. $\log(x+2)+\log(x-2)-\log(x+3)$

b. $3\log_2 3+2\log_2 y-5\log_2 x$

c. $\frac{1}{2}\log_5 16+3\log_5 y^2-\log_5 x^7-\log_5 y$

d. $4\ln 2+\ln x^6-2\ln y^3-3\ln x^2$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Change of Base Theorem

State the change of base theorem:

<u>Example</u> Show how to evaluate the following:

a. $\log_5 17$

b. $\log_2 3$

c. $\log_{\frac{1}{2}} 5$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Exponential Equations

Outline the steps to solve: $2^{x+3} = 7^{3x}$

<u>Examples</u> Solve the following.

a. $e^{x^2} = 100$

b. $e^{2x-3} * e^{3x} = 5e$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Solve.

a. $3(2)^{x+2} + 6 = 105$

b. $e^{2x} + 8e^{x} + 15 = 0$

c. $3^{2x} + 2(3^{x}) = 24$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Logarithmic Equations

Outline the steps to solve: $\log(3 - x) = 0.75$

Compare these steps to: $\log(x + 20) = \log(x + 10) + \log(6)$. How are these problems different?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Solve the following.

a. $\log_4(x^3 + 37) = 3$

b. $\log x + \log x^3 = 4$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Solve the following.

a. $\log(x^2 - 4) - \log(x - 2) = \log(8)$

b. $\log(5 + 4x) - \log(3 + x) = \log 3$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Exponential and Logarithmic Formulas

<u>Examples</u> Solve for the desired variable.

a. for b: $p = a + \frac{c}{\ln b}$

b. for n: $A = \frac{Pr}{1-(1+r)^{-n}}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

c. for A: $\log A = \log B - C \log D$

d. for x: $D = 100 + 5 \log x$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Models of Exponential Growth and Decay

Explain the equation for the general model:

Some examples of growth can come from populations or investments. Some examples of decay can come from half-life/radioactive decay, or carbon dating.

Define the following:

Doubling time:

Half-life:

What is the formula for continuous compounding?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example A sample of 400g of radioactive lead decays to polonium according to the function: $A(t) = 400e^{-0.03t}$.

a. Find the amount of lead remaining after 5 years.

b. Find the half-life.

Example The amount of carbon-14 present in a paint after t years if given by $y = y_0 e^{-0.0002t}$. The paint contains 37% of its carbon-14. How old are the paintings?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Find the doubling time of an investment earning 5% interest if interest is compounded continuously.

Example During an epidemic, the number of people who have never had a particular virus and are not immune decreases exponential, where this decline is modeled by the function: $f(x) = 20{,}000e^{-0.025t}$, where t is time in days.

a. How many people were susceptible at the beginning of the epidemic?

b. How many people are susceptible after 2 weeks?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: A Review of Angles and Degrees

Draw, label, and define how to make an angle:

Show how to create an angle of positive versus negative measure:

Degree is generally how we measure angles.

Draw the complete rotation of a ray. How many degrees is this?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Define:

- Acute angle

- Right angle

- Obtuse angle

- Straight angle

Define:

- Complementary angles

- Supplementary angles

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Find the complement and supplement of a $55°$ angle.

Example Find the measures of the marked angles shown in the video.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Converting Between Minutes and Seconds

<u>Examples</u> Convert the following to degrees.

a. $23°15'32''$

b. $38°27'15''$

<u>Examples</u> Convert the following to degrees, minutes, and seconds.

a. $32.713°$

b. $72.17°$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Adding and Subtracting Degrees, Minutes, and Seconds

Add or subtract.

a. $45°15' + 23°57'$

b. $23°18'29'' + 14°19'47''$

c. $94°16' - 27°29'$

d. $72°18'15'' - 15°20'30''$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Applications of Minutes and Seconds

Example A turntable makes 45 revolutions in a minute. How many revolutions is that per second?

Example A windmill makes 80 revolutions per minute. How many revolutions is that per second?

Example Wind power is caused by a turbine spinning 10 to 20 revolutions per minute. How many revolutions does it make per second?

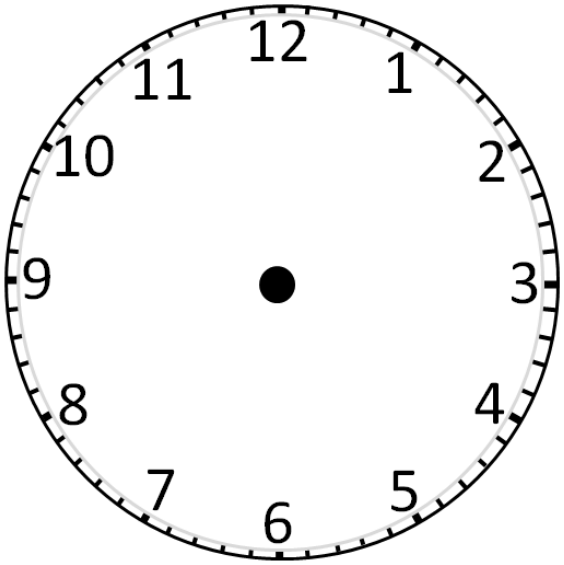
Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Angle Measure with a Clock

Explain the basics of angle measure, ie where is $90°, 180°$?

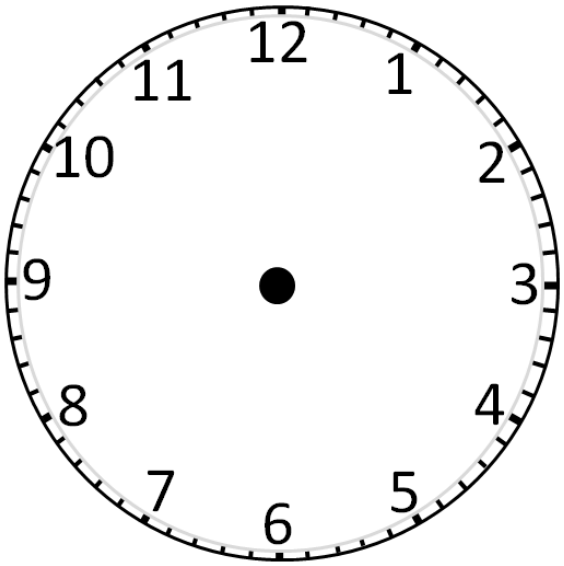
Each number represents how many degrees? SHOW how to actually determine this?

<u>Example</u> Find in the clock below, and show how to find the angle measure for 5:00. Draw pictures to help illustrate how this works, pictures are key to remembering!!!

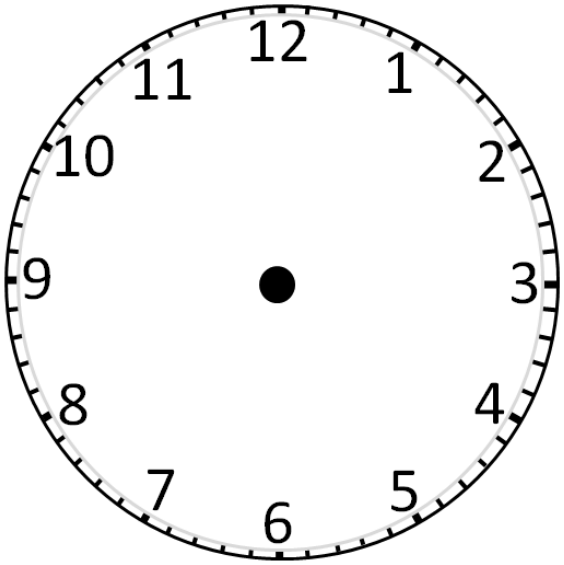


Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Find in the clock below, and show how to find the angle measure for 8:20. Draw pictures to help illustrate how this works, pictures are key to remembering!!!



Example Find in the clock below, and show how to find the angle measure for 10:05. Draw pictures to help illustrate how this works, pictures are key to remembering!!!

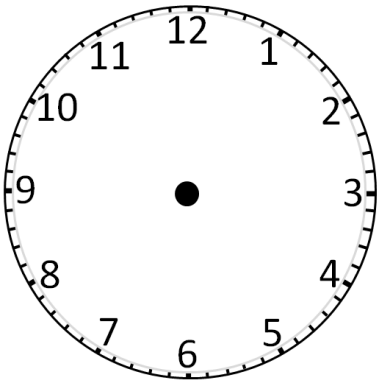


Copyright 2021, Jennifer Hill, DivideandConquerMath.com

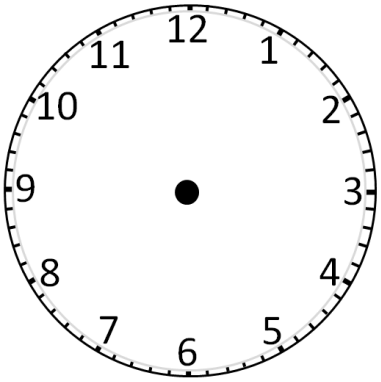
Examples: Angle Measure from a Clock

Find the angles associated with each time.

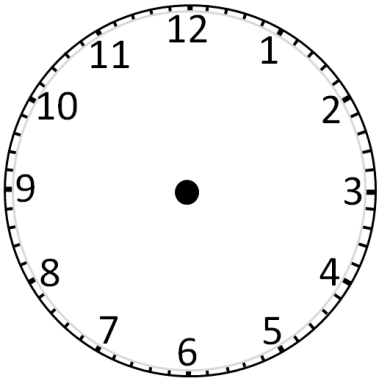
a. 6:25



b. 3:15



c. 2:38



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Standard Position and Coterminal Angles

When is an angle in standard position? Draw an example.

Show some examples of how we can determine angle measure by quadrant.

What are quadrantal angles? Draw a picture to illustrate.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What makes two angles coterminal? Draw a picture to explain.

Example Find the angle of least positive measure that is coterminal with each angle:

a. $715°$

b. $-80°$

Example How can you find all angles coterminal with $45°$? Prove it by filling in the following table.

Value of n	Angle coterminal with $45°$
2	
0	
-1	

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Fill in the table below for which angles are coterminal with the quadrantal angles

Quadrantal Angle	Coterminal Angles
$0°$	
$90°$	
$180°$	
$270°$	

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Radian Measure

Radian measure is another way to describe angle measure, as opposed to degrees.

What does radian measure allow us to do?

What is the definition of a radian?

Are radians a unit of measure like feet or meters? Circle: Yes No

What is the formula for the distance around a circle?

What does this formula show?

Write a way to relate radians to degrees:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

So what is π radians?

Show a conversion rate between radians and degrees:

<u>Examples</u> Convert each degree to radians:

a. $35°$

b. $-175°$

c. $27.8°$

Convert each radian measure to degrees:

a. $\frac{7}{4}\pi$

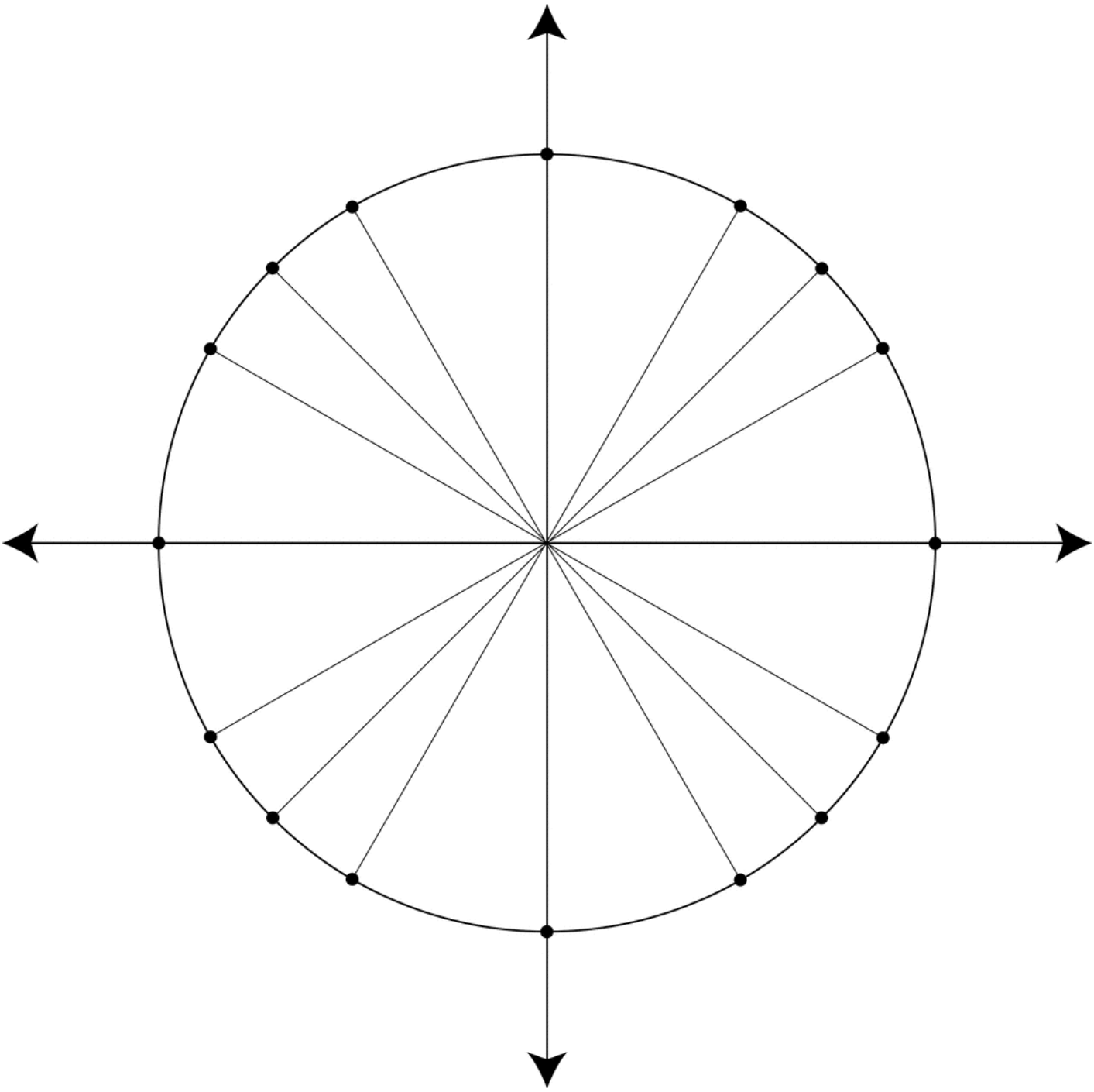
b. $-\frac{2\pi}{3}$

c. 7

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

If no unit of angle measure is specified, what should you assume?

Below we will construct an illustration of all the common degree measures we have discussed, and then write their radian equivalent. Take your time to copy this perfectly!



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

You will need to know all of those common degrees AND radian measures for the exam!

You may want to reference cheat sheets to help you with the next few problems.

<u>Examples</u> Find the value of each function.

a. $\sin\frac{5\pi}{6}$

b. $\tan\frac{3\pi}{4}$

c. $\csc\frac{5\pi}{3}$

d. $\cos\left(-\frac{2\pi}{3}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Arc Length on a Circle and Some Applications

We can determine the formula for the arc length of a circle from the definition of radians.

Regarding radians, what does θ equal? Show how this relates to arc length.

What is key about the arc length formula?

<u>Examples</u> A circle has radius 22.30 cm. Find the length of the arc intercepted by a central angle having each of the following measures.

a. $\frac{5\pi}{6}$ radians

b. $66°$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples A circle has radius 15.22 cm. Find the length of the arc intercepted by a central angle having each of the following measures.

a. $\frac{3\pi}{8}$

b. $125°$

What is the definition of latitude?

Example The latitude of Panama City, Panama is $9°$ N and the latitude of Pittsburgh, PA is $40°$ N. Find the north-south distance of these 2 cities. The radius of the Earth is 6400 km.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example A rope is being wound around a drum with a radius of 0.7525 ft. How much rope will be wound around the drum if the drum is rotated through an angle of $42.25°$?

Example Two gears are adjusted so that the smaller gear drives the larger one. If the smaller gear rotates through an angle of $150°$, through how many degrees will the larger gear rotate? Refer to the photo in the video.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Applications using Arc Length

Charleston, SC and Toronto, Canada, are 1100 km apart and lie on the same north-south line. If the latitude of Charleston is $33°$ N, what is the latitude of Toronto?

Find the radius of the larger wheel in the figure if the small wheel rotates $75.0°$ when the large wheel rotates $45°$.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

How many inches will a small weight rise if a pulley is rotated through an angle of $65°15'$? Through what angle, to the nearest minute, must the pulley be rotated to raise the weight 4 inches?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Area of a Sector of a Circle and Some Applications

What do we define as a sector of a circle?

What is the formula for the area of a section of a circle with radius r?

<u>Examples</u> Find the area of a sector of a circle having the given radius and central angle θ. Answers should be to the nearest tenth.

a. $r = 11.5$ ft, $\theta = \frac{5\pi}{6}$ radians

b. $r = 13.5$ m, $\theta = 125°$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Find the measure (in radians) of a central angle of a sector of a circle of area $25\ in^2$ in a circle of radius 2.5 in.

Example A center-pivot irrigation system provides water to a sector-shaped field as shown in the figure in the video. Find the area of the field if $\theta = 35.0°$ and $r = 155$ yd.

Example A circular sector has an area of $60\ in^2$. The radius of the circle is 6 inches. What is the arc length of the sector?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Linear Speed and Angular Speed and Applications

What is the formula for linear speed?

When are some situations where you might use linear speed?

What is the formula for angular speed?

When are some situations where you might use angular speed?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Use the formula $\omega = \frac{\theta}{t}$ to find the value of the missing variable.

a. $\omega = \frac{\pi}{3}$ radians per second, $t = 6$ seconds

b. $\theta = 3.721$ radians, $t = 18.34$ seconds

Examples Use the formula $v = r\omega$ to find the value of the missing variable.

a. $r = 10\ m$, $\omega = \frac{9\pi}{5}$ radians per second

b. $v = 10$ m per second, $\omega = \frac{3\pi}{2}$ radians per second

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Suppose a point P is on a circle with a radius of 5 cm, and the ray OP Is rotating with an angular speed of $\frac{\pi}{10}$ radian per second.

a. Find the angle generated by P in 3 seconds.

b. Find the distance traveled by P along the circle in 3 seconds.

c. Find the linear speed of P in cm per second.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

The formula $\omega = \frac{\theta}{t}$ can be rewritten as $\theta = \omega t$. We can then use the formula $s = r\theta$ to create a different formula. Show the new formula we can create:

<u>Examples</u> Use the formula above to find the value of the missing variable.

a. $r = 5\ cm, \omega = \frac{2\pi}{3}$ radians per second, $t = 8$ seconds

b. $s = \frac{3\pi}{2}$ km, $r = 4$ km, $t = 3$ seconds

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example The orbit of the earth around the sun is almost circular. Suppose that the orbit has a radius of 93 million miles.

a. We know that a year is 365 days. Find the angle formed by the Earth's movement in one day.

b. Give the angular speed in radians per hour.

C. Find the linear speed of the Earth in miles per hour.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Applications of Linear Speed and Angular Speed

Example Find the angular speed of the minute hand of a clock.

Example Find the linear speed of the tip of the second hand of a clock, if the hand is 25 mm long.

Example A bicycle has tires with a 12-inch radius. The tires are turning at the rate of 200 revolutions per minute. How fast if the bike traveling in miles per hour?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example A pulley has a radius of 13.23 cm. It took 15 seconds for 45 cm of belt to go around the pulley.

a. Find the linear speed of the belt in cm per sec.

b. Find the angular speed of the pulley in radians per second.

Example A thread is being pulled off a spool at a rate of 55.4 cm per second. Find the radius of the spool if it makes 155 revolutions per minute.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: The Unit Circle and Circular Functions

Explain how we create the unit circle:

What is the equation for the unit circle?

Recall the formula for arc length is $s = r\theta$. When the radius of the circle is 1, what does this formula become?

Why does this matter / what is cool about this?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Since r=1 on the unit circle, what can we say about our previously studied trig functions? Just summarize what we discussed in the video as best as you can.

Write out the **circular functions**

1.

2.

3.

4.

5.

6.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

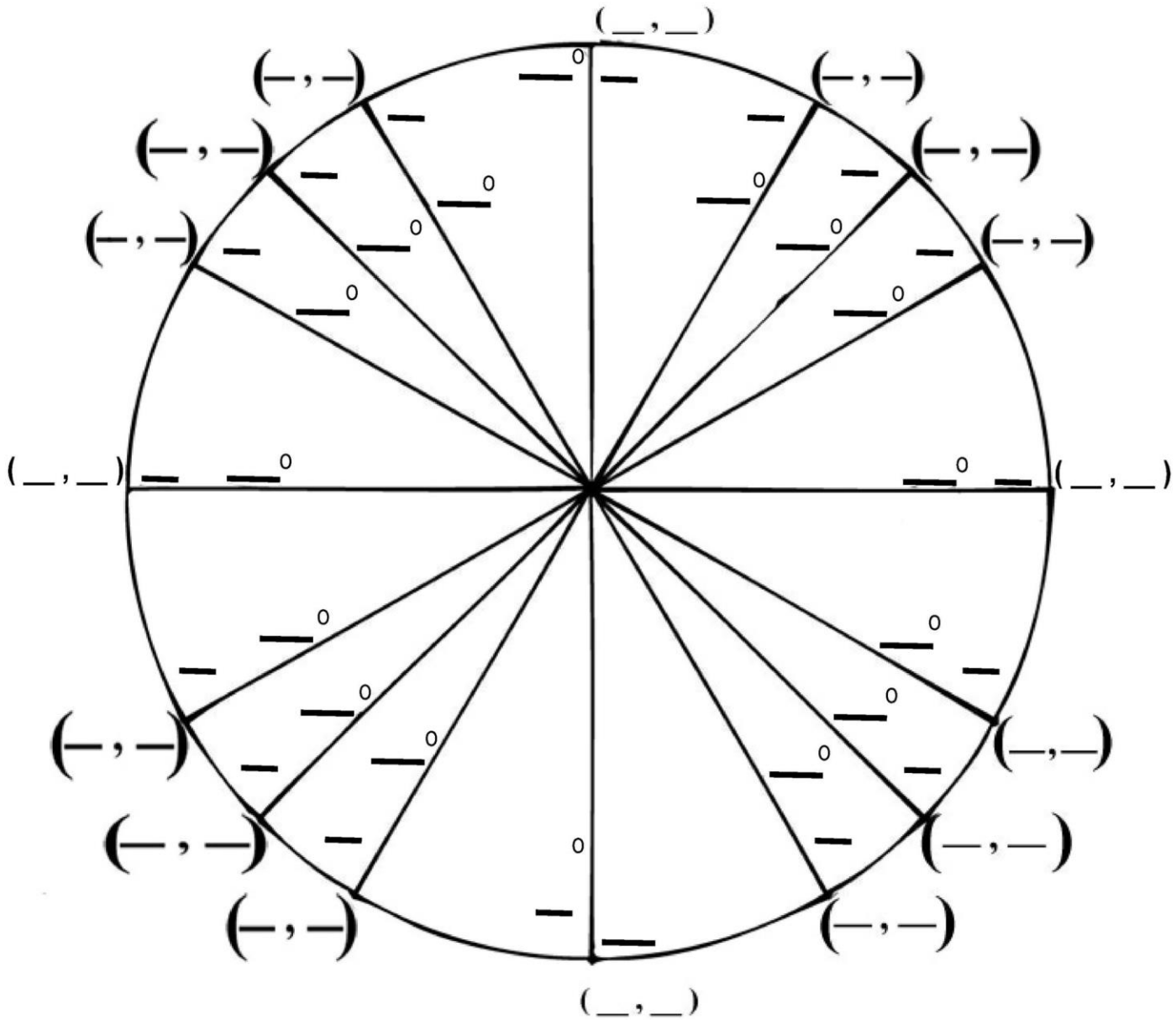
Explain the symmetry of the unit circle:

If a point (a, b) in on the unit circle, what other points are guaranteed to be on the unit circle?

How can we use symmetry with drawing the unit circle? How is it handy?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Label the unit circle. Note: you will have to have this memorized! These are the standard angle measures that you'd be expected to do without a calculator!



Now, going back to the circular functions we discussed, how do you view the x-coordinates?

How do you view the y-coordinates?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Use the unit circle to find the values of the following:

a. $\sin\frac{\pi}{3}$

b. $\cos\frac{5\pi}{4}$

c. $\sin\frac{11\pi}{6}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Arc Length on a Circle and Some Applications

Using the unit circle, we can also determine the domains of our circular functions.

What is the domain of sine and cosine, and why?

What is the domain of tangent and secant, and why?

What is the domain of cotangent and cosecant, and why?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Values of Circular Functions

We have previously discussed the unit circle (you will want to have it handy). One the most important things about the unit circle is that it will quickly tell you the value of the sine and cosine functions.

On the unit circle, what does the y-coordinate represent?

On the unit circle, what does the x-coordinate represent?

When we think about the values of these functions, what do we assume?

What does this mean for your calculator?

<u>Examples</u> Use the unit circle to find the values of:

a. $\sin\frac{\pi}{2}$ b. $\cos\frac{\pi}{2}$ c. $\tan\frac{\pi}{2}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Use the unit circle to find the values of:

a. $\sin\frac{5\pi}{4}$ b. $\cos\frac{5\pi}{4}$ c. $\tan\frac{5\pi}{4}$

Example Compare methods. Use reference angles and degrees to find the exact value of $\sin\frac{2\pi}{3}$, then confirm your answer with the unit circle.

For approximate values, we still must use a calculator. Remember, when you are not specifically told you are using degrees, then you assume you are using radians.

What must you ensure with your calculator?

Examples Use a calculator to approximate the function values.

a. $\cos(1.72)$

b. $\sin(-3.54)$

c. $\cot(1.438)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Values of Inverse Circular Functions

What do the functions $\sin^{-1} x$, $\cos^{-1} x$, $\cot^{-1} x$, etc. represent?

Your calculator makes a few key assumptions if it uses these functions (the reasons why are explained later):

1. For all x in $[-1, 1]$, using $\sin^{-1} x$ with a calculator will return a value between:

2. For all x in $[-1, 1]$, using $\cos^{-1} x$ with a calculator will return a value between:

3. For all real numbers x, using $\tan^{-1} x$ with a calculator will return a value between:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Sometimes you will be asked to find answers in different quadrants than what your calculator will return!

<u>Example</u> Solve for s, given that s is in the interval $\left[0, \frac{\pi}{2}\right]$: $\sin s = 0.3311$

<u>Example</u> Solve for s, given that s is in the interval $\left[0, \frac{\pi}{2}\right]$: $\tan s = 0.2155$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Solve for s, given that s is in the interval $\left[\pi, \frac{3\pi}{2}\right]$: $\tan s = \sqrt{3}$

Example Solve for s, given that s is in the interval $[0, 2\pi)$: $\cos^2 s = \frac{1}{2}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: The Unit Circle and Circular Functions as Line Segments

The unit circle is really just an extension of how we have previously thought about trig functions as line segments / right triangles.

Draw the illustration in the video, then show how these sides relate to our trig functions.

From your illustration, how do we define the following?

$\cos\theta =$ $\tan\theta =$ $\csc\theta =$

$\sin\theta =$ $\cot\theta =$ $\sec\theta =$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Using your illustration from the previous page, suppose the measurement of TVU is $45°$. Find the lengths of the following line segments:

OQ

OU

OV

VR

US

PQ

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: The Pythagorean Theorem, The Distance Formula, and Intro to Trig Functions

State the Pythagorean theorem:

State the distance formula:

How does the Pythagorean theorem relate to the distance formula?

Now we will introduce **trigonometric functions!** Explain what makes trig functions unique:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

How do trig functions relate to the distance formula? Draw the picture associated with this.

Explain the 6 trig functions and how they relate to what we discussed above.

Sometimes you will see expressions such as $\sin^2 90°$. What does this mean?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example The terminal side of an angle θ in standard position passed through the point (-12, -5). Find the values of the 6 trig functions of angle θ.

Example Find the 6 trig function values of an angle θ in standard position if the terminal side of θ is defined by

$3x + 5y = 0, x \geq 0$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Suppose the point (x, y) is in the indicated quadrant. Determine whether given ratio is positive or negative.

$$III, \frac{y}{r}$$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Finding Values of Trig Functions

For each given point, sketch the angle of θ in standard position such that θ has the least positive measure and the given point is on the terminal side. The find the values of the 6 trig functions.

a. $(-3, -4)$

b. $(7, 4)$

c. $\left(-\sqrt{2}, \sqrt{2}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Finding the 6 Trig Functions Given an Equation of the Terminal Side

An equation of the terminal side of an angle θ (in standard position) is given with a restriction on x. Sketch the situation with least positive angle θ, and find the values of the 6 trig functions.

a. $-6x - y = 0, x \geq 0.$

b. $x + y = 0, x \geq 0$

c. $x = 0, y \geq 0.$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Quadrantal Angles and Trig Functions

What were the quadrantal angles?

If one of the quadrantal angles is involved, how does this impact how we think about the trig functions? Draw a sketch to help illustrate this point. Explain how these are different from non-quadrantal angles, and how we choose points in these cases.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Find the indicated value of: $\sin 90°$.

How should we think about something like $\sin^2 90°$?

Example Use trigonometric function values of quadrantal angles to evaluate: $\tan 0° - 6 \sin 90°$.

Example Use trigonometric function values of quadrantal angles to evaluate: $\sin^2 180° + \cos^2 180°$.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Quadrantal Angles and Trig Functions

Find the function values:

a. $\cot 90°$

b. $\sec 180°$

c. $\sin(-270°)$

d. $3\sec 180° - 2\tan 360°$

e. $\sin^2 90° - 3\cos^2 90°$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Reference Angles and Non-Acute Angles

Review: what is a quadrantal angle?

What is associated with every non-quadrantal angle?

Show how we can find reference angles by drawing the reference graphs in the video:

<u>Example</u> Find the reference angle for each angle.

a. $274°$

b. $822°$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Draw how to find the reference angle based on the quadrant (follow along with the video and draw the diagram on your own):

Now let's talk about using $30°$, $45°$, and $60°$ as reference angles.

<u>Example</u> Find the values of the 6 trig functions for $240°$. Draw a reference!!! (This class is all about pictures!!)

Don't forget the helpful device: All Students Take Calculus 😊

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Let's outline the steps for finding trig function values for any non-quadrantal angle:

1.

2.

3.

4.

How can you avoid making an error with finding a trig function value?

<u>Examples</u> Find the exact values:

a. $\sin(-150°)$

b. $\cos(-510°)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Evaluate: $\sin^2 120° + \cos^2 120°$

Example Find all values of θ, if θ is in the interval $[0°, 360°)$ and has the given function value: $\cos\theta = \frac{\sqrt{3}}{2}$

Example Is the following true or false? $\cos 60° = 2\cos 30°$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Reference Angles and Finding Trig Value Functions

Find the exact values of all 6 trig functions of each angle. Rationalize when applicable.

a. $315°$

b. $-855°$

Evaluate

a. $\cot^2 120° + \tan^2 60° - \cos^2 330°$

b. $\cot^2 135° - \tan^4 45° - \sin^2 150°$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Finding the Value of Theta

Find all values of θ if θ is in the interval $[0°, 360°)$ and has the given value.

a. $\sec\theta = -\sqrt{2}$

b. $\cot\theta = -\frac{\sqrt{3}}{3}$

c. $\cos\theta = \frac{1}{2}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Angle Relationships

Draw and explain what vertical angles are:

Draw and explain what makes a transversal line. Draw a big picture, you will use it for the next part of your notes:

Label the diagram and note the relationships between the different angles below:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Draw and label the lines from the video. Find the measures of angles 1, 2, 3, and 4.

Example Draw and label the lines from the video. Find the measures of angles 1, 2, 3, and 4.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Review of Triangles and Similar Triangles

The sum of angle measures of any triangle is:

Draw the different types of triangles:

<u>Examples</u> Draw the triangles in the example, then classify the triangles as acute, right, or obtuse. Also classify as equilateral, isosceles, or scalene.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What makes 2 triangles similar?

What makes 2 triangles congruent?

What is the key relationship between similar and congruent triangles?

<u>What are the 2 conditions for similar triangles?</u>

In other words, triangle ABC is similar to triangle DEF if:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Draw the similar triangles in the video, and then find any missing angle measures.

Example Draw the similar triangles in the video, and then find any missing side lengths.

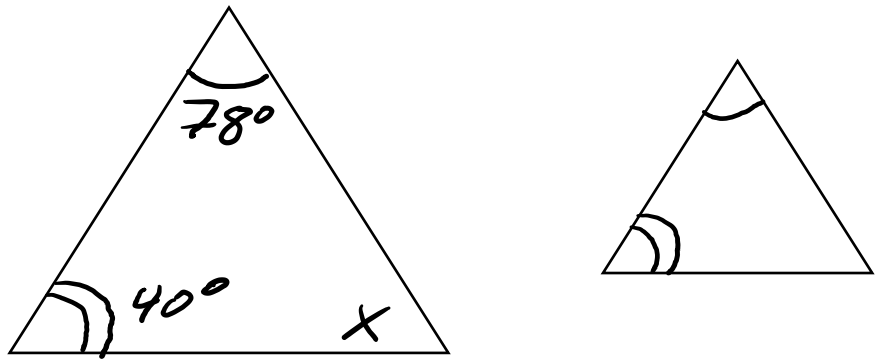
Example Draw the similar triangles in the video, and then find any missing measures.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

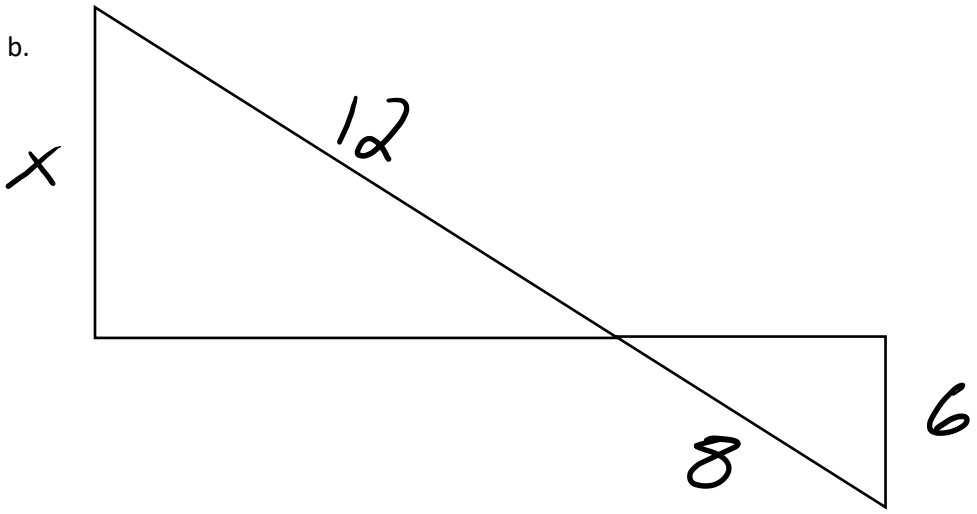
Examples: Similar Triangles

For each example, assume the triangles are similar. Triangles are not drawn to scale. Find any missing angle or side measures.

a.

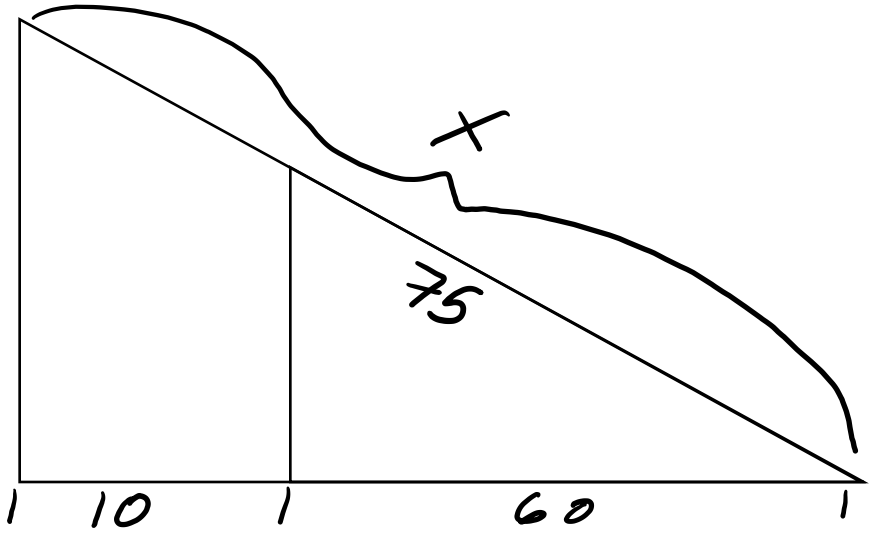


b.



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

c.



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Right-Triangle Based Definitions of Trigonometric Functions and Cofunctions

Instead of using standard position to describe trig functions, we can alternatively use right triangles.

Draw a right triangle to orient yourself.

What does A represent?

Show how to define the 6 trig functions with this right triangle.

1.

2.

3.

4.

5.

6.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What mnemonic device can help you remember the trig functions?

Example Draw the triangle in the video, and then show how to find the values of sine, cosine, and tangent.

Example Using the same example above, show how to find the values of cosecant, secant, and cotangent.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Draw a new right triangle, and show an alternative angle measure we can find from this picture:

These relationships are called the **cofunction identities,** summarize these below:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Write each function in terms of its cofunction

a. $\sin 10°$

b. $\cot 65°$

c. $\csc 34°$

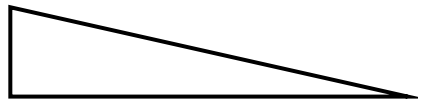
Example Find one solution for each equation. Assume all angles involved are acute angles.

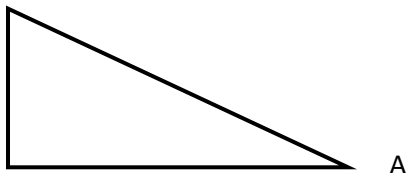
a. $\cos\theta = \sin(2\theta - 30°)$

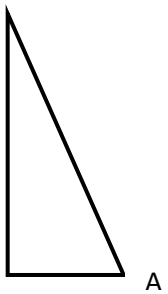
b. $\sec(\theta + 15°) = \csc(2\theta + 25°)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: More on Trig Function Values as Angles Change and Special Angles







Explain how angle A changes in these pictures:

<u>Examples</u> Determine whether each statement is true or false.

a. $\tan 25° < \tan 22°$

b. $\cos 25° < \sin 25°$

There are a few angles that occur frequently in trig and have some *nice* properties that we like to call them out. Draw out a triangle where each angle is $60°$ and describe any other properties:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

If you bisect this triangle, what happens? Draw a picture to illustrate, and then describe the trig functions associated with it:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Finally, draw an isosceles right triangle and describe any special characteristics with it:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

We have now examined some *extremely* common trig measures that will come up all the time in trig. They are so important that we will create a separate reference sheet for them.

Read this! I highly recommend that you remove the last page from this packet, and instead put it at the very front of your notes so that you can easily reference it. If you are moving on to calculus, KEEP THIS PAGE IN GOOD CONDITION FOR THE NEXT FEW YEARS. You will thank yourself later that you have this reference as these angles will come up over and over again, and students frequently forget them. I will occasionally create these handouts and make these suggestions to you as we go along in trig – you will thank yourself (and me) later if you keep these. It is extremely common for students to forget some of KEY parts of trig and then struggle in calculus – I don't want this to happen to you!

Some Function Values of Special (Common) Angles

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
$0°$						
$30°$						
$45°$						
$60°$						
$90°$						

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Sine and Cosine as Periodic Functions

What is a periodic function?

What are some examples?

What are 2 known periodic functions?

What can we say about the behavior of these functions as we move along the unit circle?

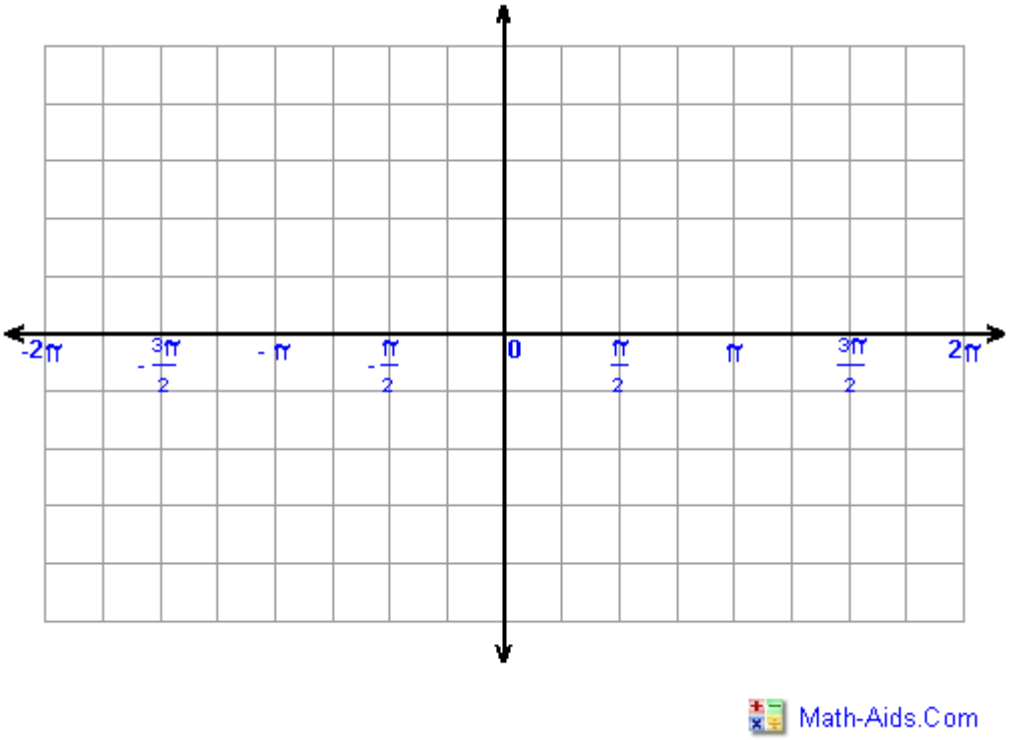
AS s CHANGES FROM	THE VALUE OF $\sin s$	THE VALUE OF $\cos s$
0 TO $\frac{\pi}{2}$		
$\frac{\pi}{2}$ TO π		
π TO $\frac{3\pi}{2}$		
$\frac{3\pi}{2}$ TO 2π		

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

But let's change s to x going forward so we don't get confused.

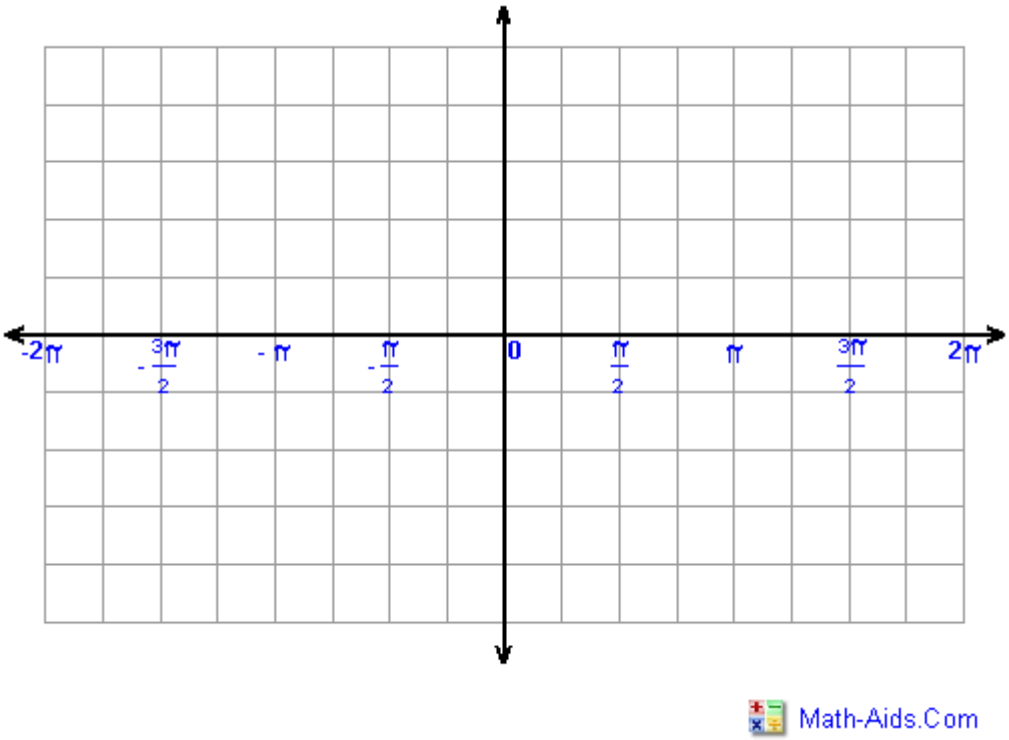
Draw the graph of sine, filling in the table and then using the graph paper.

x	$y = \sin x$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



Draw the graph of cosine, filling in the table and then using the graph paper.

x	$y = \sin x$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What term do we use to describe the graph of sine?

How do we describe the graph of cosine?

What is the period of both the sine and cosine functions?

Where are the graphs continuous?

What is the domain and range of the graphs?

How can we describe the x-intercepts of sine?

How can we describe the x-intercepts of cosine?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What is the test for even or odd functions?

What does it mean to be an even function?

What does it mean to be an odd function?

How do you know that $y = \sin x$ is odd? How would you show it?

How do you know that $y = \cos x$ is even? How would you show it?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Using Amplitude and Period to Graph Sine and Cosine

What is amplitude?

Using the equation $y = a \sin x$, when will this graph stretch or shrink? Draw 2 comparison sketches to help you remember (just a quick sketch so you remember how these visually compare)

Using the equation $y = a \cos x$, when will this graph stretch or shrink? Draw 2 comparison sketches to help you remember (just a quick sketch so you remember how these visually compare)

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What happens when a is negative? Demonstrate this with some brief sketches so that you can remember.

What is period?

Using the equation $y = \sin bx$, watch the video and write an explanation with sketches to demonstrate period.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Using the equation $y = \cos bx$, watch the video and write an explanation with sketches to demonstrate period.

What formula can you use to determine the period of a function?

<u>Examples</u> Determine the periods of the following functions:

a. $y = \sin 3x$

b. $y = \cos \frac{1}{2} x$

c. $y = \sin 2\pi x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Summarize amplitude and period when the function is of the form $y = a \sin bx$ and $y = a \cos bx$

<u>Examples</u> Graph the following over 2 period intervals, completing the table with strategic points to help you graph. Give the period and amplitude in each. You will have to sketch your own graph and create your own axes.

$y = \cos \frac{1}{3} x$

Period: **Amplitude:**

$\boldsymbol{x}$	$\boldsymbol{y = cos \frac{1}{3} x}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

$y = 2\sin x$

Period: **Amplitude:**

$\boldsymbol{x}$	$\boldsymbol{y = 2\,sin\,x}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

$y = 3\cos 2\pi x$

Period: **Amplitude:**

x	y

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Using Amplitude and Period to Graph Sine and Cosine

Graph the following over two-period intervals.

a. $y = -\frac{1}{2}\sin x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

b. $y = 3\cos 2x$

d. $y = -\sin \pi x$

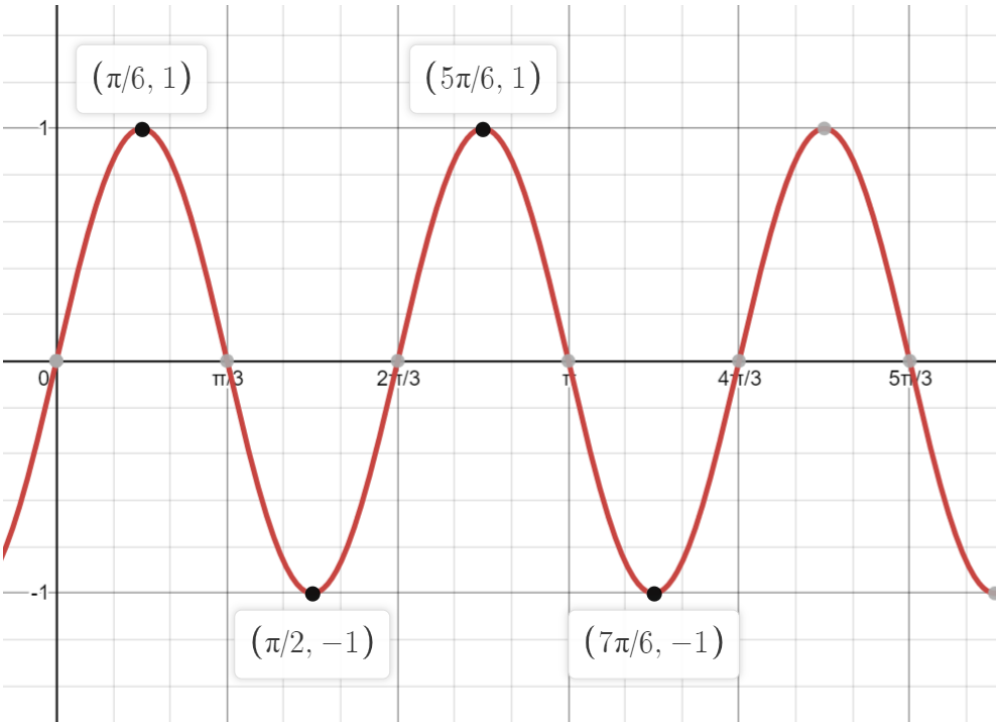
Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Determining the Equations of Sine and Cosine from Graphs

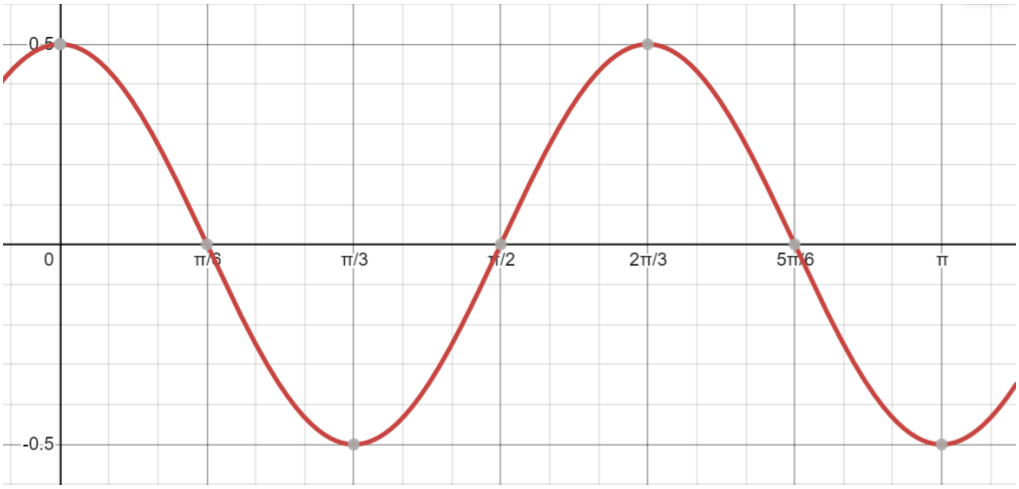
In each graph, determine the equation of the form $y = a\cos bx$ or $y = a\sin bx$, with $b > 0$, for each graph.

Desmos.com was used to create these graphs.

a.

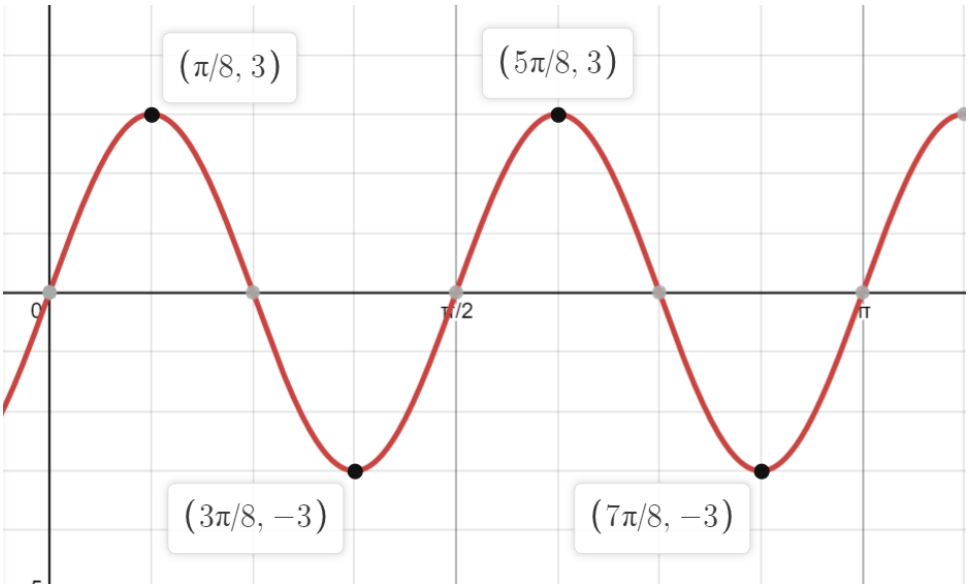


b.

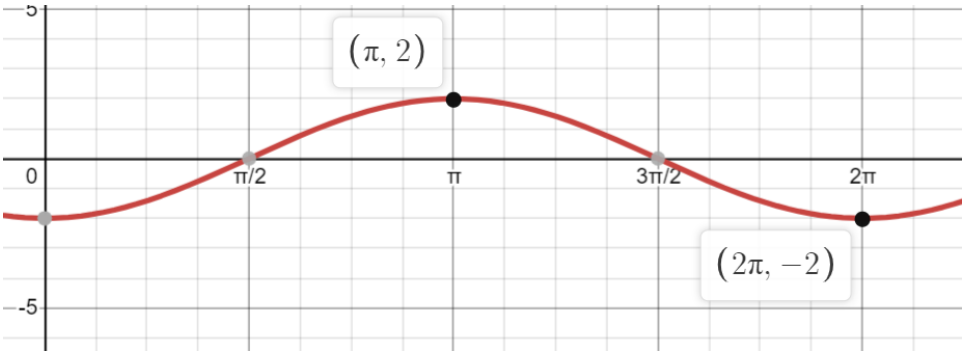


Copyright 2021, Jennifer Hill, DivideandConquerMath.com

c.



d.



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Translations of the Sine and Cosine Graphs

How do you describe a horizontal translation? In your own words / explanation, summarize with sketches how the video show these translations work.

What is phase shift?

What is an argument?

How do you describe a vertical translation? In your own words / explanation, summarize with sketches how the video show these translations work.

What happens when a function is of the form: $f(x) = a\cos(bx + bd)$ with the phase shift?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Identify the amplitude, period, phase shift, argument, and vertical translation.

a. $y = \sin\left(x + \frac{\pi}{4}\right)$

b. $y = 2 - \sin x$

c. $y = 3 + \sin(x + 2)$

d. $y = -4 + \frac{1}{2}\cos x$

e. $y = 2 + \sin(2x - 2\pi)$

f. $y = 1 - 2\cos\left(3x + \frac{\pi}{2}\right)$

g. $y = 2 - 4\cos(3x + 1)$

h. $y = 3\pi - 2\sin(\pi x - 3)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Show how to find the start and end points of an interval for us to graph.

a. $y = \sin\left(x + \frac{\pi}{2}\right)$

b. $y = 1 - 2\sin\left(3x - \frac{\pi}{2}\right)$

c. $y = 3\pi - 2\sin(\pi x - 3)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Using the graph $y = \sin\left(x + \frac{\pi}{4}\right)$, and *clearly write out the steps* required to graph any sine or cosine graph. Identify the amplitude, period, phase shift, and vertical translation.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Graph $y = 1 + \cos x$. Identify the amplitude, period, phase shift, and vertical translation.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Graph $y = -1 + \frac{1}{2}(\cos 2x - \pi)$. Identify the amplitude, period, phase shift, and vertical translation.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Translations with Sine and Cosine

Graph the following: $y = \frac{5}{2}\sin\left(4\left(x + \frac{\pi}{4}\right)\right)$

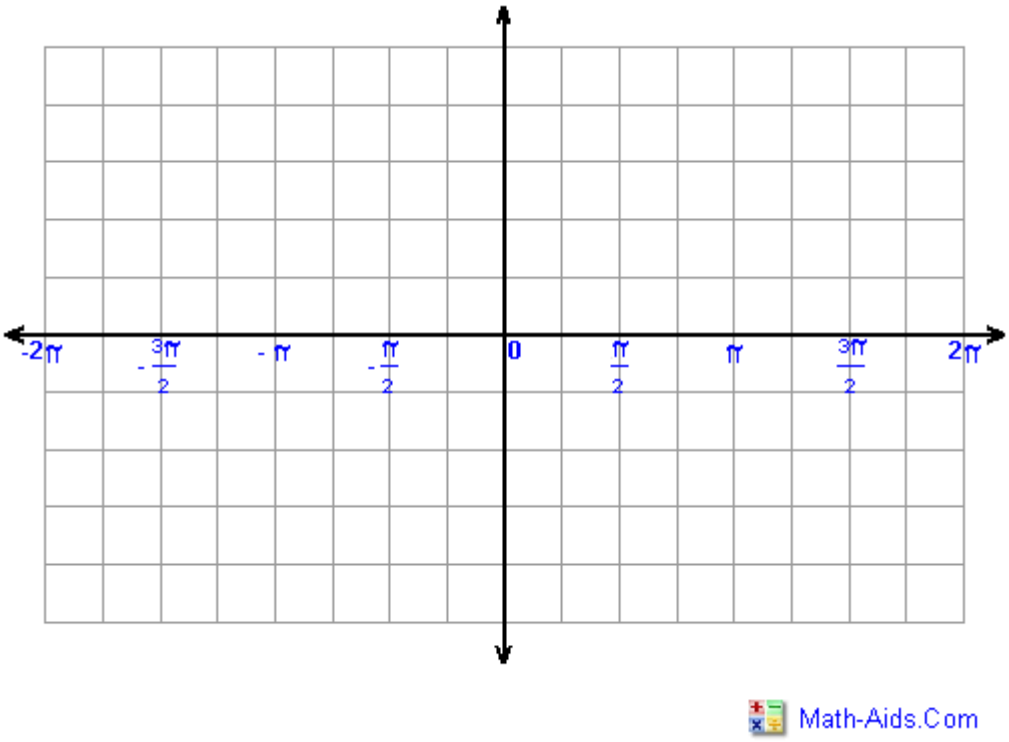
Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Graphs of Tangent and Cotangent

What is an asymptote? Draw a sketch to help explain.

Draw the graph of tangent, filling in the table and then using the graph paper.

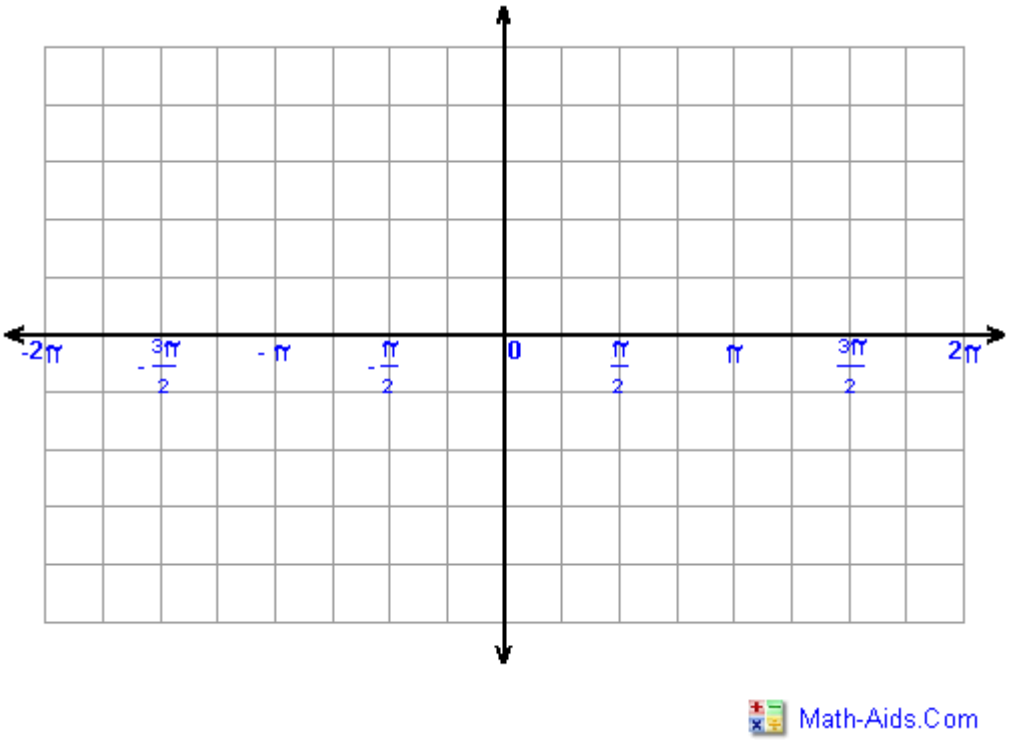
x	$y = \tan x$
$-\frac{\pi}{2}$	
$-\frac{\pi}{4}$	
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{2}$	



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Draw the graph of cotangent, filling in the table and then using the graph paper.

x	$y = \cot x$
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{2}$	
π	
$\frac{5\pi}{4}$	



How would we describe the period for these functions?

Where are the graphs continuous (and discontinuous)?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What is the domain and range of the graphs?

How can we describe the x-intercepts of tangent?

How can we describe the x-intercepts of cotangent?

Are either function even or odd?

How do you interpret amplitude with these functions and why?

What is the key for graphing? (Be specific with the equations you need to leverage)

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Graph over a one-period interval: $y = \tan 2x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Graph over a two-period interval: $y = -\frac{1}{2}\cot 4x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Translations with The Graphs of Tangent and Cotangent

Tangent and cotangent can experience the same types of translations that sine and cosine did (which we discussed in a previous lesson).

<u>Examples</u> Describe the period, phase shift, and vertical translations for each function.

a. $y = 3\tan 4x$

b. $y = -\cot(x + \pi)$

c. $y = 2\tan\left(3x - \frac{\pi}{2}\right)$

d. $y = 3 + \cot(x - 4)$

e. $y = -2 + \tan\left(2\pi x - \frac{\pi}{4}\right)$

f. $y = \frac{1}{2} - \cot(5x + 2)$

<u>Examples</u> Show how to find 2 adjacent asymptotes for graphing each of the following:

a. $y = 3\tan 4x$

b. $y = 2\tan\left(3x - \frac{\pi}{2}\right)$

e. $y = -2 + \tan\left(2\pi x - \frac{\pi}{4}\right)$

f. $y = \frac{1}{2} - \cot(5x + 2)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Graph $y = 3 + \cot\left(x + \frac{\pi}{4}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Graph $y = -1 + 2\tan\,(4x + \pi)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Translations with The Graphs of Tangent and Cotangent

Graph over a two-period interval.

a. $y = \tan\left(3x + \frac{\pi}{4}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

b. $y = -2 + 3\cot(4x - \pi)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

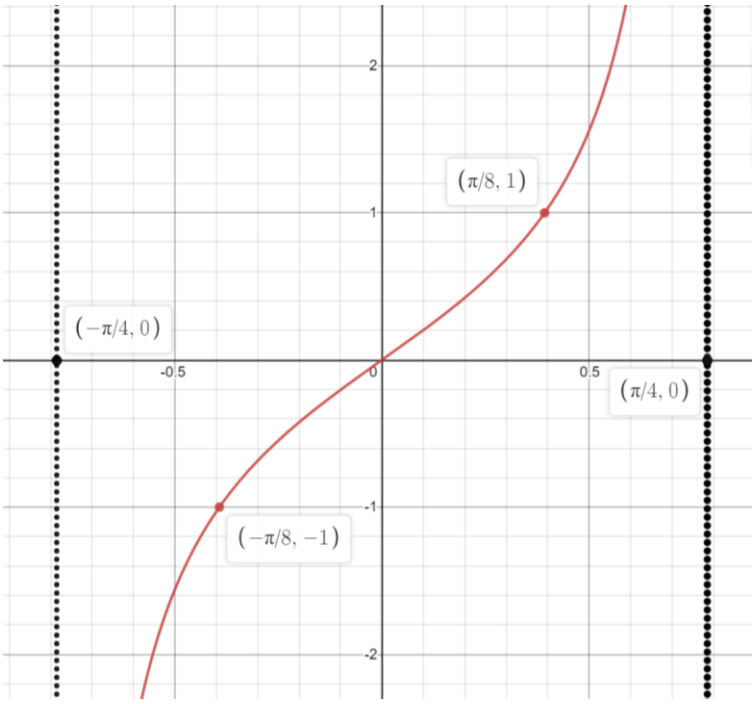
c. $y = 1 - \tan(2\pi x - \pi)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

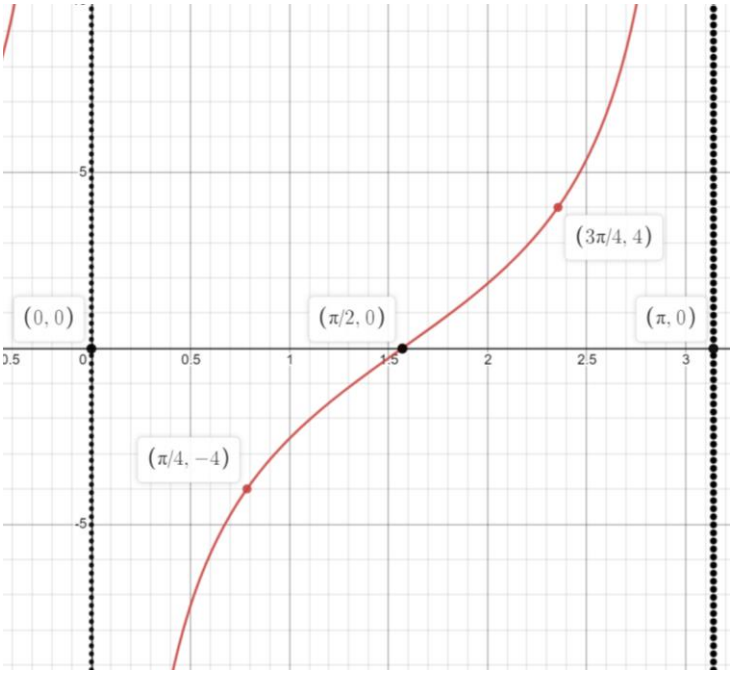
Lesson: Determining the Equations of Tangent and Cotangent from Graphs

Graphs from desmos.com of course!

a.

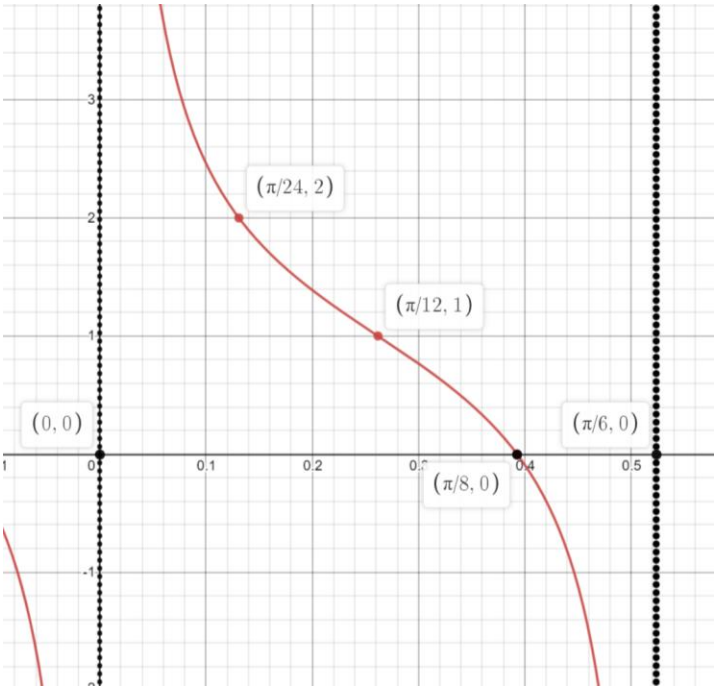


b.



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

c.

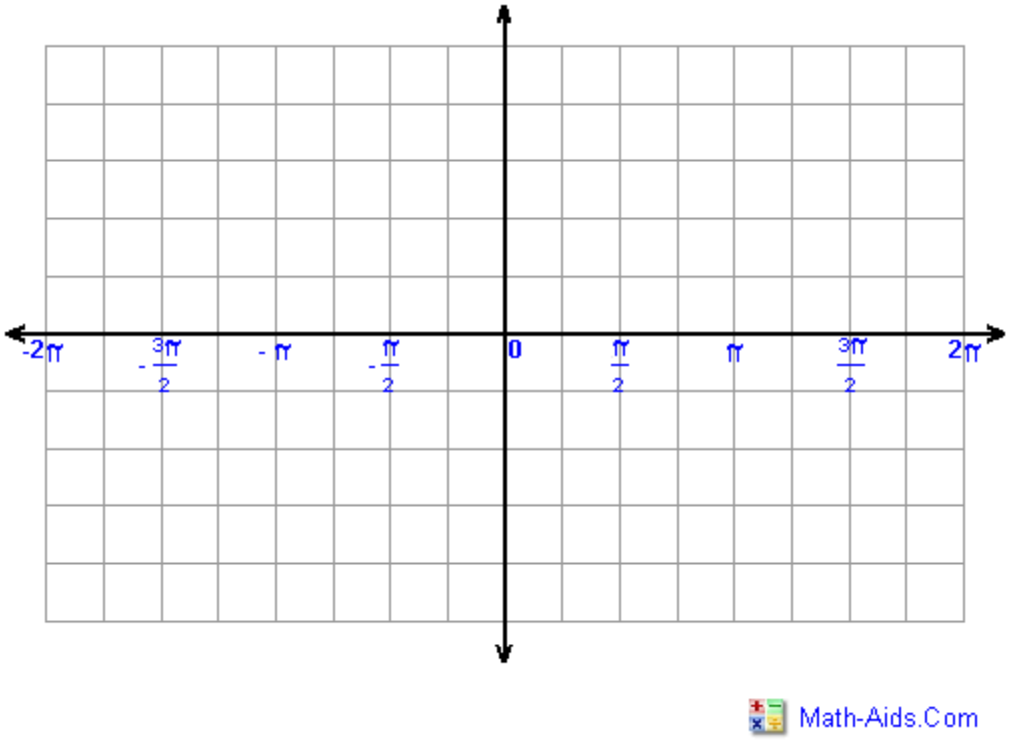


Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Translations of The Graphs of Secant and Cosecant

Draw the graph of secant, filling in the table and then using the graph paper.

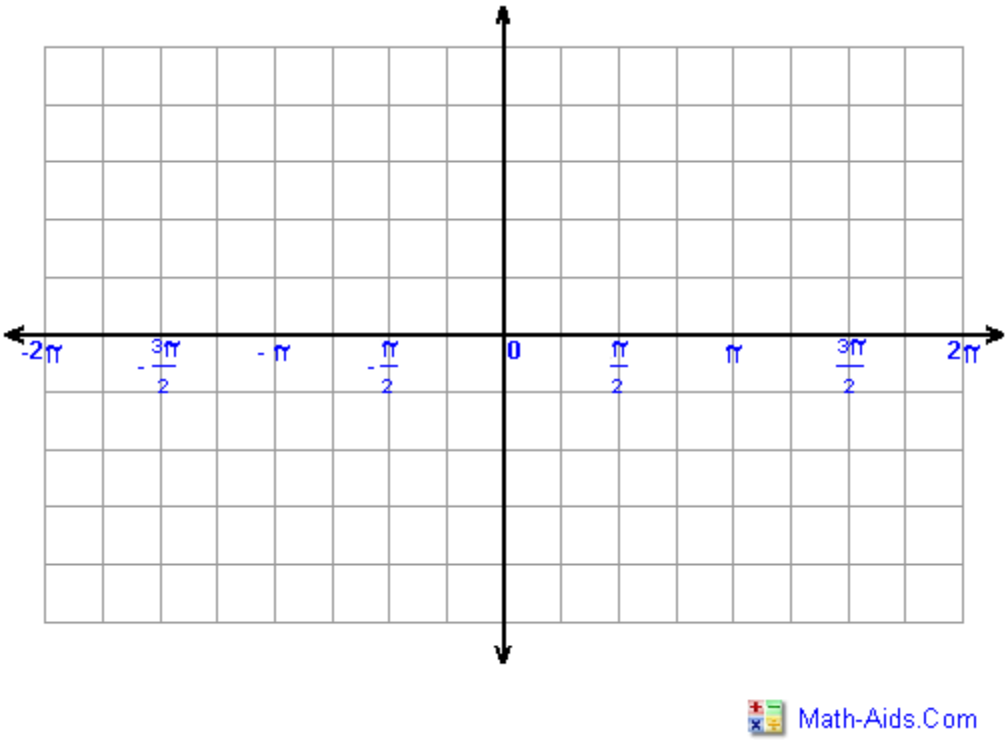
x	$y = \sec x$
$-\frac{\pi}{2}$	
$-\frac{\pi}{4}$	
0	
$\frac{\pi}{4}$	
$\frac{\pi}{2}$	
$\frac{3\pi}{4}$	
π	
$\frac{3\pi}{2}$	



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Draw the graph of cosecant, filling in the table and then using the graph paper.

x	$y = \sec x$
0	
$\frac{\pi}{6}$	
$\frac{\pi}{3}$	
$\frac{\pi}{2}$	
$\frac{2\pi}{3}$	
π	
$\frac{3\pi}{2}$	
2π	



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

How would we describe the period for these functions?

Where are the graphs continuous (and discontinuous)?

What is the domain and range of the graphs?

How can we describe the x-intercepts of both graphs?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Are either function even or odd?

How do you interpret amplitude with these functions?

What is the key for graphing? (Be specific with the equations you need to leverage)

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Graph $y = 2\csc\frac{1}{2}x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Graph $y = 1 + \frac{1}{2}\sec\left(x - \frac{\pi}{2}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Translations of The Graphs of Secant and Cosecant

Graph the following over a one-period interval.

a. $y = -2\sec\left(x + \frac{3\pi}{4}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

b. $y = \frac{1}{2}\csc(2x - \pi)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

c. $y = 2 + \csc\left(\frac{1}{2}x - \pi\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

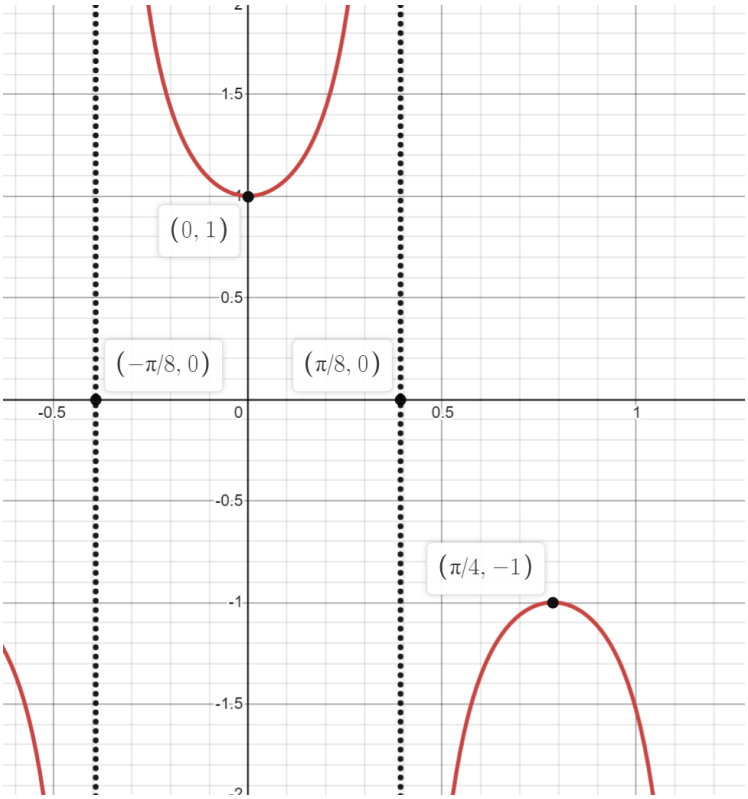
d. $y = 2 - 3\sec\left(2\pi x - \frac{\pi}{2}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

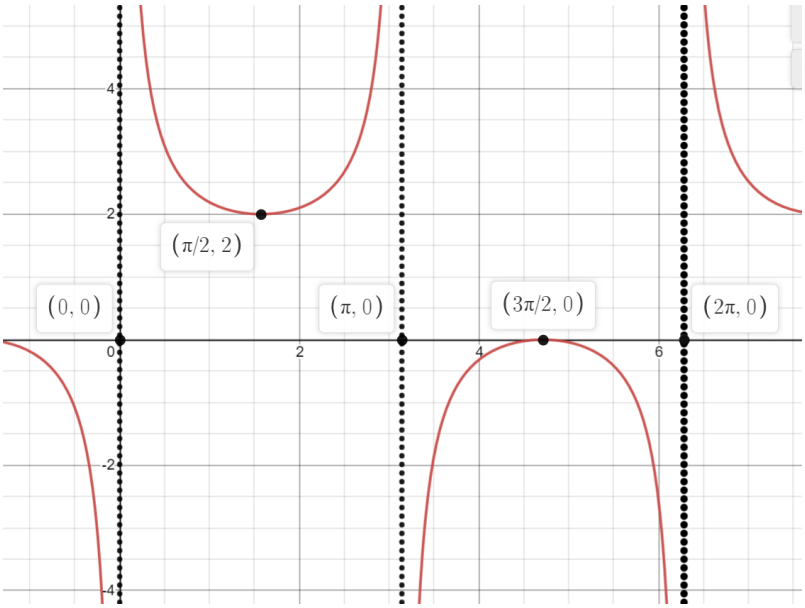
Lesson: Equations of Secant and Cosecant from Graphs

Determine the equation of secant or cosecant from the graphs.

a.

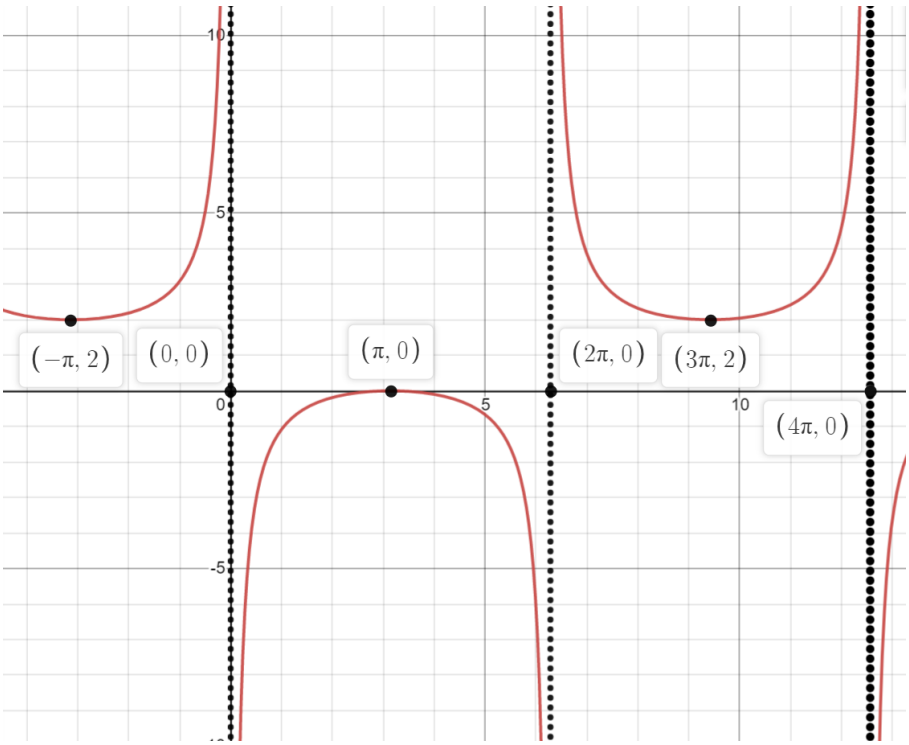


b.



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

c.



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Inverse Functions (a review)

What does it mean to be an "inverse function" ?

<u>Example</u> If a function includes the points $\{(2,3), (5,4), (-7,2), (-1,5)\}$, then what points will the inverse have?

What is the horizontal line test?

<u>Example</u> Draw the graphs from the video, then determine if they have inverses:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

How do you find the inverse of a function $f(x)$?

<u>Example</u> Find the inverse of the following:

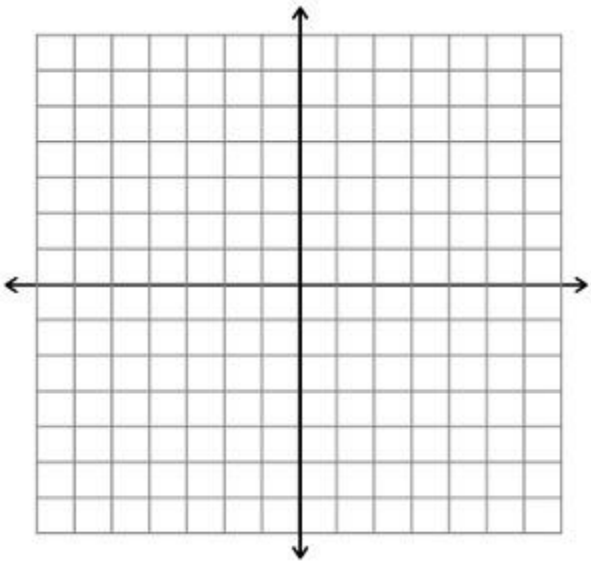
a. $f(x) = 2x + 1$

b. $f(x) = \sqrt{x}$

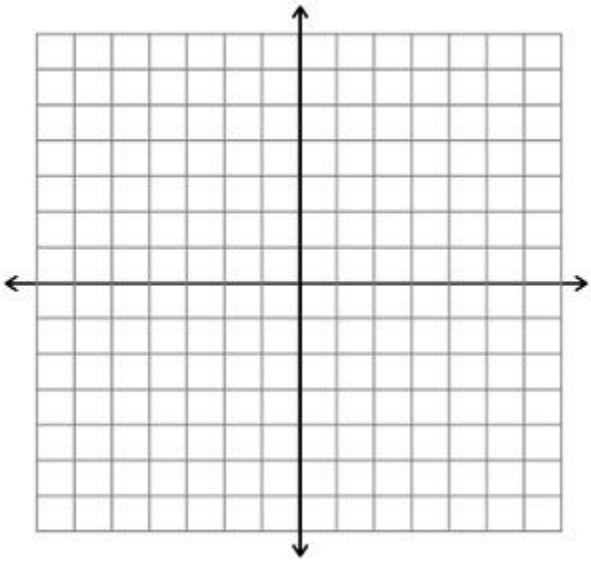
Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Graph the following functions with their inverses:

a. $f(x) = 2x + 1$



b. $f(x) = \sqrt{x}$



From these graphs, we can see that inverses of functions are really just what?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What can we say about the domain and range inverses of functions?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Inverse Trig Functions

Does the sine function have an inverse? Why or why not?

How can we manipulate the graph of the sine function to determine an inverse?

What are two ways that we denote the inverse sine function?

How do we “formally” define the inverse sine function?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Show how to graph the inverse sine function. Draw both the table and graph discussed from the video:

<u>Example</u> Find the value of each of the following:

a. $y = \arcsin\frac{\sqrt{3}}{2}$

b. $f(x) = \sin^{-1}\left(-\frac{\pi}{4}\right)$

c. $y = \sin^{-1} -\sqrt{2}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Does the cosine function have an inverse? How should we think about this?

How do we denote the inverse of cosine?

How do we define the inverse cosine function?

Show how to graph the inverse cosine function. Draw both the table and graph discussed from the video:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Find the value of each of the following:

a. $y = \arccos 1$

b. $f(x) = \cos^{-1}\left(\frac{1}{2}\right)$

How do we denote the inverse of tangent?

How do we define the inverse tangent function?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Show how to graph the inverse tangent function. Draw both the table and graph discussed from the video:

Define the other inverse circular functions:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Summarize the domains, ranges, and quadrants of the inverse trig functions below (this is important, you'll need to study this for the next exam):

Function	Domain	Interval	Quadrants

<u>Examples</u> Find the degree measure of θ:

a. $\theta = \arctan\sqrt{3}$

b. $\theta = \csc^{-1}(-\sqrt{2})$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Finding Values of Inverse Trig Functions

Here we will just examine a wealth of examples of how to find different values of inverse trig functions.

<u>Examples</u> Use a calculator to determine the appropriate values.

a. Find y in radians if $y = \sec^{-1}(-4)$

b. Find θ in degrees if $\theta = \operatorname{arccot}(-0.25431)$

Before you evaluate those, you will have to watch some explanation of how / why we evaluate these the way that we do in the calculator. Write out the explanation of how/why we evaluate $\csc^{-1}(2)$ in a calculator below:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Cheat Sheet – How to Evaluate Inverse Trig Functions That Don't Have a Button on a Calculator

Cheat Sheet (for Comparison) – How to Evaluate Regular Trig Functions That Don't Have a Button on a Calculator

<u>This might get confusing so maybe rip this sheet out and put it someplace where you can easily reference it??</u>

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Use a calculator to find each value.

a. $\sin(\cos^{-1} 0.25)$

b. $\cot\,(\cos^{-1}(0.56331233)$

Examples Evaluate each expression without using a calculator:

a. $\cos\left(sin^{-1}\left(\frac{2}{3}\right)\right)$

b. $\sec\,(\sec^{-1} 2)$

c. $\sin\left(tan^{-1}\frac{4}{3} - cos^{-1}\frac{12}{13}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Write each expression as an algebraic expression in terms of u.

a. $\cot\left(\sec^{-1} u\right)\ , u > 0$

b. $\sec\left(\operatorname{arccot}\left(\frac{\sqrt{4-u^2}}{u}\right)\right.$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Reciprocal Identities of Trig Functions

First, what is a reciprocal? Give an example.

What are the 6 reciprocal identities?

1.

2.

3.

4.

5.

6.

You should NOT write out something like _____ or _____as a shortcut for a reciprocal identities. Why not?

How can you shortcut the reciprocal identity then with notation?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What are some other ways we can relate the reciprocal identities to one another?

<u>Examples</u> Find the function values.

a. $\tan\theta$ given that $\cot\theta = 3$

b. $\sec\theta$ given that $\cos\theta = -\frac{5}{\sqrt{50}}$

c. $\sin\theta$ given that $\csc\theta = 1.25$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Signs and Ranges of Trig Functions

Review: what does r represent when we are talking about trig functions?

Since r is a distance, we know _____.

What determines if a trig function has a positive or negative value?

Draw a picture to help breakdown and analyze the signs of theta:

Now fill in this table to help you remember the sign of each function:

θ IN QUADRANT	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\cot\theta$	$\sec\theta$	$\csc\theta$
I						
II						
III						
IV						

One way to remember the signs is with the phrase All Students Take Calculus. Break down this this is helpful.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Determine the signs of the trig functions of an angle in standard position with the given measure.

a. $23°$

b. $255°$

c. $-30°$

Examples Determine which quadrant of an angle θ that satisfies the given conditions.

a. $\sin\theta > 0, \csc\theta > 0$

b. $\sec\theta > 0, \csc\theta > 0$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Now we will show how to get ranges of the six trig functions.

Draw a picture to demonstrate how angle θ increase in measure from $0° - 90°$.

As the measure of an angle _____, _____ _____ but never exceeds _____. Therefore:

Let's repeat this idea but in quadrant IV. Follow along with the video below and write down the observations again.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What can we conclude from these sketches and observations?

What can we conclude about the values of $\tan\theta$ and $\cot\theta$? And why?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What can we conclude about the values of $\sec\theta$ and $\csc\theta$? Why?

We have now discovered the ranges of all the trig functions. Let's summarize this in the table.

TRIG FUNCTION	RANGE (WRITE IN BOTH SET-BUILDER NOTATION AND INTERVAL NOTATION)
$\sin\theta, \cos\theta$	
$\tan\theta, \cot\theta$	
$\sec\theta, \csc\theta$	

Example Determine if each statement is possible or impossible.

a. $\cot\theta = -0.98$ b. $\cos\theta = -1.25$ c. $\sec\theta = 0.72$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Pythagorean and Quotient Identities

Show how we can build the first Pythagorean identity. These are actually pretty darn important, so you really do need to understand this argument!!

Now let's build another one:

And now let's build the last one:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Summarize the Pythagorean identities:

What is only key about these identities?

Now let's show how to build the quotient identities:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

By the way, similar reasoning holds for $\cot\theta$.

Summarize the Quotient Identities below:

<u>Example</u> Find the value of $\cos\theta$ and $\tan\theta$, give that $\sin\theta = -\frac{\sqrt{2}}{2}$ and $\cos\theta > 0$.

<u>Example</u> Find $\sin\theta$, given that $\cos\theta = \frac{4}{5}$ and θ is in Quadrant IV.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Give all 6 trig function values given that $\cot\theta = \frac{\sqrt{3}}{8}$ and θ is in quadrant I.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Using Pythagorean and Quotient Identities to Find Trig Function Values

Find all 6 trig function values given the information.

a. $\cos\theta = -\frac{3}{5}$, θ is in quadrant III.

b. $\csc\theta = 2, \theta$ is in quadrant II.

c. $\cos\theta = 1$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Fundamental Identities of Trig

Review the test for symmetry (and the results):

We know that the sine function is odd, show what this means:

We know that the cosine function is even, show what this means:

These are called **identities.**

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Using these identities, what other identity can we determine about the tangent function:

These are called the **even-odd identities.**

We have already seen a handful of identities, and we call these the fundamental identities. Summarize them below.

Reciprocal Identities

Quotient Identities

Pythagorean Identities

Even-Odd Identities

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example If $\cos\theta = \frac{3}{4}$ and θ is in quadrant I, find $\sin\theta$.

Example If $\tan\theta = -\frac{1}{4}$ and θ is in quadrant IV, find the values of all the other 5 trig functions.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

We use the identities to work on problems called transformations.

Example Write $\tan x$ in terms of $\sec x$.

Example Write the following in terms of sine and cosine, and then simplify until no quotients appear and all functions are of θ only.

a. $\csc\theta \cos\theta \tan\theta$

b. $\cos\ \theta(\cos\theta - \sec\theta)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Using Fundamental Trig Identities to Find Values of Other Trig Functions

Find the five remaining trig functions given the following information.

a. $\csc\theta = -\frac{5}{2}$, θ in quadrant III.

b. $\sec\theta = \frac{7}{2}$, $\tan\theta < 0$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Writing Trig Functions in Terms of Other Trig Functions (Fundamental Identities)

a. Write $\cot x$ in terms of $\sin x$

b. Write $\csc x$ in terms of $\cos x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Using Fundamental Identities to Write Expressions in Terms of Sine and Cosine

Write each expression in terms of sine and cosine, then simplify so that no quotients appear and all functions are in θ only.

a. $\cos\theta \csc\theta$

b. $\frac{(1+\cot\theta)}{\cot\theta}$

c. $\frac{\sec^2\theta-1}{csc^2\,\theta-1}$

d. $\frac{\csc\theta}{\cot(-\theta)}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Verifying Trigonometric Identities

As you progress through higher mathematics, sometimes you will need to manipulate equations in very creative ways. One way to learn this level of creativity is by verifying trigonometric identities. We will go through a few techniques of how to do this.

<u>Strategy 1: Rewrite the More Complicated Side / Work from One Side</u>

a. Verify that $\frac{1-\sin^2 x}{\cos x} = \cos x$. Show *why.*

b. Verify that $\sin^2 x + \tan^2 x + \cos^2 x = \sec^2 x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Strategy 2: Express the Functions in Terms of All Sine and Cosine

a. $\frac{\tan x}{\sec x} = \sin x$

b. $\sec x\,(\sin x + \cos x) = 1 + \tan x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Strategy 3: Factor or Complete Indicated Algebraic Operations

Examples Factor.

a. $x^2 - 4$

b. $\sin^2 x - 4$

c. $(\tan x + \cot x)^2 - (\tan x - \cot x)^2$

d. $2\sin^2 x + 3\sin x + 1$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Verify the following identities.

a. $\sin^4 x - \cos^4 x = 2\sin^2 x - 1$

b. $\frac{1-\cos x}{1+\cos x} = (\cot x - \csc x)^2$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Strategy 4: Manipulate Both Sides

a. $\frac{\cos x}{\sec x} + \frac{\sin x}{\csc x} = \sec^2 x - \tan^2 x$

b. $\tan^2 x \ \ sin^2 x = tan^2 x + cos^2 x - 1$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Strategy 5: Use the Conjugate

What is the conjugate of the following:

a. $x - 4$

b. $\sqrt{2} - \sqrt{3}$

c. $\sin x + 1$

Verify the following identities: $\frac{1-\cos x}{1+\cos x} = \csc^2 x - 2\csc x \cot x + \cot^2 x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Verifying Trigonometric Identities

a. $\frac{\cos x}{\cot x \sin x} = 1$

b. $\frac{1}{\sec x - \tan x} = \sec x + \tan x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

c. $\frac{1+\sin t}{1-\sin t} - \frac{1-\sin t}{1+\sin t} = 4\tan t \sec t$

d. $(\cos\theta - \sin\theta)(\sec\theta + \csc\theta) = \cot\theta - \tan\theta$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Proofs of the Sum and Difference Identities for Cosine

Write out the proof for $\cos(A - B) = \cos A \cos B + \sin A \sin B$. Do your best to understand the proof and then explain it in a way that you will understand.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Write out the identity for $\cos\,(A - B)$ below (yes I know I stated it above, but YOU should write it so you have a better chance of remembering it):

Write out the proof for $\cos(A + B) = \cos A \cos B - \sin A \sin B$

Write out the identity below (yes, write it out so you don't forget!):

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Using Sum and Difference Identities for Cosine

<u>Examples</u> Show how to use the cosine of a sum or difference to find the exact value of the expressions:

a. $\cos 105°$

b. $\cos \frac{\pi}{12}$

c. $\cos 40° \cos 50° - \sin 40° \sin 50°$

Show how the cofunction identities are an extension of the cosine of a difference:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Write out all the **cofunction identities**

<u>Examples</u> Find one value of θ that satisfies the following:

a. $\sec\theta = \csc 72°$

b. $\tan\theta = \cot(-44°)$

c. $\cos\theta = \sin\frac{5\pi}{6}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Write $\cos(180° - \theta)$ as a trigonometric function of θ alone.

Example Suppose $\sin s = \frac{3}{5}$ and $\sin t = -\frac{12}{13}$, where s is in quadrant I and t is in quadrant III. Find $\cos\,(s + t)$ and $\cos(s - t)$.

Example Verify that $\sec(\pi + x) = -\sec x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Sum and Difference Identities for Cosine

Write out the proof for $\sin(A + B)$

Write out the proof for $\sin(A - B)$

Write out the identities for the sum and difference of sine below:

Write out the proof for $\tan\,(A + B)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Write all identities for the sum and difference of tangent below:

<u>Examples</u> Find the exact value of each expression.

a. $\sin(-15°)$

b. $\tan\left(-\frac{5\pi}{12}\right)$

c. $\sin\left(\frac{5\pi}{9}\right)\cos\left(\frac{\pi}{18}\right) + \cos\left(\frac{5\pi}{9}\right)\sin\left(\frac{\pi}{18}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Write each function as an expression involving functions of θ alone.

a. $\cos(30° + \theta)$

b. $\sin\,(45° + \theta)$

c. $\tan\left(\frac{\pi}{4} + x\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Suppose A and B are angles in standard position such that $\sin A = -\frac{3}{5}\ \frac{3\pi}{2} < A < 2\pi$ and

$\cos B = -\frac{7}{25}$, $\pi < B < \frac{3\pi}{2}$. Find the following.

a. $\sin(A + B)$

b. $\tan(A - B)$

c. the quadrant of A-B

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Verify the identities.

a. $\sin(x+y)+\sin(x-y)=2\sin x\cos y$

b. $\frac{\sin(x-y)}{\sin(x+y)}=\frac{\tan x-\tan y}{\tan x+\tan y}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Sine of a Sum and Tangent of a Sum

Let $\cos s = \frac{5}{12}$, $\sin t = \frac{4}{5}$ where s and t are in quadrant I. Find:

$\sin(s + t)$

$\tan\,(s + t)$

The quadrant of $(s + t)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Double-Angle Identities

Show how we derive the first identity for $\cos 2A$:

Show how we derive the second identity for $\cos 2A$:

Show how we derive the third identity for $\cos 2A$:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Show how we derive the identity for $\sin 2A$:

Show how we derive the identity for $\tan 2A$:

Summarize the Double-Angle Identities Below:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Give $\sin\theta = \frac{5}{8}$ and $\cos\theta < 0$, find $\sin 2\theta$, $\cos 2\theta$, and $\tan 2\theta$.

Example Find the values of the six trig functions give $\cos 2\theta = -\frac{11}{13}$ and $180° < \theta < 270°$.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Verify the following identities:

a. $(\sin x + \cos x)^2 = \sin 2x + 1$

b. $\frac{\cot x - \tan x}{\cot x + \tan x} = \cos 2x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Simplify the expressions.

a. $\sin^2 15° - \cos^2 15°$

b. $\sin^2 \frac{3\pi}{5} - \cos^2 \frac{3\pi}{5}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Write $\cos 3x$ in terms of $\cos x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Product-to-Sum Identities

Show what happens when we add the identities $\cos(A+B)$ to $\cos(A-B)$

Now show what happens when we subtract them:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Write the following as the sum or difference of 2 trig functions.

a. $2\sin 48° \cos 104°$

b. $8\sin 9x \sin 5x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Write the expression as a product of trigonometric functions.

a. $\cos 6x + \cos 9x$

b. $\sin 106° - \sin 96°$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Half-Angle Identities

Show how we derive the identity for $\sin\frac{A}{2}$:

Show how we derive the identity for $\cos\frac{A}{2}$:

Show the first way we can derive an identity for $\tan\frac{A}{2}$:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Show the second way we can derive an identity for $\tan\frac{A}{2}$:

Show the third way we can derive an identity for $\tan\frac{A}{2}$:

Summarize the half-angle identities below:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Show how to use the half-angle identity to evaluate: $\sin 12.5°$

Example Find the exact value of $\cos 165°$

Example Find the exact value of $\tan 75°$ using the identity $\tan A = \frac{\sin A}{1+\cos A}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Given $\cos s = \frac{2}{7}$ with $0 < s < \frac{\pi}{2}$, find $\sin\frac{s}{2}$, $\cos\frac{s}{2}$, and $\tan\frac{s}{2}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Simplify each expression.

a. $\sqrt{\frac{1-\cos 40°}{2}}$

b. $\frac{1-\cos 64.72°}{\sin(64.72°)}$

c. $\pm\sqrt{\frac{1+\cos 10x}{2}}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Verify the identities.

a. $\cot^2 \frac{x}{2} = \frac{(1+\cos x)^2}{\sin^2 x}$

b. $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Summarizing All of the Identities

Using all the trig identities that we've discussed, verify the following:

a. $\frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

b. $1 - \tan^2 \frac{x}{2} = \frac{2\cos x}{1+\cos x}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

c. $\tan\left(\frac{a}{2}+\frac{\pi}{4}\right)=\sec a+\tan a$

d. $\sin 7x+\sin 5x+\sin 3x+\sin x=4\sin 4x\cos 2x\cos x$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

e. $\tan 4x = \frac{2\tan 2x}{2-\sec^2 2x}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Trig Equations, part 1 – linear methods

Recall how linear methods work first by solving: $2x + 4 = 8$

Now compare this approach with the equation: $3\tan\theta - \sqrt{3} = 0$, over the interval $[0°, 360°)$

How does this answer change if we want to solve for *all* solutions?

<u>Example</u> Solve: $2\sin x + 3 = 4$ over $[0, 2\pi)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Trig Equations, part 2 – quadratic methods

Using quadratic methods to solve trig equations relies on using the zero-factor property. Show how the zero-factor property works by solving $(x+5)(x-3)=0$.

Compare this approach by solving $\cos\theta\sin\theta = 0 = -\cos\theta$, over $[0°, 360°)$.

<u>Example</u> Solve $(\csc x + 2)\left(\csc x - \sqrt{2}\right) = 0$ for all solutions.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Recall how to use quadratic methods by solving $x^2 + 5x + 6 = 0$.

Compare this approach by solving $\cos^2 x + 2\cos x + 1 = 0$ over $[0°, 360°)$.

<u>Example</u> Solve $2\cos^2 x + \cos x - 1 = 0$ for all possible solutions.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Solve $2\cos^2 x - \sqrt{3}\cos x = 0$ over $[0, 2\pi)$

Example Solve $\tan x \csc x - \sqrt{3}\csc x = 0$ for all solutions.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Solve $\csc^2 x - 2\cot x = 0$ over $[0°, 360°)$

Example Solve $\tan x - \sqrt{3} = \sqrt{3}\cot x - 1$ for all possible solutions.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Trig Equations, part 3 – equations with half-angles

<u>Example</u> Solve $2\cos\frac{x}{2} - \sqrt{2} = 0$ over the interval $[0, 2\pi)$

How would the answers change if you needed to find all solutions?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Solve $2\sqrt{3}\sin\frac{x}{2} = 3$ for all possible solutions.

Example Solve $\sin^2\frac{x}{2} - 2 = 0$ for all possible solutions.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Trig Equations, part 4 – equations with multiple angles

<u>Example</u> Solve $\cos 2x = \sin x$ over $[0, 2\pi)$

<u>Example</u> Solve $2\cos^2 x - 2\sin^2 x + 1 = 0$ over $[0°, 360°)$

How would this change for all solutions?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Solve $\cot 2x - \csc 2x = 1$ over $[0, 2\pi)$

Example Solve $\tan 2x - \sec 2x = 2$ over $[0, 2\pi)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Solve $\cos x - 1 = \cos 2x$ for all solutions.

Example Solve $\sin 2x = 2\cos^2 x$ for all solutions.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Equations Involving Inverse Trig Functions

<u>Example</u> Solve $y = 4\tan 3x$, where x is restricted to $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$.

<u>Example</u> Solve $y = \tan\,(2x - 1)$ for x in $\left(\frac{1}{2} - \frac{\pi}{4}, \frac{1}{2} + \frac{\pi}{4}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Solve $3\tan^{-1} x = \pi$

Example Solve $6\arccos\left(x - \frac{\pi}{3}\right) = \pi$

Example Solve $\cot^{-1} x = \tan^{-1}\frac{4}{3}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Solve $\arccos x + 2\arcsin\frac{\sqrt{3}}{2} = \frac{\pi}{3}$

Example Solve $\sin^{-1} x + \tan^{-1} x = 0$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: How to Approximate Trigonometric Function Values

We will discuss using a calculator to do approximations of trig functions.

If you are going to use a calculator, what is key?

<u>Examples</u> Use a calculator to evaluate the following expressions.

a. $\cos 32°15'$

b. $\sec 55.321°$

c. $\sin(-105°)$

d. $\frac{1}{\csc(233.45°)}$

To find the value of θ (certain angle measures), we can use inverse trig functions. What are the three inverse trig functions that a calculator will typically have?

What do these functions means?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Show how to use completely proper notation to find the angle θ in the interval $[0°, 90°)$.

a. $\cos\theta = 0.931145$

b. $\cot\theta = 1.7538932$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Solving for Theta with a Calculator

Find all the values of θ in $[0, 360°)$ where $\tan\theta = 0.9845331$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Significant Digits and Solving Triangles

What is an exact number?

We've seen that many values for trig functions are not exact. Consider this: a room is 12ft x 18ft. We are doing a project where we need to know the length of the diagonal. What is it?

What's the issue with reporting this number?

What do we call the number of digits obtained by actual measurement?

Show some examples of significant digits:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Significant Digits for Angles

Angle Measure to Nearest	Number of Significant Digits Used	Write an Example

How do you perform calculations in this case?

What should you keep in mind about the answer in this case?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

We can practice significant digits by solving triangles.

Example Solve the right triangle presented in the video.

Example Solve the right triangle presented in the video.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples: Significant Digits and Rounding

How many significant digits are there in each?

a. 421	e. 400.01
b. 420	f. 400.00
c. 400	g. $72°31'$
d. 400.	h. $72°30'$

Your answer needs to be 2 significant digits. You calculate the answer 421. What should your final answer be for significant digits?

Your answer needs to be 4 significant digits. You calculate the answer 421. What should your final answer be for significant digits?

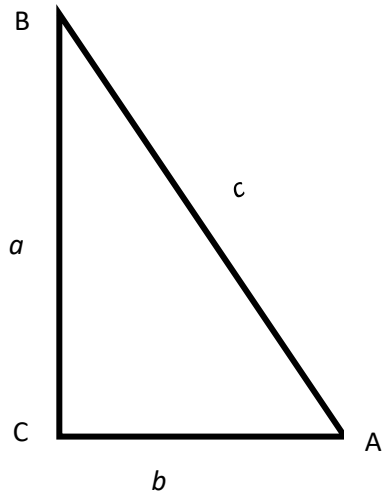
Your answer needs to be 3 significant digits. You calculate the answer $42°18'$. What should your final answer be for significant digits?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

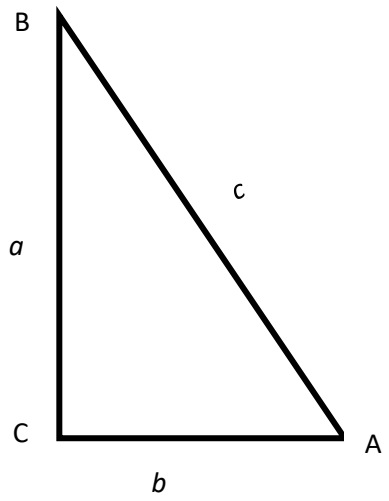
Examples: Solving Triangles

Find all missing lengths and angle measures for the given triangles. Use significant figures.

a. $A = 28.3°, c = 18.5$



b. $b = 85m, c = 112m$



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Oblique Triangles and the Law of Sines

First, we should review some facts about congruent triangles. When are 2 triangles congruent?

<u>Congruence Axioms</u>

Side-Angle-Side (SAS)

Angle-Side-Angle (ASA)

Side-Side-Side (SSS)

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

When does Side-Angle-Angle (SAA) hold?

When a triangle is <u>not</u> a right triangle, what is it it called?

When can a triangle be solved?

What data is required for solving oblique triangles?

Case 1:

Case 2:

Case 3:

Case 4:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Is there an Angle-Angle-Angle congruency?

When Case 1 or Case 2 is involved, what is required?

When Case 3 or Case 4 is involved, what is required?

This Law of Cosines will be discussed in another video.

Show how we derive the Law of Sines:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

State the Law of Sines:

What is an alternative form of the law of sines?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Solve Triangle ABC if $A = 28.5°, C = 102.3°, c = 25.1$ in.

Recall the formula for the area of a triangle:

Using SAS, how else can you find the area of a triangle?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Reference the sketch in the video. Find the area of the triangle.

Example Find the area of a triangle DEF if $D = 58°10', e = 31.2in, F = 73°30'$.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Applications of the Law of Sines

<u>Example</u> An engineer wishes to measure the distance across a river. Reference the sketch in the video.
If $C = 117.4°, A = 30.0°, b = 75.8$ ft

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Review the two methods of bearing:

<u>Example</u> The bearing of a lighthouse from a ship was found to be $N\ 52°$ W. The ship sailed 12.3 miles south, and the new bearing was $N\ 23°W$. Find the distance, to the nearest mile, between the ship and the lighthouse at each location.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example A balloonist is directly above a straight road, 2 miles long that joins the 2 villages. The closer town to him is an angle of depression of $45°$, and the farther town is at angle of depression of $41°$. How high above the ground is the balloon?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: The Ambiguous Case of the Law of Sines

Describe the ambiguous case of the law of sines as describe in the video. There's going to be quite a bit of information presented – watch the entire explanation of why we need the law of sines and do your best to describe the various cases to sum up. Summarize the explanation so that it makes sense to YOU.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What were some tips for applying the law of sines?

<u>Example</u> Solve triangle ABC if $a = 16.2cm, c = 14.1\ cm$, and $C = 72°30'$.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Solve triangle DEF if $D = 62°8', a = 34.5in, b = 38.1in.$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Explain the number of triangles satisfying the ambiguous case (SSA):

<u>Examples</u> Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

a. $B = 80°, C = 35°, b = 27$

b. $A = 60°, B = 70°, c = 5$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

c. $a = 15, b = 24, B = 33°$

d. $A = 25°, b = 8, a = 11$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Outline the steps to solve these triangles. You don't have to solve completely, but do enough work to determine how many possible triangles there are. Include sketches.

e. $A = 61.4°, b = 35.5, b = 39.1$

f. $a = 17.1, c = 13.1, C = 76°$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Without using the law of sines, explain why no triangle ABC exists satisfying $B = 95°, b = 40cm, c = 46cm$.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: The Law of Cosines

What is the property of triangle side length restriction?

Show how to derive the law of cosines:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

State the Law of Cosines:

Where is one place where we've seen this?

<u>Example</u> Two boats leave a harbor at the same time, traveling on courses that make an angle of $81°30'$ between them. When the slower boat has traveled 65.2 miles, the faster boat has traveled 84.3 miles. At that time, what is the distance between the boats?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Solve triangle ABC if $B = 72.4°, a = 31.4ft, c = 44.3ft$.

Example Solve triangle DEF if $d = 24.2in, b = 42.6in, c = 59.1in$.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example The simplest type of roof truss is a triangle. Refer to the sketch in the video. Find angle C to the nearest degree for the truss shown.

Explain the 4 Cases of Solving Oblique Triangles

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example: The Law of Cosines with SSS Triangles

Find the angles of the triangles given the following sides:

a. $a = 3ft, b = 5ft, c = 6ft$

b. $a = 324m, b = 421m, c = 298m$

Copyright 2024, Dr. Jennifer Hill, DivideandConquerMath.com

Lesson: Heron's Formula for the Area of a Triangle

Explain how to use Heron's formula (as explained in the video):

<u>Example</u> Find the area of triangle ABC where $a = 12m, b = 16m, c = 25m$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Find the area of the Bermuda Triangle if the sides of the triangle have approximate lengths 800mi, 875mi, and 1250mi.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

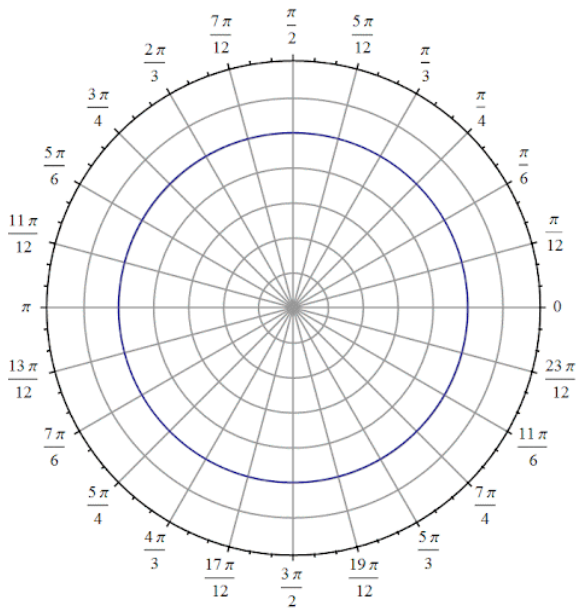
Lesson: Polar Equations and Graphs

There are other coordinate systems besides the rectangular coordinate system.

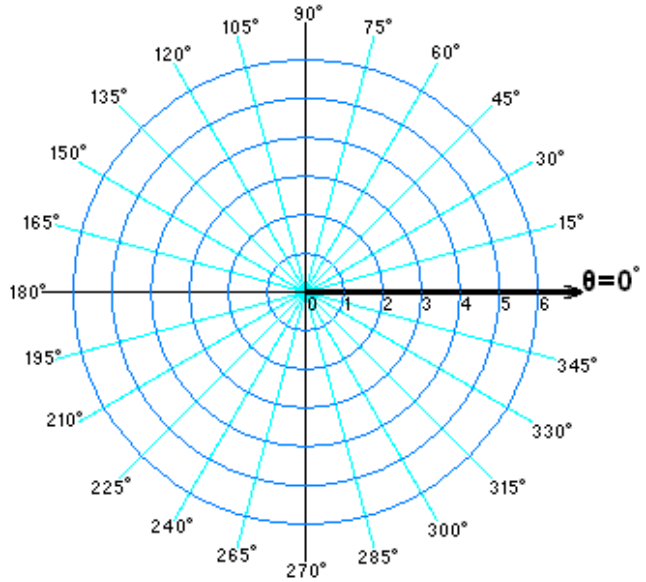
We will discuss the polar coordinate system!

What is a pole? What is the polar axis? Draw a sketch.

Here is what the polar coordinate system looks like:



It can also be in degrees:

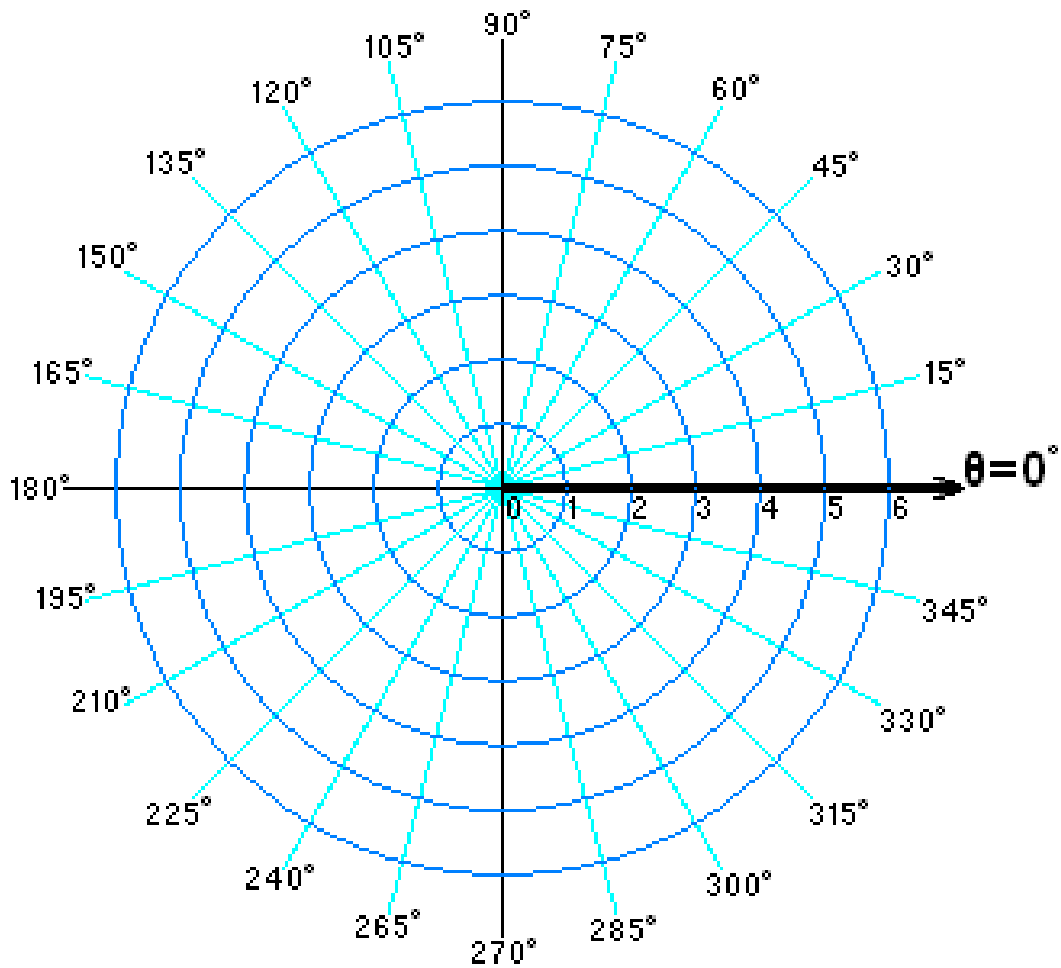


Copyright 2021, Jennifer Hill, DivideandConquerMath.com

How do we convert between rectangular and polar coordinates?

<u>Examples</u> Show how to plot each point, and then determine the rectangular coordinates of each point.

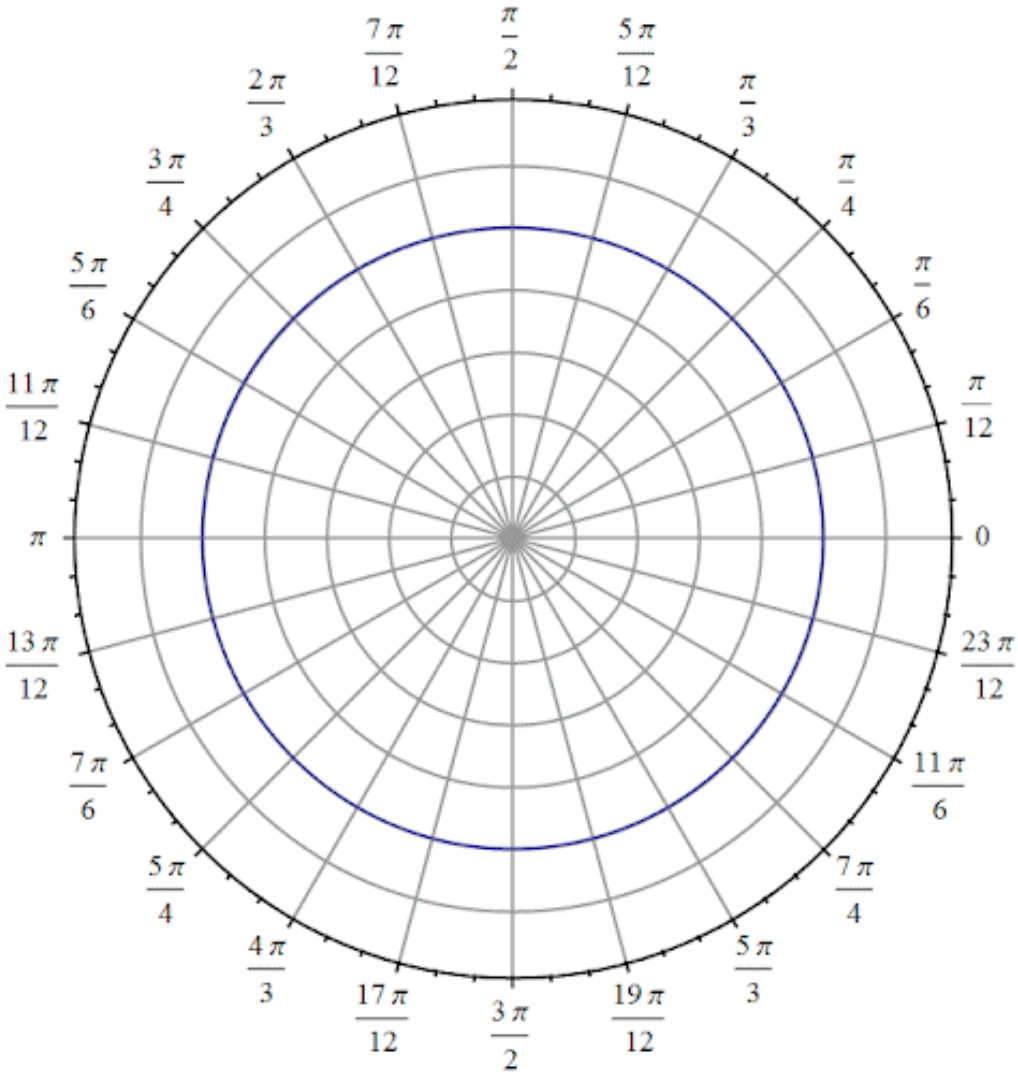
a. $P(3, 45°)$



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Show how to plot each point, and then determine the rectangular coordinates of each point.

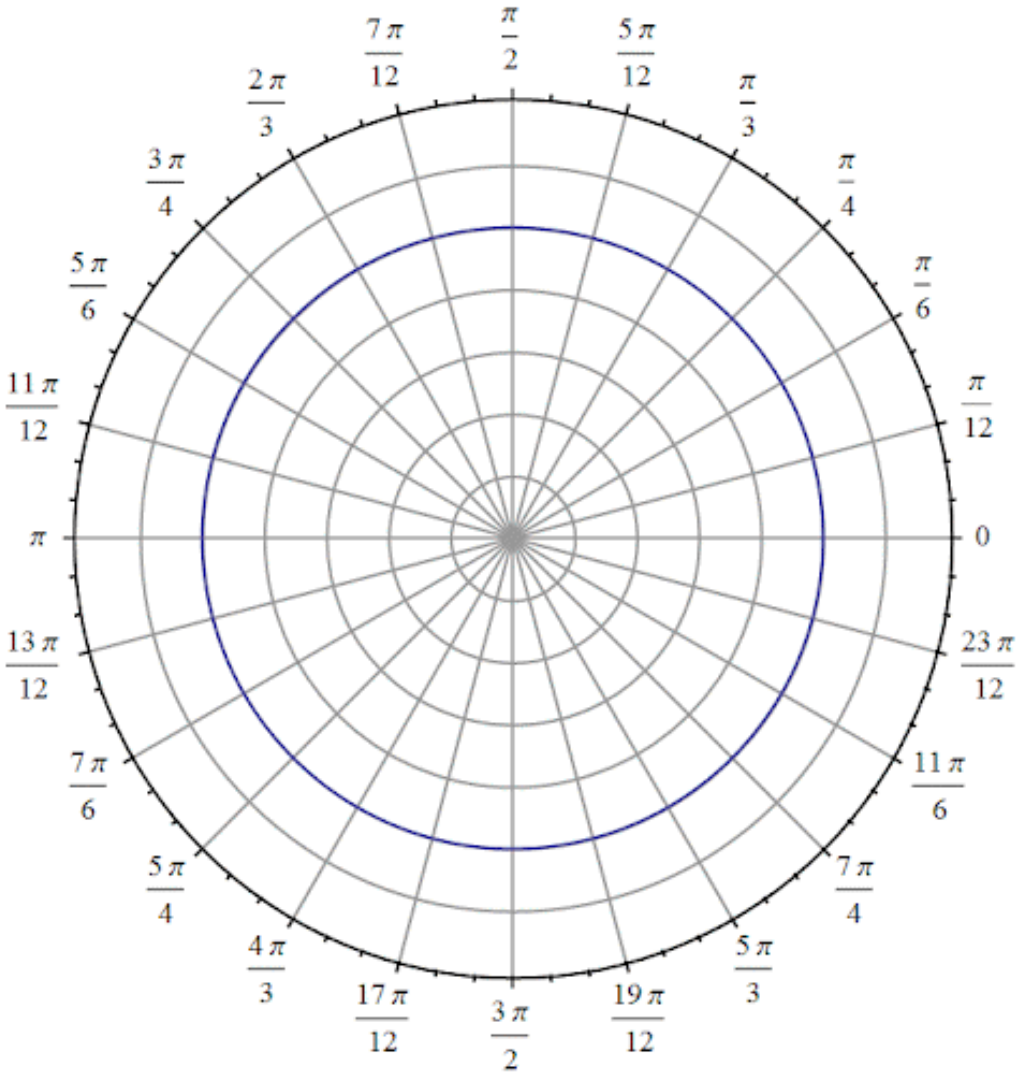
b. $P\left(-4, \frac{\pi}{3}\right)$



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples Show how to plot each point, and then determine the rectangular coordinates of each point.

c. $R\left(3, -\frac{\pi}{2}\right)$



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

While a given point in the plane can only have 1 pair of rectangular coordinates, how many pairs of polar coordinates can it have?

<u>Examples</u> Determine the following:

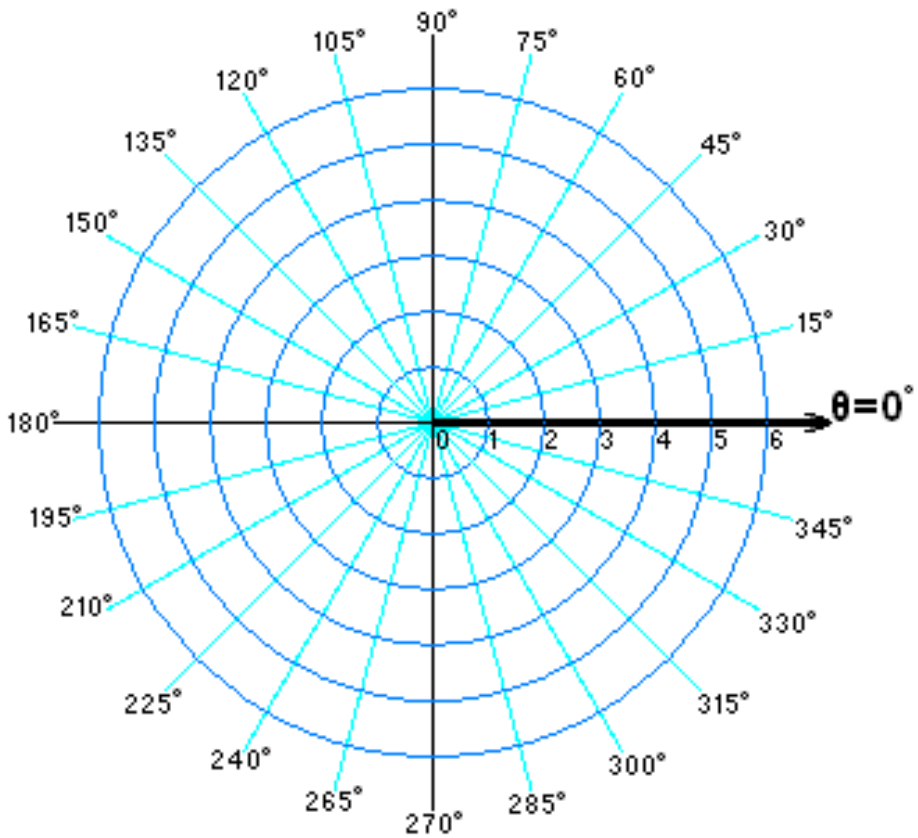
a. Three other pairs of coordinates for the point $P(3, -110°)$.

b. Two pairs of polar coordinates for the point with rectangular coordinates $\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

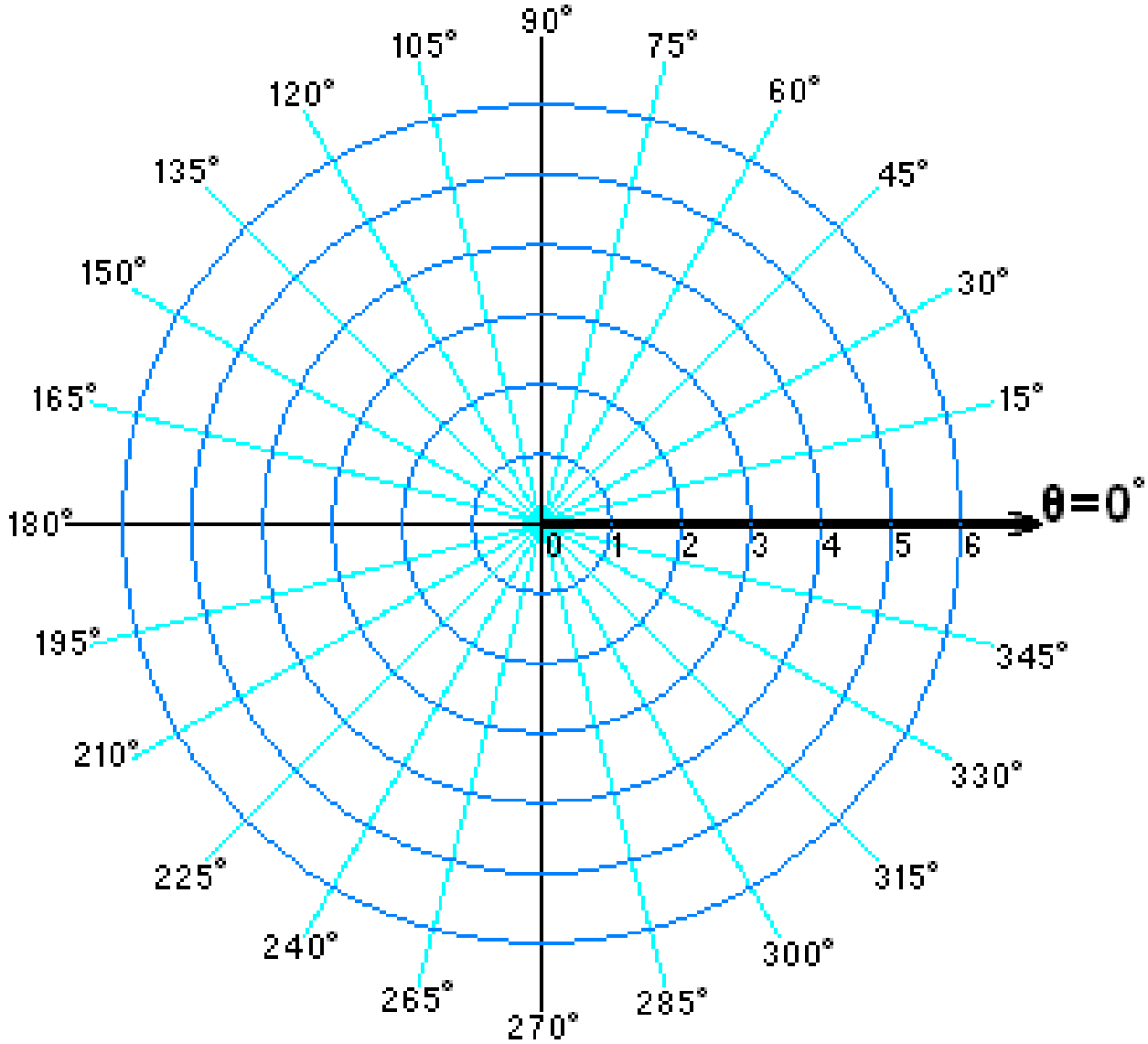
What makes an equation a “polar equation” ?

<u>Example</u> Convert the equation to a polar equation, and then graph it: $y = x - 1$



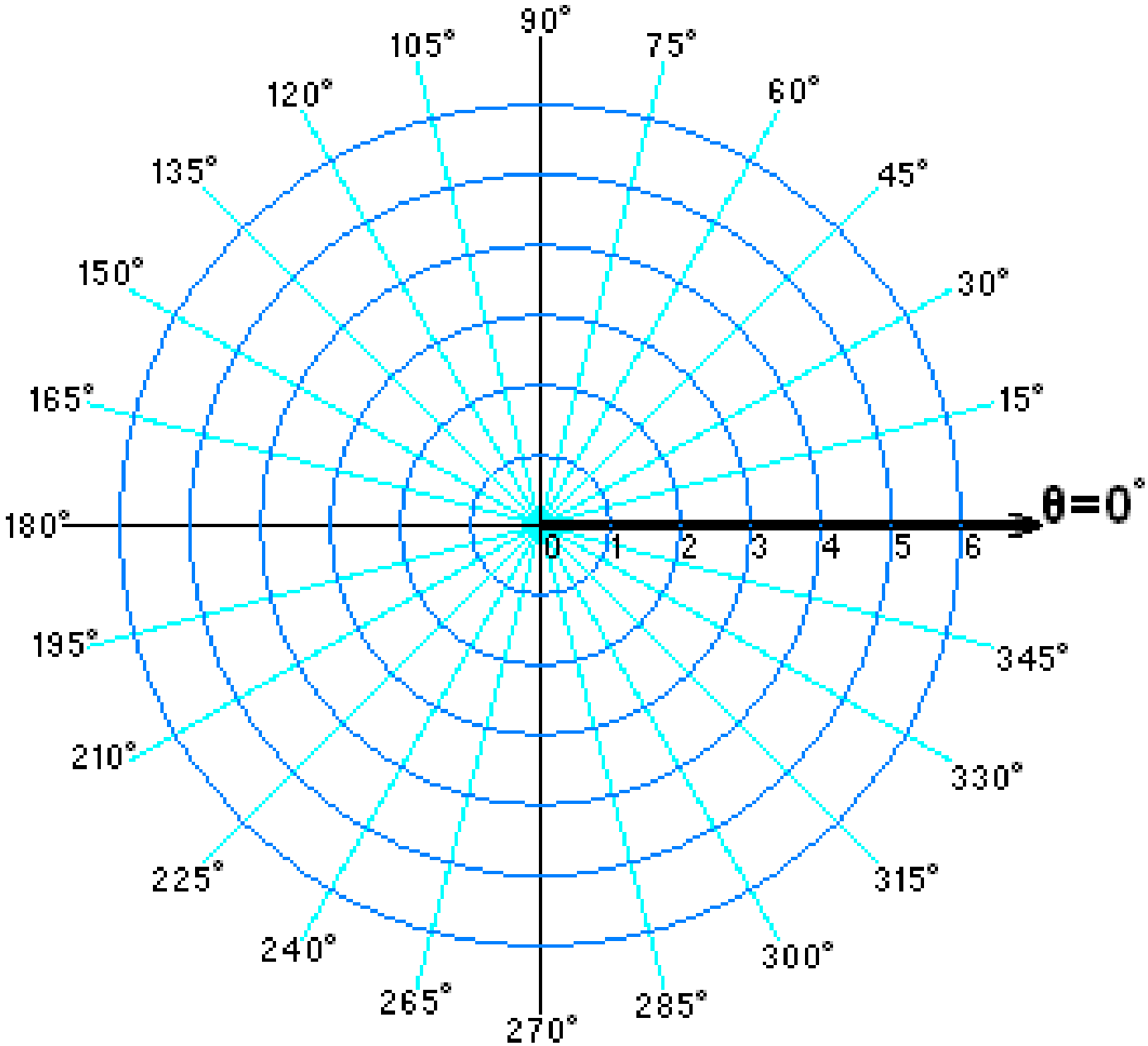
Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Convert the equation to a polar equation, and then graph it: $x^2 + y^2 = 25$



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Graph $r = 1 - \sin\theta$



Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Trigonometric (Polar) From of Complex Numbers

Complex numbers cannot be ordered like real numbers can. What is one way that we can organize complex numbers?

Show how we can graph complex numbers:

Complex numbers correspond to vectors. For a complex number $x + yi$, show how this corresponds to a vector (sketch), then label the sketch.

<u>Examples</u> Add $3 + 2i$ and $2 - 5i$, then graph both numbers and their resultant.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Define the relationships between x, y, r, θ below (with a sketch)

What is the trigonometric form (or polar form) of a complex number?

How do we refer to the number r?

How do we refer to θ?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Write $3(\cos 300° + i \sin 300°)$ in rectangular form.

How do you convert from rectangular from to trigonometric form?

Examples Write in trigonometric form.

a. $-\sqrt{2} + i$

b. -12

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Write the complex number in its alternative form.

a. $8(\cos 200° + i \sin 200°)$

b. $-8 + 5i$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: The Product and Quotient Theorems

First, let's review how to multiply $\left(2+i\sqrt{3}\right)(-2\sqrt{3}+2i)$

Let's rewrite both of those complex numbers in their trigonometric form:

Now multiply their trigonometric forms together:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What do you notice about the result?

This problem demonstrates the product theorem.

State the Product Theorem:

<u>Example</u> Find the product of $3(\cos 45° + i \sin 45°)$ and $5(\cos 120° + i \sin 120°)$. Then write the answer in rectangular form.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

(PART A) Show how to find the quotient of $(2\sqrt{6} - 2i\sqrt{2})$ and $\left(\sqrt{2} - i\sqrt{6}\right)$

Now rewrite these complex numbers in their trigonometric form.

How can the "answer" to PART A be rewritten in trigonometric form?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What do you notice about this result?

This demonstrates the quotient theorem.

State the Quotient Theorem.

<u>Example</u> Find the quotient $\frac{35\ cis\ 45°}{7\ cis\ (-90°)}$. Write the result in rectangular form.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Use a calculator to find the following. Put your answer in rectangular form.

a. $(7.2\ cis\ 38.5°)(3.5\ cis\ 49.8°)$

b. $\dfrac{15.42\ (\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6})}{5.21\ (\cos\frac{\pi}{4}+i\sin\frac{\pi}{4})}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: De Moivre's Theorem and Powers and Roots of Complex Numbers

Find the product: $[r\ (\cos\theta + i\sin\theta)]^2$

This is one example of De Moivre's theorem.

State De Moivre's Theorem:

<u>Example</u> Find $\left(\sqrt{3} + i\sqrt{5}\right)^8$ and write the answer in rectangular form.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

We can extend this theorem to find all nth roots of complex numbers.

Explain the concept of finding the nth root of a complex number.

State the nth Root Theorem

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Find the three cube roots of -8

Example Find all complex number solutions to $x^4 + 1 = 0$.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Vectors

What is a scalar?

What are vector quantities?

What is a vector?

Draw some vectors showing the differences with magnitude and directions.

Explain some of the notation we use regarding these concepts:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

When are two vectors equal?

Is the sum of two vectors still a vector? (circle) Yes No

How do you find the sum of two vectors?

Vector addition is commutative. What does this mean?

For every vector, there is an opposite vector. Explain what this means:

The sum of **v** and **-v** results in what?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What happens when you multiply a vector and a scalar?

Explain the geometric properties of parallelograms:

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Examples For each pair of vectors and the angle θ between them, sketch the result.

a. $|\boldsymbol{u}| = 12, |\boldsymbol{v}| = 22, \theta = 25°$

b. $|\boldsymbol{u}| = 20, |\boldsymbol{v}| = 40, \theta = 30°$

Examples Use the parallelogram in the video to find the magnitude of the resultant force for the 2 forces shown in the figure in the video.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Vectors and Applications of Magnitude

Example A boat is being towed by two other boats, with the first boat having a force of 840lbs and the second have 925lb. The angle between these forces is 26°. Find the direction and magnitude of the equilibrant.

Example Two forces of 150 lbs and 275 lbs act at a point. The resultant force between them is 350 lbs. Find the angle between the forces.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Vectors and Applications of Navigation

<u>Example</u> A ship leaves port on a bearing of $27°$ and travels 62.4miles. The ship then turns due east and travels 85.2 miles. How far is the ship from the port? What is its bearing from port?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Vectors and Applications of Incline

<u>Example</u> Find the force required to keep 2700-lb car parked on a hill that makes a $13°$ angle with the horizontal.

<u>Example</u> A force of 18.00 lb is required to hold a 74.0 lb crate on a ramp. What angle does the ramp make with the horizontal?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: Vectors (Algebraic Interpretation)

Vectors are defined algebraically in the rectangular coordinate system with what notation?

Draw how we interpret this vector below.

In a previous video, we discussed how to define vectors geometrically. How is this definition different from our algebraic interpretation?

Show how to define a direction angle.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

How do we calculator the magnitude of a vector?

How do we calculation the direction angle of the same vector $\boldsymbol{u} = \langle a, b \rangle$?

<u>Example</u> Calculate the magnitude and direction for $\boldsymbol{u} = \langle 5, 2 \rangle$.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Draw how we can view the horizontal and vertical components of vectors.

Example A vector has magnitude 36.0 and direction angle $35°$. Find the horizontal and vertical components.

Example Copy the sketch in the video. Write the vectors in the form $\langle a, b \rangle$.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What are the vector operations?

<u>Example</u> Let $\boldsymbol{u} = \langle -3, 4 \rangle, \boldsymbol{v} = \langle 5, 7 \rangle$. Find and illustrate each of the following.

a. $\boldsymbol{u} + \boldsymbol{v}$

b. $-3\boldsymbol{v} + 2\boldsymbol{u}$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

What is the i,j form of vectors?

<u>Example</u> Write $\langle 3, 5 \rangle$ in i,j form

<u>Example</u> Write $2\boldsymbol{i} - 4\boldsymbol{j}$ in bracket vector form.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: The Dot Product

What is the dot product?

What is another name for the dot product?

Which is true: the dot product results in a vector the dot product results in a real number

<u>Example</u> Find the dot products.

a. $\langle 3, 5\rangle \cdot \langle 2, -1\rangle$

b. $\langle 2, -3\rangle \cdot \langle -4, -5\rangle$

What are the properties of the dot product?

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Let $\boldsymbol{u} = \langle 1, 3 \rangle, \boldsymbol{v} = \langle 2, 4 \rangle, \boldsymbol{w} = \langle -1, 2 \rangle$. Evaluate the following.

a. $\boldsymbol{u} \cdot \boldsymbol{v} - \boldsymbol{u} \cdot \boldsymbol{w}$

b. $(3\boldsymbol{u}) \cdot \boldsymbol{w}$

What are the possible outcomes of the dot product?

What is the geometric interpretation of the dot product? Draw a sketch.

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Example Find the angle between the vectors.

a. $\boldsymbol{u} = \langle 1, 3 \rangle, \boldsymbol{v} = \langle 2, 4 \rangle$

b. $\boldsymbol{u} = \langle 1, -2 \rangle, \boldsymbol{v} = \langle -1, -3 \rangle$

Explain how to interpret the dot product in the following scenarios …

The Dot Product is …	The Angle between the Vectors is …
Positive	
Negative	
Zero	

When the dot product is 0, what does this mean?

What's the better word for perpendicular? (fancier, makes you sound smart, you should definitely use it to impress people at parties):

Example Determine if the pair of vectors is orthogonal: $\boldsymbol{u} = \langle 1, 2 \rangle, \boldsymbol{v} = \langle -6, 3 \rangle$

Copyright 2021, Jennifer Hill, DivideandConquerMath.com

Lesson: A Quick Review of the Basics of Solving Systems of Linear Equations

Systems of equations are 2 or more equations in 2 or more variables. They can look like:

$\begin{cases} 2x + 3y = 6 \\ x - 3y = 3 \end{cases}$ or $\begin{cases} 2x - y + z = 8 \\ 5x + 2y - z = 9 \\ x - 4y + z = 10 \end{cases}$ or $\begin{cases} 3x + 2y - z = 7 \\ x - 4y + 2z = 8 \end{cases}$

A solution of a system is an ordered pair that is true in each equation of the system. Some systems have no solutions, and some systems have infinite solutions.

In this lesson, we will briefly review the main techniques to solve systems. We will focus on systems that have a solution.

<u>Example</u> Use the substitution method to solve: $\begin{cases} 3x - y = -4 \\ x + 3y = 12 \end{cases}$

<u>Example</u> Use the elimination method to solve: $\begin{cases} 3x - y = -4 \\ x + 3y = 12 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Solve: $\begin{cases} \frac{1}{2}x + \frac{1}{3}y = 4 \\ \frac{3x}{2} + \frac{3y}{2} = 15 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Solve: $\begin{cases} 2x + y + z = 9 \\ -x - y + z = 1 \\ 3x - y + z = 9 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Systems of 3 Linear Equations with 3 Variables

Solve: $\begin{cases} x + y + z = 2 \\ 2x + y - z = 5 \\ x - y + z = -2 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Solve: $\begin{cases} x + 2y + 3z = -11 \\ 3x - y + z = 0 \\ -2x + 3y - z = 4 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Systems with No Solutions or Infinite Solutions

<u>Example</u> Solve: $\begin{cases} x + 4y - 3z = 2 \\ 2x - 5y + 2z = -8 \\ -3x - 12y + 9z = 7 \end{cases}$

<u>Example</u> Solve: $\begin{cases} 7x + 2y = 6 \\ 14x + 4y = 12 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Solve: $\begin{cases} 5x - 4y + z = 9 \\ y + z = 15 \end{cases}$

<u>Example</u> $\begin{cases} 3x - 2y + z = 15 \\ x + 4y - z = 11 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Solve: $\begin{cases} 2x + y - 3z = 0 \\ 4x + 2y - 6z = 0 \\ x - y + z = 0 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Systems of Equations by Graphing

We will now turn our attention to systems of equations.

System of Equations

A system of equations refers to 2 or more equations that share 2 or more variables.

Solution of a System of Equations

A solution of a system of equations is an ordered pair that is true in all equations.

Determine if the following pairs are solutions to the systems.

a. $(1,3)$
$x - 2y = -5$
$2x - 3y = -4$

b. $(-4, 3)$
$x + 3y = 5$
$2x + 3y = 1$

There are several ways to solve systems. The first method we will focus on is how to solve by graphing.

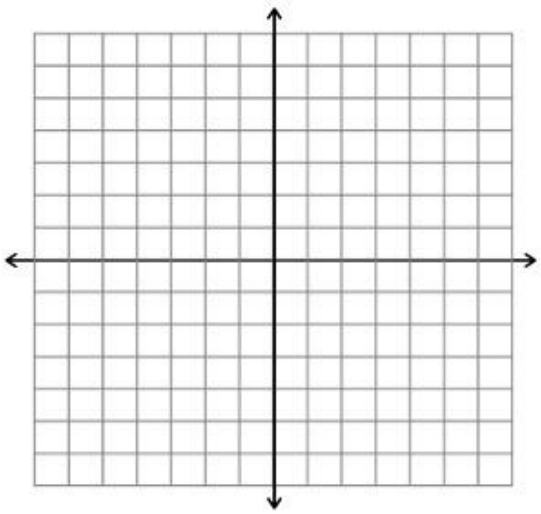
Solving by Graphing

In this method, we graph both lines and determine the point of intersection if possible. We will examine all the possibilities.

Example

Graph each line and determine the point of intersection to find the solution.

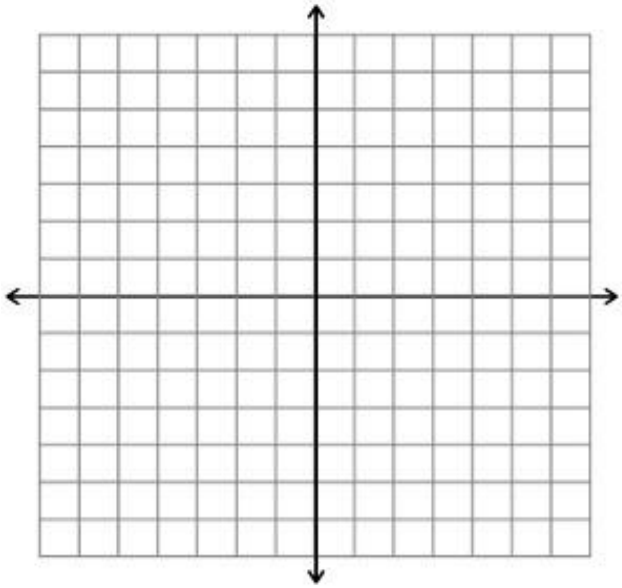
$y = 2x + 3$
$y = -\frac{1}{2}x - 2$



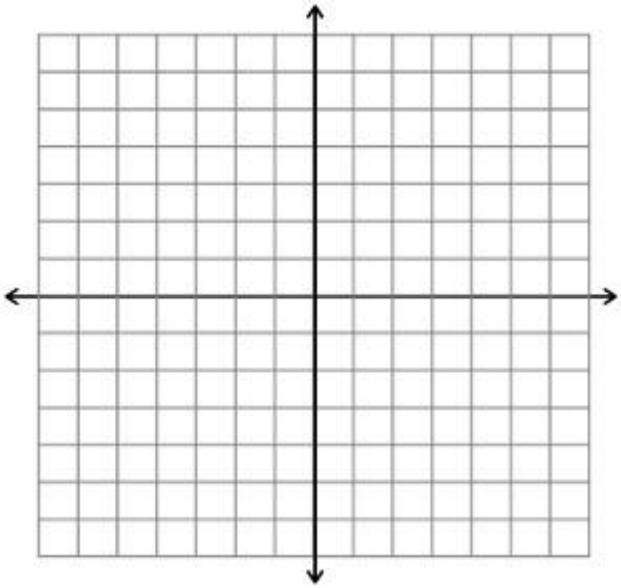
Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example

a. $y = -3x + 1$
$y = 2x - 4$



b. $y - \frac{1}{2}x = -4$
$y + 2x = 1$

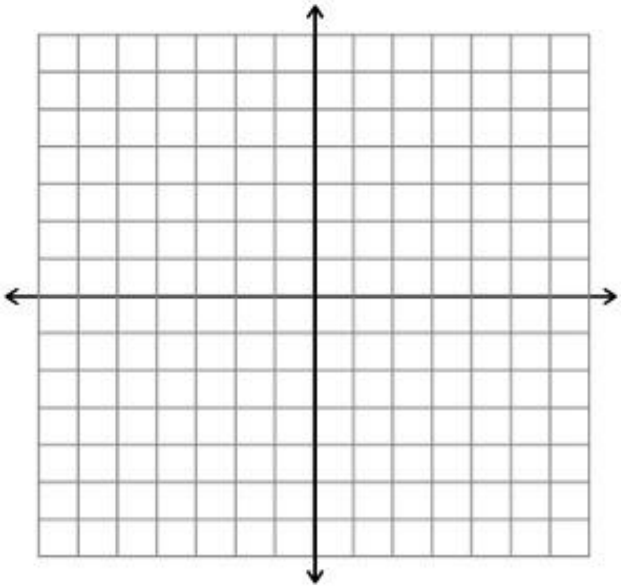


Copyright 2020, Jennifer Hill, DivideandConquerMath.com

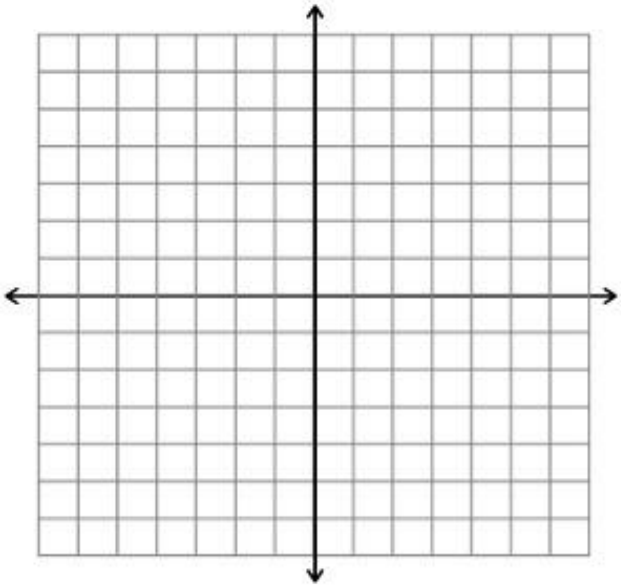
There are two special situations that can occur with graphing.

Example

a. $y = -3x + 2$
$9x + 3y = 6$



b. $y = -\frac{1}{2}x + 4$
$2x + 4y = 12$



Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Systems of Equations by Substitution

In this lesson, we will focus on the second method for solving systems: substitution.

Show how to use substitution with this example:

$2x + y = 6$

$3x + 5y = 9$

<u>Example</u> Solve the system using substitution

$x - 2y = 10$

$5x - 7y = 20$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Below are 2 more complicated problems using substitution. Outline the steps shown in the video:

$-2x + 4y = 4$

$-2x + 6y = 6$

$2x - 5y = -4$

$8x - 9y = 6$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Now let's see what happens when we get a problem with no solution or infinite solutions.

a. $9y - 18x = 5$

$2x - y = 3$

b. $x + 2y = -3$

$4x + 12 = -8y$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Systems of Equations by Elimination

In this lesson, we will focus on the third method for solving systems: elimination.

Show how to use elimination with this example:
$3x - y = 8$

$2x + y = -3$

<u>Example</u> Use elimination to solve the system.

$4x + 2y = -6$
$3x - 2y = 6$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Now let's examine a more complicated case of elimination. Write down how to approach these problems:

a. $3x - 6y = 12$

$5x - 2y = -4$

b. $5x - 10y = 5$
$2x - 3y = 6$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Now let's investigate what happens in the situation where there is no solution or infinite solutions.

a. $5x - 2y = 1$
$-10x + 4y = 3$

b. $3x + y = 5$
$-9x - 3y = -15$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Systems of Equations that Require Clearing of Fractions

<u>Examples</u>

a. $\frac{1}{6}x - \frac{1}{3}y = \frac{1}{2}$
$\frac{2}{5}x - \frac{3}{5}y = -\frac{6}{5}$

b. $-\frac{1}{2}x - \frac{3}{4}y = 1$
$\frac{1}{3}x + \frac{3}{2}y = -\frac{2}{3}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

c. $\frac{1}{2}x + \frac{3}{5}y = 6$

$\frac{1}{4}x - \frac{1}{2}y = -9$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Solving Systems of Equations that Require Clearing of Decimals

<u>Examples</u>

a. $0.02x + 0.1y = 0.05$
$4x + 20y = 10$

b. $0.1x - 0.05y = 0.2$
$0.3x - 0.25y = 0.2$

c. $-0.4x + 1.3y = 1.$
$1.6x - 0.9y = 6.9$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Applications of Systems

The top of a rectangular desk is twice as long as it is wide. The perimeter of the desk is 180 inches. What are the dimensions of the desk?

Sarah bought 3 shirts and a scarf for $53. Marie bought 2 shirts and 3 scarves for $54. Find the cost of each item.

How many milliliters of a 5% acid solution and how many milliliters of a 17% acid solution must be mixed to obtain a 60 mL solution that is 15% acid?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Systems of Equations with 3 Equations

Show how to solve the system:

$x + 2y + 3z = -11$

$3x - y + z = 0$

$-2x + 3y - z = 4$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Sometimes you will have missing variables. Show how to solve the following system that is missing some variables:

$x - 2y = -3$

$x + 6z = 3$

$2y - 4z = 6$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Now let's look at an example where there is no solution versus infinite solutions:

$8x + 20y - 4z = 16$

$-2x - 5y + z = -4$

$10x + 25y - 5z = 20$

$x + 4y - 3z = 2$

$2x - 3y + 2z = -5$

$3x + 12y - 9z = 2$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: More Examples of Systems of 3 Equations

<u>Example</u> Solve the system:

$x + 3y - z = -2$

$-x + 3y - 2z = 5$

$2x + 4y + 2z = 4$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Solve the following system:

$x - 2z = 2$

$-4y + 2z = -2$

$-x + 3y = -3$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Solve the following system:

$x - 4y + 2z = -7$

$3x - 8y + z = 7$

$6x - 12y + 3z = 12$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Using Matrices to Solve Systems of Linear Equations

What is a matrix?

Show how to setup the associated **augmented matrix** for the system: $\begin{cases} 3x + 4y = 4 \\ x - y = 13 \end{cases}$

What are the dimensions of this matrix?

In general, how do we state dimensions of a matrix (circle): rows x columns columns x rows

<u>Example</u> What is the dimension of

a.

$$\begin{bmatrix} 3 & 2 & 4 & 1 \\ 7 & 8 & 1 & 2 \\ 9 & 5 & -2 & -3 \end{bmatrix}$$

b.

$$\begin{bmatrix} 2 & 4 \\ 5 & 3 \\ 8 & 3 \\ -1 & -4 \end{bmatrix}$$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

With matrices, there are 3 **row transformations** that we can make. We are not allowed to do anything else to a matrix. Write down these 3 transformations.

Now, recall, our goal in this lesson is to use a matrix to solve systems of linear equations. What do we call this specific solving process?

What do we call the “ideal form” of a matrix in its solved form?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

The Gauss-Jordan method has a goal of transforming a matrix into reduced-row echelon form or diagonal form. Draw a few examples of how the FINAL matrix should look after we transform it:

When we use the Gauss-Jordan method, which direction do we want to work on the matrix?

What is the general strategy to remember with this method?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Outline the steps for the Gauss-Jordan method for the system: $\begin{cases} -2x + 4y = -4 \\ x - y = 4 \end{cases}$

Example Use Gauss-Jordan method for the system: $\begin{cases} 3x + 4y = 4 \\ x - y = 13 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Using The Gauss-Jordan Method for a System with 3 Equations

<u>Example</u> Use the Gauss-Jordan method for the system: $\begin{cases} x + y + z = 2 \\ 2x + y - z = 5 \\ x - y + z = -2 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Another page for you to show your work …

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Using Matrices to Solve Systems of Linear Equations when There is No Solution or Infinite Solutions

<u>Example</u> Use the Gauss-Jordan method to solve: $\begin{cases} 3x + 5y - z = -2 \\ 4x - y + 2z = 1 \\ -6x - 10y + 2z = 0 \end{cases}$

<u>Example</u> Use the Gauss-Jordan method to solve: $\begin{cases} 3x + 5y = -2 \\ 9x + 15y = -6 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Use the Gauss-Jordan method to solve: $\begin{cases} x - 2y + 3z = 6 \\ 2x - y + 2z = 5 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Use the Gauss-Jordan method to solve: $\begin{cases} 5x - 4y + z = 0 \\ x + y = 0 \\ -10x + 8y - 2z = 0 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Using Matrices to Solve Systems of Linear Equations (any type of solutions)

Use the Gauss-Jordan method to solve the following.

a. $\begin{cases} x + 4y - z = 6 \\ 2x - y + z = 3 \\ 3x + 2y + 3z = 9 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

b. $\begin{cases} 2x + y - 3z = 0 \\ 4x + 2y - 6z = 0 \\ x - y + z = 0 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

c. $\begin{cases} 3x - 2y + z = 15 \\ x + 4y - z = 11 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Nonlinear Systems of Equations

Nonlinear systems of equations involve at least one equation that is not linear. Some examples look like this:

$\begin{cases} x^2 - y = 8 \\ x + y = -4 \end{cases}$ or $\begin{cases} x^2 + y^2 = 4 \\ |x| + y = 2 \end{cases}$

Similar to systems of linear equations, we can still use substitution or elimination to solve the system. However, the best method to use tends to depend on the type of system you are trying to solve. Additionally, you might have to use both methods.

What is one potential issue that can happen with solving systems of nonlinear equations and what should you do about this?

When one equation is linear and the other is not, it is usually best to use the substitution method.

<u>Example</u> Solve the system: $\begin{cases} x^2 + y = 8 \\ x - y = -2 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

When both variables in both equations are squared, it's usually best to use elimination.

<u>Example</u> Solve the system: $\begin{cases} x^2 + y^2 = 25 \\ x^2 + 2y^2 = 9 \end{cases}$

Sometimes you will have to use both the substitution and elimination methods.

<u>Example</u> Solve the system: $\begin{cases} x^2 - xy + y^2 = 5 \\ 2x^2 + xy - y^2 = 10 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Now let's look at a few other examples to get a sense of how to use these methods!

<u>Example</u> Solve the system: $\begin{cases} x^2 + y^2 = 9 \\ -|x| + y = 0 \end{cases}$

<u>Example</u> Solve the system: $\begin{cases} 3x^2 + y^2 = 3 \\ 4x^2 + 5y^2 = 26 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Properties of Matrices: Equality, Addition, Subtraction, and Scalar Multiplication

When are 2 matrices equal?

<u>Example</u> What values of x, y, z, and w will make the matrices equal?

$$\begin{bmatrix} 2+x & 3-w \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 2-y & 2+z \end{bmatrix}$$

When can you add 2 matrices?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What is the definition of matrix subtraction?

What is scalar multiplication with a matrix?

List the properties of scalar multiplication for matrices:

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}, B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 7 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 7 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Complete the following operations, if possible.

a. $A + C$

b. $B - D$

c. $B - E$

d. $2B + E$

e. $2A - 3C$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Matrix Multiplication

First, let's review dimension. What is the dimension of the following matrices?

$A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}$

$B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}$

$D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}$

To understand how to multiply matrices, let's use the following 2 examples:

a. $\begin{bmatrix} 2 & 7 \\ -4 & 3 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 3 & 2 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ 7 & 2 \end{bmatrix}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}, B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 7 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 7 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Complete the following operations, if possible.

a. AC

b. CA

Using the above example, what can we say about matrix multiplication and commutativity?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

List the properties of matrix multiplication:

<u>Example</u> Let $A = \begin{bmatrix} 3 & 2 \\ 1 & 8 \end{bmatrix}, B = \begin{bmatrix} 7 & 2 & 1 \\ 0 & 4 & 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 7 \\ 1 & 3 \end{bmatrix}, D = \begin{bmatrix} 2 & 3 \\ 1 & 2 \\ 1 & 0 \end{bmatrix}, E = \begin{bmatrix} 7 & 2 & 1 \\ 1 & 0 & 0 \end{bmatrix}$. Complete the following operations, if possible.

a. BC

b. ED

c. CB

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Inverse Matrices

What is the 2x2 identity matrix?

What is the 3x3 identity matrix?

How do we describe the general nxn identity matrix?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Multiply $A = \begin{bmatrix} 3 & 2 & 1 \\ 8 & 1 & 7 \\ 3 & 1 & 0 \end{bmatrix}$ by the identity matrix as follows:

a. AI

b. IA

What can we generalize about the identity matrix?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

How do we describe a multiplicative inverse of a real number?

Some matrices have inverses. For a matrix A, what notation denotes its inverse?

What is the property of inverse matrices?

What are the steps for finding an inverse matrix?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Find the inverses of the following matrices. Prove your answer by checking it: $\begin{bmatrix} 6 & -2 \\ 2 & 4 \end{bmatrix}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Find the inverses of the following matrices. Prove your answer by checking it: $\begin{bmatrix} 3 & 1 & 2 \\ 1 & -1 & 1 \\ 2 & 1 & -1 \end{bmatrix}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Find the inverses of the following matrices. Prove your answer by checking it: $\begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Using Inverse Matrices to Solve Systems

We've previously talked about using matrices to solve systems. We will look at another way to do that in this lesson. Show another way to set up the system using matrices (as well as the relevant notation):

$$\begin{cases} x + y + z = 6 \\ 2x + 3y - z = 7 \\ 3x - y - z = 6 \end{cases}$$

Now, using our setup above, explain the reasoning for how inverse can help solve a system:

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Use inverse matrices to solve: $\begin{cases} -x + y = 1 \\ 2x - y = 1 \end{cases}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Solve the system: $\begin{cases} x + y + z = 6 \\ 2x + 3y - z = 7 \\ 3x - y - z = 6 \end{cases}$ using inverse matrices.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: How to Calculate the Determinate of a Matrix

Every n x n matrix has a real number associated with it called the **determinant.** What is the notation to represent this?

<u>Example</u> Show, in detail, how to calculate the determinants of the following matrices.

a. $A = \begin{bmatrix} 2 & 4 \\ -5 & 7 \end{bmatrix}$

b. $B = \begin{bmatrix} -5 & 1 \\ -3 & 4 \end{bmatrix}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Show, with illustrations, how to calculate the determinant of $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 1 & -3 \\ 3 & 4 & -2 \end{bmatrix}$

The 2x2 "sub" matrices that we calculate are called minors. The symbol M_{ij} represents the minor that results when row i and column j are eliminated. Using the matrix above, describe the following minors.

$M_{11} =$

$M_{32} =$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

The number that we multiply the minor against is called the **cofactor.** Draw an illustration to show whether a cofactor will be positive or negative:

<u>Example</u> Using the matrix below, describe the minor associated with each cofactor AND determine whether the cofactor is positive or negative. You do not need to calculate the determinant.

$$A = \begin{bmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ -3 & -5 & -1 \end{bmatrix}$$

a. 2

b. 3

c. -3

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Show how to efficiently calculate the determinants of the following matrices. EXPLAIN YOUR REASONING FOR WHY YOU DID IT THE WAY THAT YOU DID.

a. $A = \begin{bmatrix} 2 & 7 & 8 \\ -1 & 0 & 1 \\ 3 & 5 & 1 \end{bmatrix}$

b. $B = \begin{bmatrix} 9 & -3 & 1 \\ 2 & 3 & 0 \\ 5 & 2 & 0 \end{bmatrix}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Determinant Theorems

Theorem 1: If every element (or column) of a matrix is 0, then the determinant will be 0.

<u>Example</u>: find the determinant of $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 4 & 2 & 8 & 9 \\ 3 & 2 & 8 & 4 \\ 9 & 3 & 1 & 7 \end{bmatrix}$

Theorem 2: If the rows of a matrix A are the corresponding columns of a matrix B, then the determinants are equal.

<u>Example:</u> Calculate both determinants: $A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 3 & 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 0 \end{bmatrix}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Theorem 3: If any 2 rows (or columns) of a matrix are interchanged, then the determinants are negatives of one another.

<u>Example</u> Find the determinants of: $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 6 \\ 2 & 4 \end{bmatrix}$

Theorem 4: Suppose matrix B is formed by multiplying every element in a row or columns in a matrix A by a real number k. Then the determinant is $|B| = k|A|$.

<u>Example:</u> Find the determinants of: $A = \begin{bmatrix} 2 & 1 \\ 3 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 8 & 4 \\ 3 & 6 \end{bmatrix}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Theorem 5: If two rows (or columns) of a matrix are identical, then the determinant will be 0.

<u>Example</u> Find the determinant of: $A = \begin{bmatrix} 4 & 4 \\ 4 & 4 \end{bmatrix}$

Theorem 6: Changing a row (or column) or a matrix by adding it to a constant times another row (or column) does not change the determinant of the matrix.

<u>Example</u> Find the determinant of $A = \begin{bmatrix} 1 & 2 \\ 5 & 8 \end{bmatrix}$. Then create a matrix B by taking adding $3 *$ (row 3) to row 1. Find the new determinant.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Theorem 7: If a matrix is in triangular form (all 0s either above or below the diagonal), then the determinant is found by multiplying the elements down the diagonal.

<u>Example</u>: Find the determinant of $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 5 & 0 & 0 \\ 2 & 1 & 2 & 0 \\ 8 & 7 & 3 & 1 \end{bmatrix}$

<u>Examples</u> Use the determinant theorems to find the following determinants.

a. $\begin{vmatrix} -4 & 1 & 4 \\ 2 & 0 & 1 \\ 0 & 2 & 4 \end{vmatrix}$

b. $\begin{vmatrix} 1 & 0 & 8 \\ 4 & 0 & 2 \\ 0 & 0 & 8 \end{vmatrix}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Ellipses as Conic Sections

Give the geometric definition of an ellipse. Draw an illustration to accompany this.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What are the standard forms of an ellipse? Describe the parts and draw VERY CLEAR illustrations.

What is important to note with this equation?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Graph each ellipse. Give the domain, range, center, vertices, endpoints of the minor axis, and focus.

a. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

b. $9x^2 + y^2 = 81$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Graph each ellipse. Give the domain, range, center, vertices, endpoints of the minor axis, and focus.

c. $\frac{(x-1)^2}{9} + \frac{(y+2)^2}{4} = 1$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Writing Equations of Ellipses

Example Write the equation of an ellipse with the following characteristics.

a. major axis 6, focus at (0,2) and (0, -2)

b. center at (-2, 7), major axis vertical with length 10, and c=2

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Eccentricity with Ellipses

To discuss the concept of eccentricity, we need to discuss what the formal mathematical definition of conic sections is.

Define a conic:

The intuitive idea behind eccentricity can change slightly with each conic section, but it is generally a ratio of different measures within the conic section.

What does the eccentricity of an ellipse tell us?

How do we represent eccentricity?

How do you calculate the eccentricity of an ellipse?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

How does the eccentricity of a parabola compare to the eccentricity of an ellipse? Summarize the explanation from the video.

<u>Examples</u> Find the eccentricity of each ellipse.

a. $\frac{x^2}{9} + \frac{y^2}{4} = 1$

b. $x^2 + 25y^2 = 25$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Find the equation of an ellipse with $e = \frac{3}{4}$ with foci at $(-2,0), (2,0)$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Using Complete the Square to Write an Ellipse Equation in Standard Form

Rewrite the following equations in standard form.

a. $x^2 + 12y^2 - 14x - 48y = -85$

b. $9x^2 + 36x + 16y^2 - 160y + 292 = 0$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Hyperbolas as Conic Sections

What is the geometric definition of a hyperbola? Draw an illustration with your definition.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

State the equations for the standard form of the hyperbola. Draw illustrations with it.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Graph each hyperbola. Give the domain, range, center, vertices, foci, and equations of the asymptotes.

a. $\frac{x^2}{9} - \frac{y^2}{16} = 1$

b. $9x^2 - 25y^2 = 225$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Graph each hyperbola. Give the domain, range, center, vertices, foci, and equations of the asymptotes.

c. $\frac{(y-1)^2}{9} - \frac{(x+3)^2}{4} = 1$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Writing Equations of Hyperbolas

<u>Examples</u> Write the equation of the hyperbola with the following characteristics.

a. vertices at (0, 6) and (0, -6) with asymptotes $y = \pm\frac{1}{2}x$

b. foci at $\left(-3\sqrt{5}, 0\right), (3\sqrt{5}, 0)$ with asymptotes $y = \pm 2x$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Write the equation of the hyperbola with the following characteristics.

c. center at (1, -2) and focus at (-2, -2) and vertex at (-1, -2)

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Eccentricity of Hyperbolas

How do you calculate the eccentricity of a hyperbola?

What can we say about the eccentricity of the hyperbola? What does it mean?

<u>Example</u> Find the eccentricity of the hyperbolas.

a. $\frac{x^2}{36} - \frac{y^2}{9} = 1$

b. $8y^2 - 16x^2 = 16$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Find the equation of a hyperbola with $e = \frac{25}{9}$ and foci at (9, -1) and (-11, -1)

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Writing the Standard Form of a Hyperbola

Write each hyperbola in standard form.

a. $x^2 - 6x - y^2 + 4y = 1$

b. $y^2 - 14y - x^2 + 14x = 16$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: An Intro to Conic Sections

What are the 4 conic sections we will discuss? Draw an illustration with it.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Parabolas as Conic Sections

What is the previous equation we used for parabolas opening upwards or downwards? Draw a illustration to go with this.

How can we represent parabolas in similar notation for parabolas that open sideways?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Give the domain and range of the following parabolas, then graph the parabola.

a. $x + 1 = (y + 1)^2$

b. $-x = 3y^2 + 6y + 2$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

When dealing with conic sections, we shift how we think about things.

What is the geometric definition of a parabola? Draw an illustration to accompany this definition.

What equations do we use that align with the geometric definition?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Give the focus, directrix, vertex, and axis of symmetry for each parabola. Then use this information to graph the parabola.

a. $x^2 = 4y$

b. $(y-2)^2 = 24(x-3)$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Writing Equations of Parabolas

<u>Example</u> Write the equation of a parabola with the following characteristics.

a. focus $\left(0, -\frac{1}{2}\right)$ with the vertex at the origin

b. vertex (4, 3) and focus is (4, 5)

c. vertex (1, 2), directrix x=4

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Sequences

Define a sequence.

<u>Examples</u> Write the first five terms of each sequence.

a. $a_n = \frac{1}{n}$

b. $a_n = \frac{n\mp 1}{n+3}$

c. $a_n = (-1)^{n+1} * (2n + 1)$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What does it mean for a sequence to converge?

What does it mean for a sequence to diverge?

What is a recursively defined sequence?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Find the first four terms in the sequence.

a. $a_1 = 3; a_n = 2a_{n-1} - 4$

b. $a_1 = 2; a_n = a_{n-1}^2 + n - 2$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Series

What is the summation sign and what does it mean? What do we called the “i” in this formula?

Define a finite series.

Define an infinite series.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Examples</u> Evaluate the following sums.

a. $\Sigma_{i=1}^{4}\left(3^i + 1\right)$

b. $\Sigma_{j=3}^{6} a_j$

c. $\Sigma_{k=1}^{4}(4x_k - 1)$ if $x_1 = 2, x_2 = 3, x_3 = 5, x_4 = 8$

d. $\Sigma_{i=1}^{3} f(x_i)\Delta x$, if $f(x) = 3x, x_1 = 1, x_2 = 4, x_3 = 7, \Delta x = 3$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

What are the summation properties?

What are the summation rules for summing up terms?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Use the summation properties and rules to evaluate each series.

a. $\Sigma_{i=1}^{50} 3$

b. $\Sigma_{k=1}^{25} 3k$

c. $\Sigma_{j=1}^{33} (4j^2 + 1)$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Arithmetic Sequences

What is an arithmetic sequence?

Give some examples of an arithmetic sequence.

<u>Example</u> Determine the common difference, d, for the arithmetic sequence:

$$-7, -5, -3, -1, \ldots$$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Find the first five terms of each arithmetic sequence.

a. The first term is 3, and the common difference is -2.

b. $a_1 = -5, d = 7$

How do we define the general nth term of an arithmetic sequence?

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example Determine a_{11} and a_n for the arithmetic sequence $-2, 2, 6, 10, \ldots$

Example Determine a_n and a_{20} for the arithmetic sequence having $a_3 = 9$, and $a_4 = 16$.

Example An arithmetic sequence has $a_7 = -15$ and $a_{14} = -35$. Determine a_1.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Arithmetic Series

When we sum up the terms of an arithmetic sequence, how can we define that sum? (sum of an arithmetic sequence)

<u>Example</u> Consider the arithmetic sequence $-7, -3, 1, 5, \ldots$

a. Evaluate S_{10}

b. Evaluate the sum of the first 50 positive integers.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Example The sum of the first 15 terms of an arithmetic sequence is 345. If $a_{15} = 65$, find a_1 and d.

Examples Evaluate each sum.

a. $\Sigma_{i=1}^{19}(-3 - 5i)$

b. $\Sigma_{k=4}^{15}(2 + 4k)$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: Geometric Sequences and Series

Define a geometric sequence:

<u>Example</u> Determine a_6 and a_n for the geometric sequence 4, 12, 36, 108, ….

<u>Example</u> Determine r and a_1 for the geometric sequence with $a_2 = -18$ and $a_5 = 486$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

How do we define the sum of the first n terms of a geometric sequence?

<u>Example</u> Evaluate: $\Sigma_{i=1}^{8} 3 * 2^i$

If we start with 2 and use a common ratio of $\frac{1}{2}$, what eventually happens as the series continues infinitely? Summarize the explanation below.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples Evaluate the following infinite sums.

a. The first term is 1 and the common ratio is $\frac{1}{3}$

b. $\Sigma_{i=1}^{\infty}\left(-\frac{1}{2}\right)\left(-\frac{3}{4}\right)^{i-1}$

c. $\Sigma_{i=1}^{\infty}\left(\frac{3}{7}\right)^{i}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Lesson: The Binomial Theorem

Below we can observe the pattern for binomial expansion:

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = x^2 + 2xy + y^2$

$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

Focusing on the VARIABLES x and y only, how can we generalize the exponents for the variables?

Pascal's Triangle can explain how to find the coefficient. Draw out Pascal's triangle below.

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Having to draw out this pattern would be very difficult for larger expansions, so we will explore a few ways to simplify how to expand this pattern.

First, we need to understand factorials.

n-factorial: for any positive integer n,

$$n! = n(n-1)(n-2)\dots(3)(2)(1)$$

By definition: $0! = 1$

<u>Examples</u> Evaluate:

a. $5!$

b. $8!$

Now that we understand factorials, we can discuss the formula to find binomial coefficients.

For nonnegative integers n and r, $r \leq n$, the binomial coefficient is defined as follows:

<u>Examples</u> Evaluate each binomial coefficient:

a. $\binom{5}{2}$

b. $\binom{6}{0}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

We can summarize how to expand $(x+y)^n$ as follows:

1. There are n+1 terms in the expansion

2. The first and last terms are x^n and y^n respectively

3. In each succeeding term, the exponent on x decreases by 1 and the exponent on y increase by 1

4. The sum of the exponents on x and y in any term is n.

5. The coefficient of the term with $x^r y^{n-r}$ or $x^{n-r} y^r$ is $\binom{n}{r}$

These observations are captured in the binomial theorem.

State **The Binomial Theorem:**

<u>Example</u> Find the first, last, and seventh term of the binomial expansion: $(3x+2y)^{10}$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

<u>Example</u> Use the binomial theorem to expand: $(a-b)^5$

Copyright 2022, Jennifer Hill, DivideandConquerMath.com

Examples: Finding Average Rates of Change

Calculate the average rate of change of the functions over the given intervals.

a. $f(x) = x^2 + 3x; [0, 3]$

b. $f(x) = 2 + \sin x; \left[\frac{\pi}{2},\ \pi\right]$

c. $f(x) = \sqrt{4x + 1}; [0, 2]$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Determining the Slope at a Point and the Tangent Line

Find the slope of the curve at the given point and find the equation of the tangent line at that point. Do this only using the formula for the slope of the secant line.

a. $f(x) = x^2 - 1, (2, 3)$

b. $f(x) = x^2 - 4x; (2, -4)$

c. $y = \frac{1}{x}; (2, \frac{1}{2})$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: An Intro to Limits

Graph $f(x) = \frac{(x^2-4)}{x-2}$ below. What does it look like this graph should equal when x=2?

Explain this previous question now in mathematical terms

<u>Example</u> Find $\lim_{x \to 1} f(x)$ when $f(x) = \begin{cases} x^2 + 1 & x \neq 1 \\ -2 & x = 1 \end{cases}$. Draw a picture of this situation.

What is the value of $f(1)$?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example What is $\lim_{x \to 2} f(x)$ when $f(x) = x$? Draw a graph to accompany your answer.

What do we call the function $f(x) = x$?

What can we say about the limit of these types of functions?

Example What is $\lim_{x \to -3} f(x)$ when $f(x) = 4$? Draw a graph to accompany your answer.

What do we call the function $f(x) = c$, where c is a constant?

What can we say about the limit of these types of functions?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Write out all the limit laws on this space:

Write out the other 2 handy theorems:

Theorem 1: Limits of Polynomials

Theorem 2: Limits of Rational Functions

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example Using the limit laws, calculate the following limits:

a. $\lim_{x \to 3}(2x + 1)$

b. $\lim_{x \to 1} \frac{5x+2}{x-4}$

c. $\lim_{x \to 2} \sqrt{5x + 4}$

d. $\lim_{x \to -3} 5\sqrt{2 - 3x}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Intro to Calculating Limits

Calculate the following limits:

a. $\lim_{x \to 3}(x^2 + 2x + 1)$

b. $\lim_{x \to 0}(8 - 2x)^{\frac{2}{3}}$

c. $\lim_{x \to \frac{2}{5}}(5x + 2)(3x - 1)$

d. $\lim_{x \to \frac{1}{3}} \sqrt[3]{3x + 7}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Troubleshooting Limits With Algebraic Tricks (When You Get 0/0)

Trick #1 (name this trick here):

Note what happens if we try to evaluate $\lim_{x \to 2} \frac{x^2-4}{x-2}$

Draw what this situation looks like graphically.

How can we solve this problem?

<u>Example</u> Evaluate $\lim_{x \to 3} \frac{(x^3-27)}{x-3}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Trick #2 (name it):

<u>Examples</u> Evaluate:

a. $\lim_{x \to 0} \frac{\sqrt{x+9}-3}{x}$

b. $\lim_{x \to 1} \frac{2-\sqrt{x^2+3}}{x-1}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Trick #3 (name it):

<u>Examples</u> Evaluate

a. $\lim_{x\to 3} \frac{\left(\frac{1}{x}-\frac{1}{3}\right)}{x-3}$

b. $\lim_{x\to 1} \frac{\left(1-x^{-1}\right)}{1-x}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Algebraic Tricks for Limits

Evaluate the following:

a. $\lim_{x \to -4} \frac{\left(\frac{1}{\sqrt{12+x}} - \frac{1}{3}\right)}{x+4}$

b. $\lim_{x \to 0} \frac{\left(\frac{1}{3+x} - \frac{1}{3}\right)}{x}$

c. $\lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Limits that Do Not Exist

List the types of limits that do not exist, as well as an example and graph

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: The Sandwich Theorem and Basic Trig Limits

Explain the idea of the Sandwich Theorem:

State the Sandwich Theorem

<u>Example</u> Suppose there is a function $f(x)$ where $3 - x^2 \leq f(x) \leq 3\cos x$. Find $\lim_{x \to 0} f(x)$.

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Show how to use the Sandwich Theorem to evaluate the following limits:

a. $\lim_{x \to 0} \sin x$

b. $\lim_{x \to 0} \cos x$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Basic Trig Limits

Evaluate the following:

a. $\lim_{x \to 0} \frac{3+x+\sin x}{4\cos x}$

b. $\lim_{x \to \frac{\pi}{3}} \sec x$

c. $\lim_{x \to \pi} \sin^2 x$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: One-Sided Limits

Use the following function to describe one-sided limits: $f(x) = \begin{cases} 3, & x > 2 \\ 1, & -1 \leq x \leq 2 \\ -2, & x < -1 \end{cases}$

Evaluate the following:

a. $\lim_{x \to 2^+} f(x)$

b. $\lim_{x \to 2^-} f(x)$

c. $\lim_{x \to 2} f(x)$

d. $\lim_{x \to -1^+} f(x)$

e. $\lim_{x \to -1^-} f(x)$

f. $\lim_{x \to -1} f(x)$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Draw the graph presented in the video then evaluate the following:

a. $\lim_{x \to 3^+} f(x)$

b. $\lim_{x \to 3^-} f(x)$

c. $f(3)$

d. $\lim_{x \to -2^+} f(x)$

e. $\lim_{x \to 4^-} f(x)$

f. $\lim_{x \to 4^+} f(x)$

g. $\lim_{x \to 4} f(x)$

h. $f(4)$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Write down the theorem that explains when $\lim_{x \to c} f(x)$ exists:

<u>Examples</u> Evaluate the following:

a. $\lim_{x \to 2^+} \sqrt{\frac{x-2}{x+1}}$

b. $\lim_{x \to 2^-} \left(\frac{1}{x+1}\right)\left(\frac{x-3}{x}\right)\left(\frac{4}{4-x}\right)$

c. $\lim_{x \to -3^-} \frac{(x+1)|x+3|}{x+3}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Basic One-Sided Limits

a. $\lim_{x \to 2^+} \sqrt{x^2 - 4}$

b. $\lim_{x \to 0^+} \frac{\sqrt{2x^2+3x+1}-1}{x}$

c. $\lim_{x \to 3^-} \begin{cases} 4, & x > 3 \\ -2, & x \leq 3 \end{cases}$

d. $\lim_{x \to 5^-} \frac{x-5}{|x-5|}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: One-Sided Limits with Absolute Value

a. $\lim_{x \to 2^+} \frac{2-x}{|x-2|}$

b. $\lim_{x \to 5^-} \frac{(x-4)|x-5|}{x-5}$

c. $\lim_{x \to 2^+} \frac{\sqrt{3x}(x-2)}{|x-2|}$

d. $\lim_{x \to 0^-} \frac{\sin x}{|\sin x|}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: The $\boldsymbol{lim_{x\to 0}\frac{sin\ x}{x} = 1}$

This lesson is about the tricky but very important limit:

$$\lim_{x\to 0}\frac{\sin x}{x} = 1$$

Or equivalently:

$$\lim_{x\to 0}\frac{x}{\sin x} = 1$$

The proof is interesting and worth noting, you can write it down if you wish but you should minimally attempt to understand the reasoning behind it. It's difficult, but difficult things will expand your mind!

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: The $\boldsymbol{lim_{x\to 0}\frac{sin\ x}{x} = 1}$

a. $\lim\limits_{x\to 0}\frac{\sin 3x}{5x}$

b. $\lim\limits_{x\to 0} 12x^2(\cot 2x)\ (\csc 3x)$

c. $\lim\limits_{x\to 0}\frac{x^3-x+\sin 5x}{3x}$

d. $\lim\limits_{x\to 0}\frac{\sin x}{\sin 3x}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Step-by-Step Examples: Limits using $\boldsymbol{lim_{x\to 0}\frac{sinx}{x}=1}$

a. $\lim\limits_{x\to 0}\frac{\sin 7x}{x}$

b. $\lim\limits_{x\to 0} x\cot 3x$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

More Examples: The $\boldsymbol{lim_{x\to 0}\frac{sin\,x}{x} = 1}$

a. $\lim\limits_{x\to 0}\frac{1}{x^2}\sin^2\left(\frac{x}{2}\right)$

b. $\lim\limits_{x\to 0}\frac{\sin x}{\sqrt{x}}$

c. $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x}$

d. $\lim\limits_{x\to 0}\frac{\sin 3x}{\mathrm{x}\cos 5x}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Troubleshooting with $\lim_{x \to 0} \frac{\sin x}{x}$ problems – examples of what not to do

a. $\lim_{x \to 0} \frac{\tan 5x}{x}$

b. $\lim_{x \to 0} \frac{\sin 3x}{\sin 7x}$

Lesson: Continuity

The idea of continuity is probably intuitive at the point. Draw the example in the video to demonstrate continuity.

For a point c, define when a function is:

1. Continuous

2. Right – continuous

3. Left – continuous

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Draw the graph in the video to demonstrate these ideas.

<u>Example</u> Draw the graph in the video, then describe the continuity at the following points:

$x = 1$ $x = 3$ $x = -4$ $x = -2$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

A function is continuous at a point c is it meets these 3 conditions:

1. f(c) exists

2. the limit $\lim_{x \to c} f(x)$ exists

3. $\lim_{x \to c} f(x) = f(c)$

Describe the different kinds of discontinuities below:

What does it mean for a function to be continuous?

If a function is not continuous, then it is _____

If $f(x)$ and $g(x)$ are continuous functions at the point x=c, then the following are continuous at c:

1. sum/differences:

2. constant multiples (for any number k):

3. products:

4. quotients:

5. powers:

6. roots:

7. compositions:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example Describe where the following functions are continuous:

a. $f(x) = \frac{x+3}{x-2}$

b. $f(x) = \sqrt{2x}$

What can we say about the limits of a composition of continuous functions $f(x)$ and $g(x)$?

Example Find the limit $\lim_{x \to 0} (x+2)e^{\sin x}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Continuity

Determine where the following functions are continuous:

a. $f(x) = \frac{1}{(x+3)^2} - 2$

b. $g(x) = \begin{cases} 1 + x, & x \geq 3 \\ 4, & -2 \leq 0 < 3 \\ 2 - x, & x < -2 \end{cases}$

c. $h(x) = \begin{cases} |x + 1|, & x \geq -2 \\ -5, & x < -2 \end{cases}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Using Continuity with Limits

Find the limits.

a. $\lim_{x \to 0} \cos\left(x + \cos\left(\frac{\pi}{2} + x\right)\right)$

b. $\lim_{x \to \frac{\pi}{2}} \sin(\cos(\tan x))$

c. $\lim_{x \to 0} \sin^{-1}\left(e^{\cos\left(\frac{\pi}{2} + x\right)}\right)$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: The Intermediate Value Property and Continuous Functions

What is the Intermediate Value Property?

There are 2 consequences of this:

1. Connectedness:

2. Root Finding:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example Determine the value of a that would make the function continuous at every x:

$$f(x) = \begin{cases} x^2 + 1, & x > 2 \\ ax, & x \leq 2 \end{cases}$$

Example Show that $x^3 - 14x + 2 = 0$ has 3 solutions in the interval $[-4, 4]$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Continuous Extensions

Determine the value of a that would make the following functions continuous:

a. $f(x) = \begin{cases} 2x, & x > 1 \\ ax^2, & x \leq 1 \end{cases}$

b. $g(x) = \begin{cases} \frac{2x+a}{a+2}, & x > 0 \\ x^2 + a, & x \leq 0 \end{cases}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Step-by-Step Examples: Continuous Extensions

Use the space below to find a (or a and b) to make the following functions continuous:

a. $f(x) = \begin{cases} x + a, & x \geq 1 \\ 3, & x < 1 \end{cases}$

b. $f(x) = \begin{cases} 3, & x \geq 1 \\ ax + b, & -2 < x < 1 \\ -3, & x \leq -2 \end{cases}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: The Formal Definition of the Derivative

One of the things we learn in pre-calculus is about secant lines. You can write a review of this below:

When the distance between two points for a secant line goes to zero, what do we get?

What is the formal definition of the derivative?

What is the alternate formula for the derivative?

Which formula should you use?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

How do you find the derivative at a specific point?

What 4 things does the derivative at a point represent?

1.

2.

3.

4.

<u>Example</u> Calculate the derivative of $f(x) = x^2 + 2$

<u>Example</u> What is the value of the derivative of $f(x) = x^2 + 2$ at $(1, 3)$?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example Find the equation of the tangent line for $f(x) = x^2 + 2$ at $(1, 3)$

List all the different ways to represent a derivative below:

Write out the theorem that discusses differentiability and continuity

What types of functions will not have a derivative at a point?

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Calculating Derivatives Using the Formal Definition Only and Finding Tangent Lines

Use the formal definition of a derivative to calculate the follow derivatives of each function. Then determine the tangent line at the given point.

a. $f(x) = 4 - x^2; (-1, 3)$

b. $y = \frac{1-x}{2x}, (1, 0)$

c. $g(x) = x^{\frac{3}{2}}, (4, 8)$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Examples: Using the Alternative Formula for Derivatives

Use the alternative formula for derivatives to calculate the derivative of the following function. Compare this calculation to the original formula.

a. $y = x^3 - x^2$

b. $y = \frac{1}{x+3}$

c. $f(x) = 2 + \sqrt{x}$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Step-by-Step Examples: Comparing the Definition of the Derivative to the Alternative Formula

a. $f(x) = x + \frac{3}{x}$

b. $g(x) = x^3 - 5x^2 + 1$

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Example: Finding the Derivative of a Square Root with the Definition of the Derivative

Find the tangent line at (6,5) for $y = \frac{5}{\sqrt{7-x}}$ using only the definition of the derivative.

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Lesson: Other Notes about the Derivative

Define right-hand derivatives

Define left-hand derivatives

Explain how $y = |x|$ is differentiable on each side

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Interpreting Graphs of Derivatives

The derivative of $f(x) = x^2$ is $f'(x) = 2x$. Compare their graphs below:

Copyright 2020, Jennifer Hill, DivideandConquerMath.com

Not included (no notes file):

112 (Lesson Introducing Where Calculus Starts)